Chapter 5: Shaft Design

Topics

- Introduction
- Shaft Materials
- Shaft Layout
- Design based on stress
- Design based on deflection
- Critical speed of shafts
- Keys
- Fits and tolerances

Introduction: What is a shaft?

- A shaft is a rotating member, usually of circular cross section, used to transmit power or motion.
- A shaft provides the axis of rotation, or oscillation, of elements such as gears, pulleys, flywheels, cranks, sprockets, and so on, and controls the geometry of their motion.

Difference with an axle:
➥ An axle is a non-rotating member that carries no torque and is used to support rotating wheels, pulleys, and so on.

➥ The automotive axle is not a true axle; the term is a carryover from the horse-and-buggy era, when the wheels rotated on non-rotating members.

Introduction ... contd.

In shaft design: *First* stress; *then* deflection considerations

Stress:

- Stress analysis depends on the local geometry around a point.
- Geometry of the entire shaft is *not* necessary.
- Possible to locate the critical areas; size these to meet the strength requirements; and then size of the rest of the shaft to meet the requirements of the shaft-supported elements

Deflection:

- Deflection and slope analyses cannot be made until the geometry of the entire shaft has been defined.
- Function of the geometry *everywhere*

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Shaft Materials

Deflection is not affected by strength, but rather by geometry and stiffness, represented by modulus of elasticity.

Modulus of elasticity is essentially constant for all steels. So, effectively, deflection not controlled by material decisions, but only by geometric decisions.

Strength required to resist loading stresses → Choice of materials and their treatments.

Many shafts are made from low carbon, cold-drawn or hot-rolled steel: AISI 1020-1050

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Shaft Materials ... contd.

Fatigue failure is reduced moderately by increase in strength.

Beyond a certain level of increase in strength, adverse effects start on endurance limit and on notch sensitivity $\rightarrow$ bad for fatigue, effectively.

Higher strength allows lower shaft diameters ...

... but lower diameters increase deflections!

Number of other considerations (see Sec. 7.2 in Shigley)

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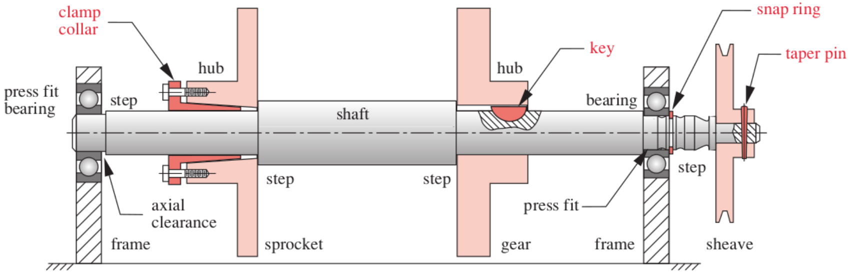
Shaft Layout

The general layout of a shaft must be specified *early in the design process* to accommodate various shaft elements (gears, bearings, pulleys, and so on).

Issues to consider for shaft layout:

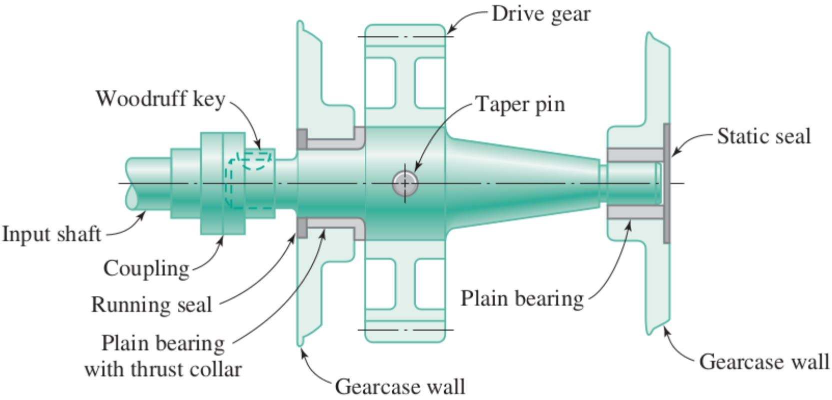
- Axial layout of components
- Supporting axial loads
- Providing for torque transmission
- Assembly and Disassembly

Shaft Layout



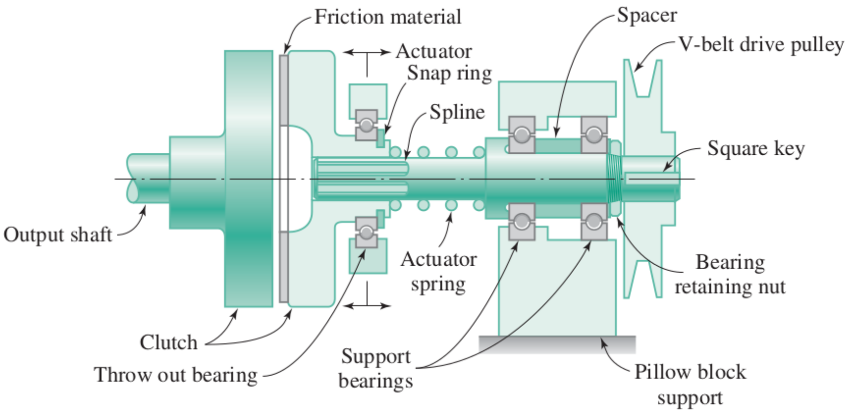
Various methods to attach rotating elements to shafts

Shaft Layout



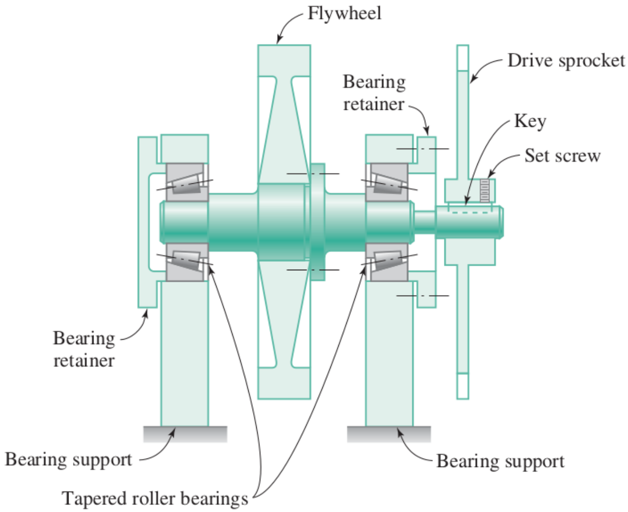
Gear Support Shaft

Shaft Layout



Clutch Drive Shaft

Shaft Layout



Flywheel Drive Shaft

Shaft Layout

Axial layout of components

- Axial positioning of components dictated by layout of the housing and other meshing components.
- Load-carrying components → between bearings (instead of cantilevered outboard of bearings), and near bearings (to reduce moments & minimize deflections)
- Pulleys and sprockets → outboard (ease of installation of the belt or chain)
- Preferably two bearings only
- Shafts to be kept short (as much as possible)
- Axial space b/w components desirable
  - allow lubricant flow
  - provide access space for disassembly

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Shaft Layout

Axial layout of components ... contd.

- Shaft components must be accurately located → to align with other mating components
- Primary means of locating components: Position them against **shoulders**, also called **steps**
- Shoulders (or, steps) provide solid support to minimize deflection and vibration of the component

Shaft Layout

Supporting Axial Loads

- Axial loads must be supported through a bearing to the frame
  - ➥ Particularly necessary with helical or bevel gears
  - ➥ Tapered roller bearings
- Best to have only one bearing carry the axial load

Shaft Layout

Providing for torque transmission

Most shafts transmit torque from an input gear or pulley, through the shaft, to an output gear or pulley.

Common means of transmitting torque between shaft and gears:

- Keys
- Splines
- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

Many of these torque-transfer elements are **designed to fail** if torque exceeds acceptable operating limits, protecting more expensive components. (Similar to an electrical fuse)

Shaft Layout

Assembly and Disassembly

Assembly:

- Generally large diameter in the centre of the shaft, with progressively smaller diameters towards the ends to allow components to be slid on from the ends.
- One cannot have shoulders on both sides of a shaft component. Instead retaining rings and sleeves are used.
- When components are to be press-fit to the shaft, the shaft should be designed so that it is not necessary to press the component down a long length of shaft.

Disassembly

- Consideration of issues such as accessibility of retaining rings, space for pullers to access bearings, openings in the housing to allow pressing the shaft or bearings out, and so on.

Design Based on Stress

Shaft design based on stress considerations basically follows the previous two chapters: **Static failure**, and **Fatigue failure**

Stresses in the shaft need to be evaluated only at a few critical locations.

Sometimes, neither the material is known nor the shaft dimensions!
➥ Guess-estimate → Analyse → Revise → Iterate

Design Based on Stress

For torque:

- Typically, torque comes into the shaft at one gear and leaves the shaft at another gear
- FBD of the shaft → determine torque at any section
- Torque relatively constant at steady state operation
- Shear stress due to torque greatest on outer surfaces

Design Based on Stress

For bending moment:

- Gears and pulleys → forces on two planes → SFD and BMD in two planes
- Determine resultant moments at points of interest along the shaft
- Normal stresses due to bending moments greatest on the outer surface
- For end-of-shaft bearings, bending moments are small → stresses not critical

Design Based on Stress

For axial loads:

- Axial stresses on shafts transmitted through helical gears or tapered roller bearings are usually negligible compared to bending moment stresses
- Such axial stresses are often also constant → hardly contribute to fatigue
- Usually acceptable to neglect axial stresses induced by gears and bearings when bending is also present
- **Caution:** Other forms of axial load: Don't neglect them without checking their magnitudes!

Design Based on Stress

Analysis Procedure

- Basically the procedure is to follow the whole of the Fatigue Failure chapter together with Static Failure (usually against yielding).
- Common to neglect axial stresses. But see the **Caution** in the previous slide.
- Shoulders and other discontinuities always present → Stress concentration factors are really important!
- For actual shaft design, proper knowledge of shaft components (gears, bearings) is also necessary

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Design Based on Stress

Shaft Stresses

For Fatigue Failure:

$$\sigma_a = K_f \frac{32M_a}{\pi d^3}, \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3},$$
$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3}, \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3},$$

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2}$$
$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2}$$

Then, compare using Goodman, Gerber, ASME-elliptic, and so on.

For Static Failure (yielding):

$$\sigma'_{\max} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2}$$

For a quick, conservative check: use $\sigma'_a + \sigma'_m$.

It is conservative because it is always larger than the $\sigma_{\max}$ defined above.

Design Based on Deflection

To know the deflection/slope at any point of interest, we must first know the complete geometry information for the entire shaft.

Procedure:

- Design the dimensions at critical locations to handle the stresses
- Fill in reasonable estimates for all other dimensions (yes, this means guess work, but *educated* guesses)
- Then perform a deflection analysis
- Check against allowable deflections/slopes: from bearing and gear catalogs
- Revise and iterate

Design Based on Deflection

Allowable Deflections and Slopes

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	$<$0.0005 rad

Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

Design Based on Deflection

Revising diameters

If any deflection magnitude is larger than the allowable deflection at that point, a new diameter can be found by noting that I is proportional to d^4:

$$d_{\mathrm{new}} = d_{\mathrm{old}} \left| \frac{n_d y_{\mathrm{old}}}{y_{\mathrm{all}}} \right|^{1/4}$$

y_{all}: allowable deflection
n_d: design factor

Similary, for slope: $d_{\mathrm{new}} = d_{\mathrm{old}} \left| \frac{n_d (\mathrm{d}y/\mathrm{d}x)_{\mathrm{old}}}{(\mathrm{slope})_{\mathrm{all}}} \right|^{1/4}$

➥ Determine the largest $d_{\mathrm{new}}/d_{\mathrm{old}}$ ratio, then multiply all diameters by this ratio.

Design Based on Deflection
Angular deflection due to torsion

For a stepped shaft shaft with individual cylinder length l_i and torque T_i, the angular deflection can be estimated from:

$$\theta = \sum \theta_i = \sum \frac{T_i l_i}{G_i J_i}$$

Note: This is only an estimate. Experimental data shows that the actual θ is larger than this.

Let the torsional stiffness be denoted by $k_i = \dfrac{T_i}{\theta_i} = \dfrac{G_i J_i}{l_i}$
So, for a case of constant torque $T_i = T$, we have:

$$\frac{T}{k} = \theta = \sum \theta_i = \sum \frac{T_i l_i}{G_i J_i} = T \sum \frac{1}{k_i},$$

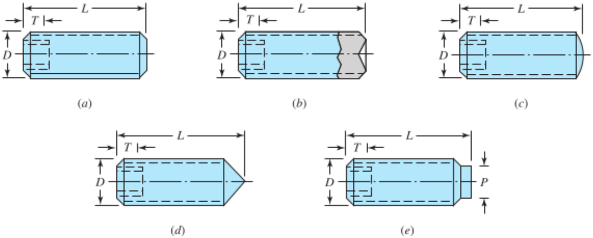
$$\text{or,} \quad \frac{1}{k} = \sum \frac{1}{k_i}$$

Miscellaneous Shaft Components

Setscrews

Setscrews are also known as grub screws.
Setscrews are different from countersunk screws.

Socket setscrews: (*a*) flat point; (*b*) cup point; (*c*) oval point; (*d*) cone point; (*e*) half-dog point.

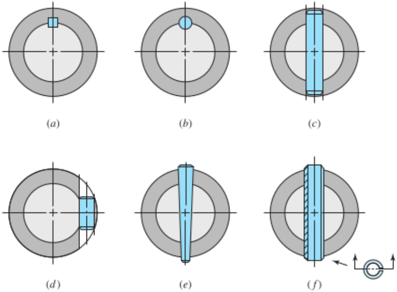


Setscrews should have a length of about half of the shaft diameter. This practice also provides a rough rule for the radial thickness of a hub or collar.

Keys and Pins

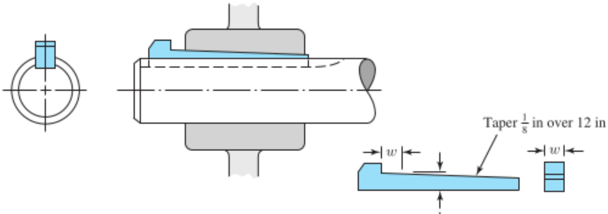
Keys are used to enable the transmission of torque from the shaft to the shaft-supported element. Pins are used for axial positioning and for the transfer of torque or thrust or both.

(*a*) Square key; (*b*) round key; (*c* and *d*) round pins; (*e*) taper pin; (*f*) split tubular spring pin. The pins in parts (*e*) and (*f*) are shown longer than necessary, to illustrate the chamfer on the ends, but their lengths should be kept smaller than the hub diameters to prevent injuries due to projections on rotating parts.

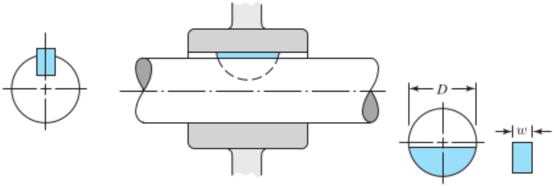


The maximum length of a key is limited by the hub length of the attached element, and should generally not exceed about 1.5 times the shaft diameter.

Keys and Pins ... contd.



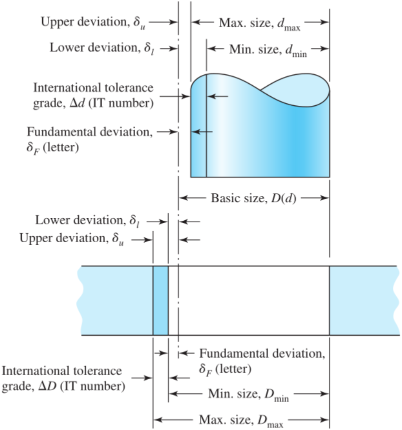
(a) Gib-head key



(b) Woodruff key

Limits and Fits

- **Basic size:** Size to which limits or deviations are assigned; same for both members of the fit
- **Deviation:** algebraic difference b/w a size and corresponding basic size
- **Upper deviation:** Algebraic difference b/w max. limit and corresponding basic size
- **Lower deviation:** Algebraic difference b/w min. limit and corresponding basic size
- **Fundamental deviation:** either upper or lower deviation, depending on which is closer to the basic size
- **Tolerance:** difference between max. and min. size limits of a part
- **International Tolerance (IT) grade numbers:** designate groups of tolerances such that the tolerances for a particular IT number have the same relative level of accuracy but vary depending on the basic size



Clearance Fit

Type of Fit	Description	Symbol
Clearance	*Loose running fit:* for wide commercial tolerances or allowances on external members	H11/c11
	Free running fit: not for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures	H9/d9
	Close running fit: for running on accurate machines and for accurate location at moderate speeds and journal pressures	H8/f7
	Sliding fit: where parts are not intended to run freely, but must move and turn freely and locate accurately	H7/g6
	Locational clearance fit: provides snug fit for location of stationary parts, but can be freely assembled and disassembled	H7/h6

The maximum size of the shaft is smaller than the minimum size of the hole.

There is *never* overlap between the shaft and the hole for all size ranges.

Interference Fit

Type of Fit	Description	Symbol
Interference	*Locational interference fit:* for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements	H7/p6
	Medium drive fit: for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron	H7/s6
	Force fit: suitable for parts that can be highly stressed or for shrink fits where the heavy pressing forces required are impractical	H7/u6

Minimum size of the shaft is larger than the maximum size of the hole.

There is *always* overlap between the shaft and the hole for all size ranges.

Transition Fit

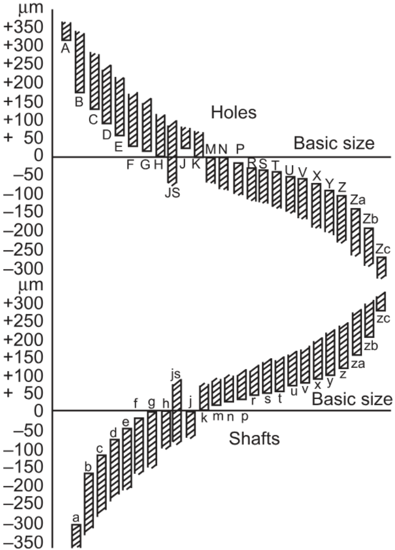
Type of Fit	Description	Symbol
Transition	*Locational transition fit:* for accurate location, a compromise between clearance and interference	H7/k6
	Locational transition fit: for more accurate location where greater interference is permissible	H7/n6

The maximum size of the shaft is greater than the minimum size of the hole, BUT the minimum size of the shaft is smaller than the maximum size of the hole.

Transitions from interference fit to clearance fit.
Sometimes there is overlap, and *sometimes* there is no overlap.

Fundamental Deviations and Tolerances

Holes and Shafts



Fundamental Deviations

- The letters A, B, C, . . . , a, b, c, . . . represent the fundamental deviations.
- Capital letters represent fundamental deviations of holes; small letters represent fundamental deviations of shafts.
- H or h represents 0 fundamental deviation.
- If the fundamental deviation of hole is given as H, then for various kinds of clearance fit, the shaft can have fundamental deviations as a, b, . . . g.
- If the fundamental deviation of shaft is given as h, then for various kinds of clearance fit, the hole can have fundamental deviations as A, B, . . . G.
- Note that A represents a positive fundamental deviation for holes, whereas a represents a negative fundamental deviation for shafts. Similarly for other letters.

Tolerance

Some tolerance values are enlisted in Table A-11 of Shigley.

Alternatively, some formulae can also be used.

A standard tolerance unit is defined as: $i = 0.45D^{1/3} + 0.001D$, where i is in microns and D is in mm.

Then some examples of tolerances in terms of international tolerance (IT) grades are:

IT grade	5	6	7	8	9	10
Magnitude	$7i$	$10i$	$16i$	$25i$	$40i$	$64i$

Example

Find the shaft and hole dimensions for a loose running fit with a 34-mm basic size.

From Table 7–9, the ISO symbol is 34H11/c11. From Table A–11, we find that tolerance grade IT11 is 0.160 mm. The symbol 34H11/c11 therefore says that $\Delta D = \Delta d = 0.160$ mm. Using Eq. (7–36) for the hole, we get

$$D_{\max} = D + \Delta D = 34 + 0.160 = 34.160 \text{ mm}$$

$$D_{\min} = D = 34.000 \text{ mm}$$

The shaft is designated as a 34c11 shaft. From Table A–12, the fundamental deviation is $\delta_F = -0.120$ mm. Using Eq. (7–37), we get for the shaft dimensions

$$d_{\max} = d + \delta_F = 34 + (-0.120) = 33.880 \text{ mm}$$

$$d_{\min} = d + \delta_F - \Delta d = 34 + (-0.120) - 0.160 = 33.720 \text{ mm}$$

Stress and Torque in Interference Fits

This topic is not needed for Mid-Sem Exam.

The stresses due to an interference fit can be obtained by treating the shaft as a cylinder with a uniform external pressure, and the hub as a hollow cylinder with a uniform internal pressure.

The pressure p at the interface of an interference fit is:

$$p = \frac{\delta}{\dfrac{d}{E_o}\left(\dfrac{d_o^2 + d^2}{d_o^2 - d^2} + \nu_o\right) + \dfrac{d}{E_i}\left(\dfrac{d^2 + d_i^2}{d^2 - d_i^2} - \nu_i\right)}$$

and, for the same outer and inner materials:

$$p = \frac{E\delta}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$

d : nominal shaft diameter

d_i : inside diameter (if any) of the shaft

d_o : outside diameter of the hub

δ : diametral interference between the shaft and the hub

Stress and Torque in Interference Fits ... contd.

The diametral interference is:

$$\delta = d_{\text{shaft}} - d_{\text{hub}}$$

Considering tolerances:

$$\delta_{\min} = d_{\min} - D_{\max}$$
$$\delta_{\max} = d_{\max} - D_{\min}$$

The maximum interference should be used in the interface pressure formulas (previous slide) to determine the maximum pressure.

Stress and Torque in Interference Fits ... contd.

The tangential stresses at the interface of the shaft and hub are:

$$\sigma_{t,\mathrm{shaft}} = -p\frac{d^2 + d_i^2}{d^2 - d_i^2},$$

$$\sigma_{t,\mathrm{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2}$$

The radial stresses at the interface are:

$$\sigma_{r,\mathrm{shaft}} = -p,$$

$$\sigma_{r,\mathrm{hub}} = -p$$

The tangential and radial stresses are mutually perpendicular, and should be combined using a failure theory to compare with the yield strength.

Stress and Torque in Interference Fits

The torque that can be transmitted through an interference fit can be estimated from the friction at the interface.

Given the coefficient of friction (μ), the maximum friction force (F) at the interface is obtained from the normal force due to the pressure (p) acting over a length L of the interface:

$$F = \mu N = \mu p 2\pi(d/2)L = \pi \mu p L D$$

The moment arm of this friction force is $d/2$, so that the torque capacity of the joint due to the interference fit is:

$$T = Fd/2 = \frac{\pi}{2}\mu p L d^2$$

To check for the max. amount of torque the joint should be designed to transmit without slipping, the minimum pressure obtained from $\delta_{\min}$ should be used.