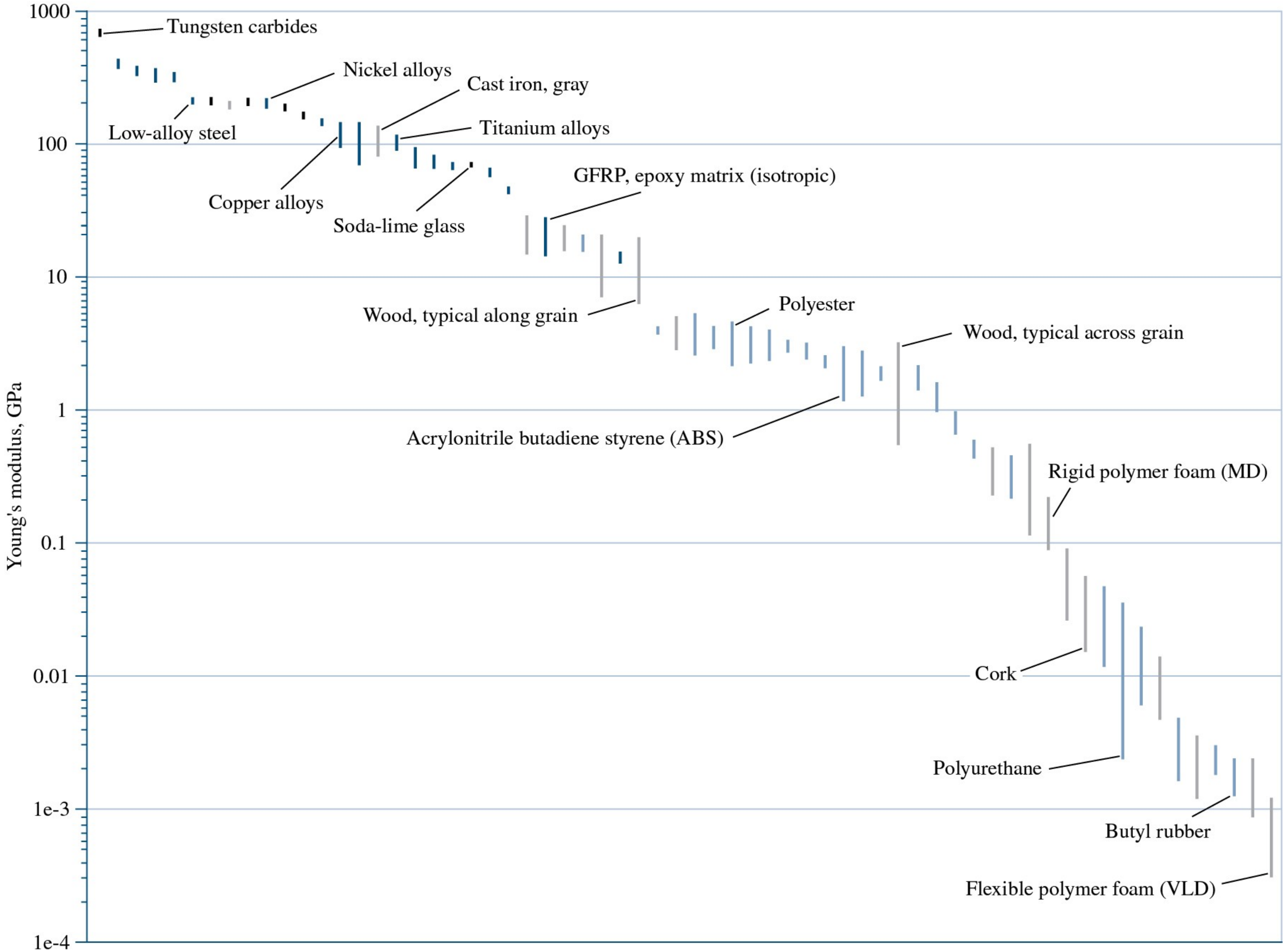
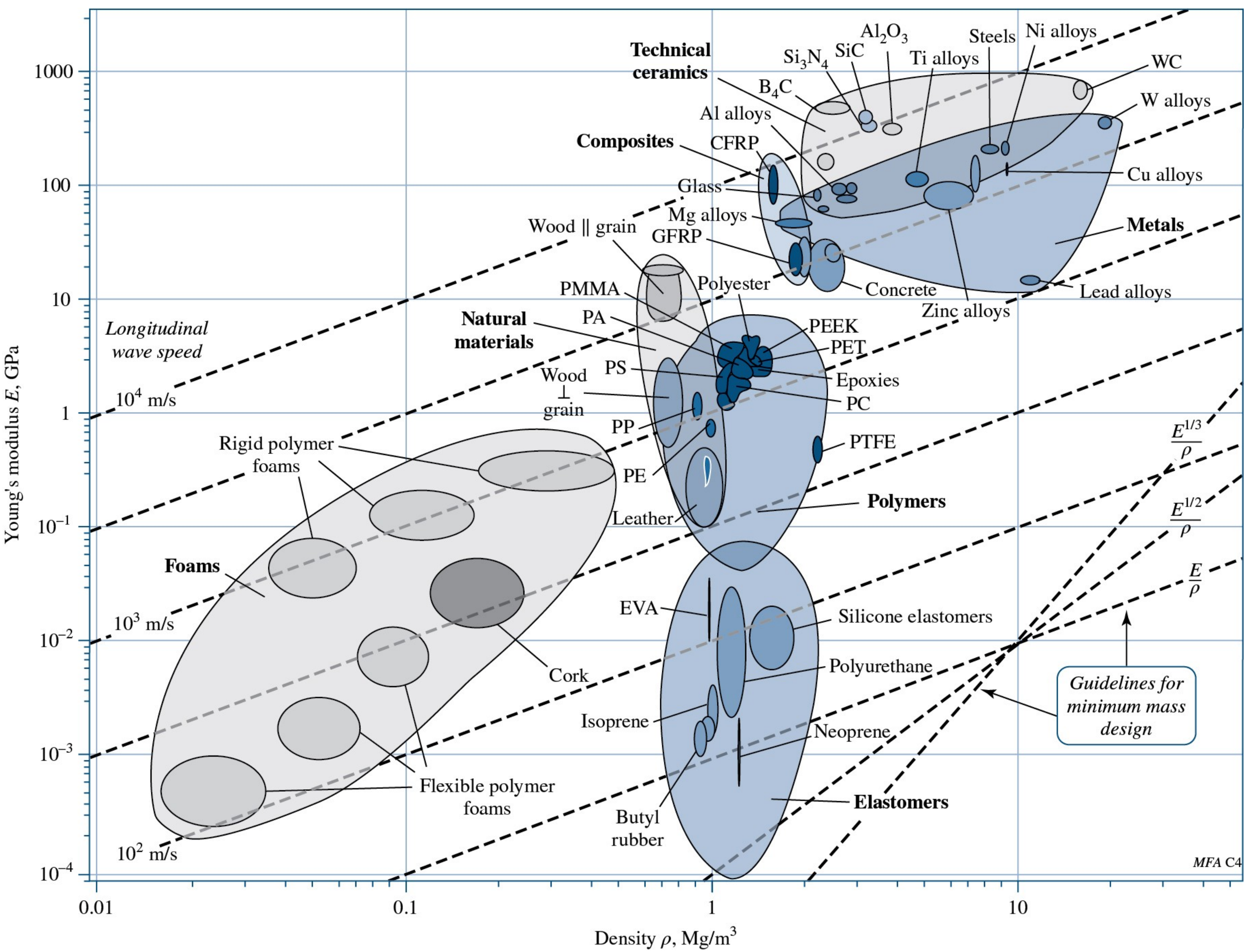
Chapter 2: Materials

Young’s Modulus for Various Materials

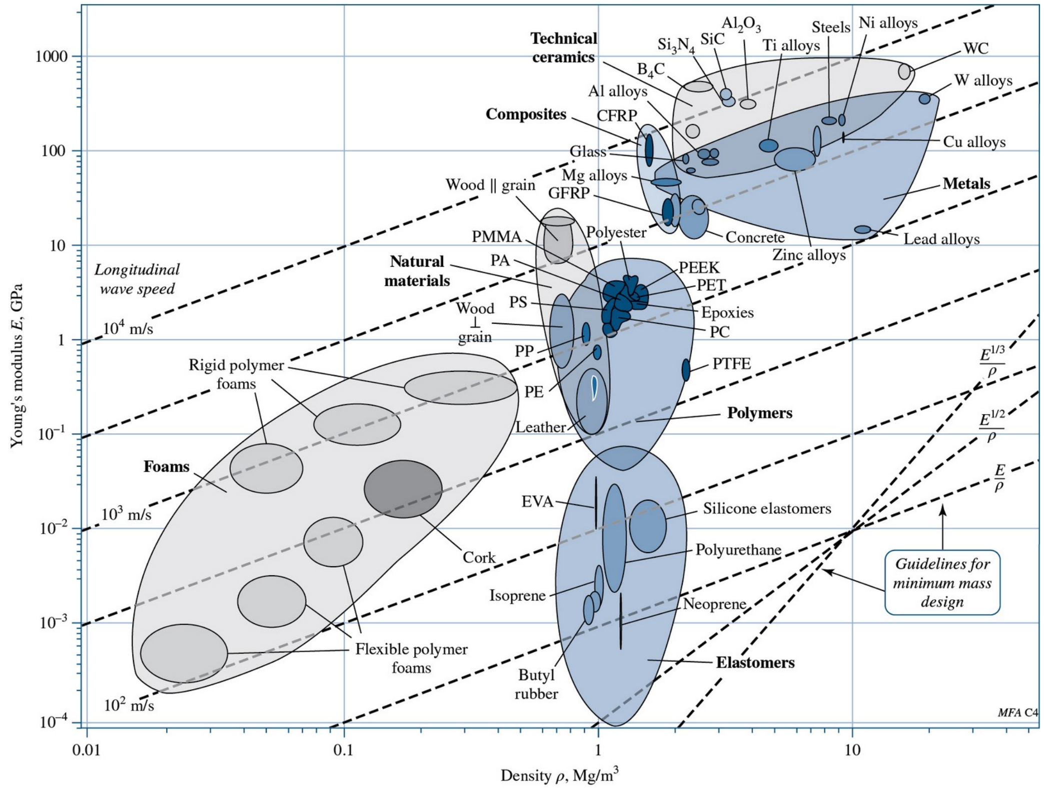


Young's Modulus vs. Density



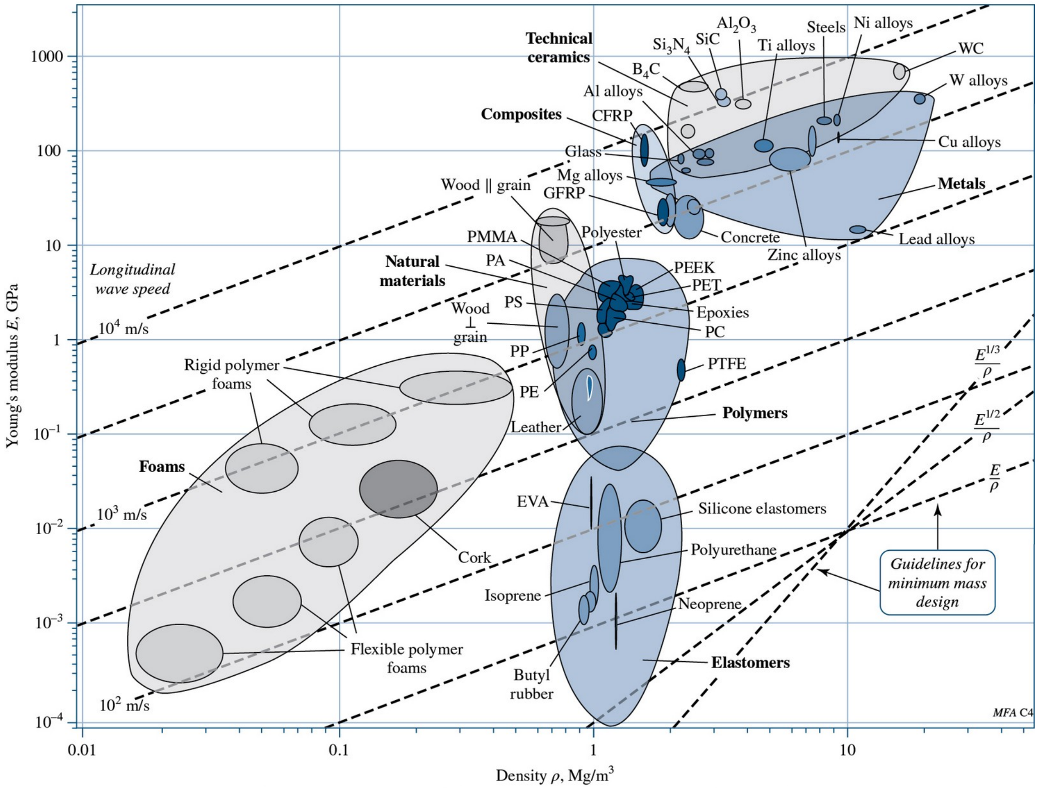
Specific Modulus

- *Specific Modulus* – ratio of Young's modulus to density, E/ρ
- Also called *specific stiffness*
- Useful to minimize weight with primary design limitation of deflection, stiffness, or natural frequency
- Parallel lines representing different values of E/ρ allow comparison of specific modulus between materials



Minimum Mass Guidelines for Young's Modulus-Density Plot

- Guidelines plot constant values of E^{β}/ρ
- β depends on type of loading
- $\beta = 1$ for axial
- $\beta = 1/2$ for bending



Example, for axial loading,

$$k = AE/l \quad \Rightarrow \quad A = kl/E$$

$$m = Al\rho = (kl/E)\, l\rho = kl^2\rho/E$$

Thus, to minimize mass, maximize E/ρ $\qquad (\beta = 1)$

The Performance Metric

The *performance metric* depends on (1) the functional requirements, (2) the geometry, and (3) the material properties.

$$P = \left[\begin{pmatrix} \text{functional} \\ \text{requirements } F \end{pmatrix}, \begin{pmatrix} \text{geometric} \\ \text{parameters } G \end{pmatrix}, \begin{pmatrix} \text{material} \\ \text{properties } M \end{pmatrix} \right]$$

$$P = f(F, G, M) \qquad (2\text{–}38)$$

The function is often separable,

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \qquad (2\text{–}39)$$

$f_3(M)$ is called the *material efficiency coefficient*.

Maximizing or minimizing $f_3(M)$ allows the material choice to be used to optimize P.

Performance Metric Example

- Requirements: light, stiff, end-loaded cantilever beam with circular cross section
- Mass m of the beam is chosen as the performance metric to minimize
- Stiffness is functional requirement
- Stiffness is related to material and geometry

$$k = \frac{F}{\delta}$$

Performance Metric Example (continued)

From beam deflection table, $\delta = \dfrac{Fl^3}{3EI}$

$$k = \frac{F}{\delta} = \frac{3EI}{l^3} \qquad (2\text{–}40)$$

$$I = \frac{\pi D^4}{64} = \frac{A^2}{4\pi} \qquad (2\text{–}41)$$

Sub Eq. (2–41) into Eq. (2–40) and solve for *A*

$$A = \left(\frac{4\pi k l^3}{3E} \right)^{1/2} \qquad (2\text{–}42)$$

The performance metric is

$$m = Al\rho \qquad (2\text{–}43)$$

Sub Eq. (2–42) into Eq. (2–43),

$$m = 2\sqrt{\frac{\pi}{3}} \left(k^{1/2}\right)\left(l^{5/2}\right)\left(\frac{\rho}{E^{1/2}} \right) \qquad (2\text{–}44)$$

Performance Metric Example (continued) 1

$$m = 2\sqrt{\frac{\pi}{3}}\left(k^{1/2}\right)\left(l^{5/2}\right)\left(\frac{\rho}{E^{1/2}}\right) \quad (2\text{–}44)$$

Separating into the form of Eq. (2–39),

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \quad (2\text{–}39)$$

$$f_1(F) = 2\sqrt{\pi / 3}(k^{1/2})$$

$$f_2(G) = (l^{5/2})$$

$$f_3(M) = \frac{\rho}{E^{1/2}} \quad (2\text{–}45)$$

To minimize m, need to minimize $f_3(M)$, or maximize

$$M = \frac{E^{1/2}}{\rho} \quad (2\text{–}46)$$

Performance Metric Example (continued) 2

- M is called *material index*
- For this example, $\beta = ½$
- Use guidelines parallel to $E^{1/2}/\rho$
- Increasing M, move up and to the left
- Good candidates for this example are certain woods, composites, and ceramics

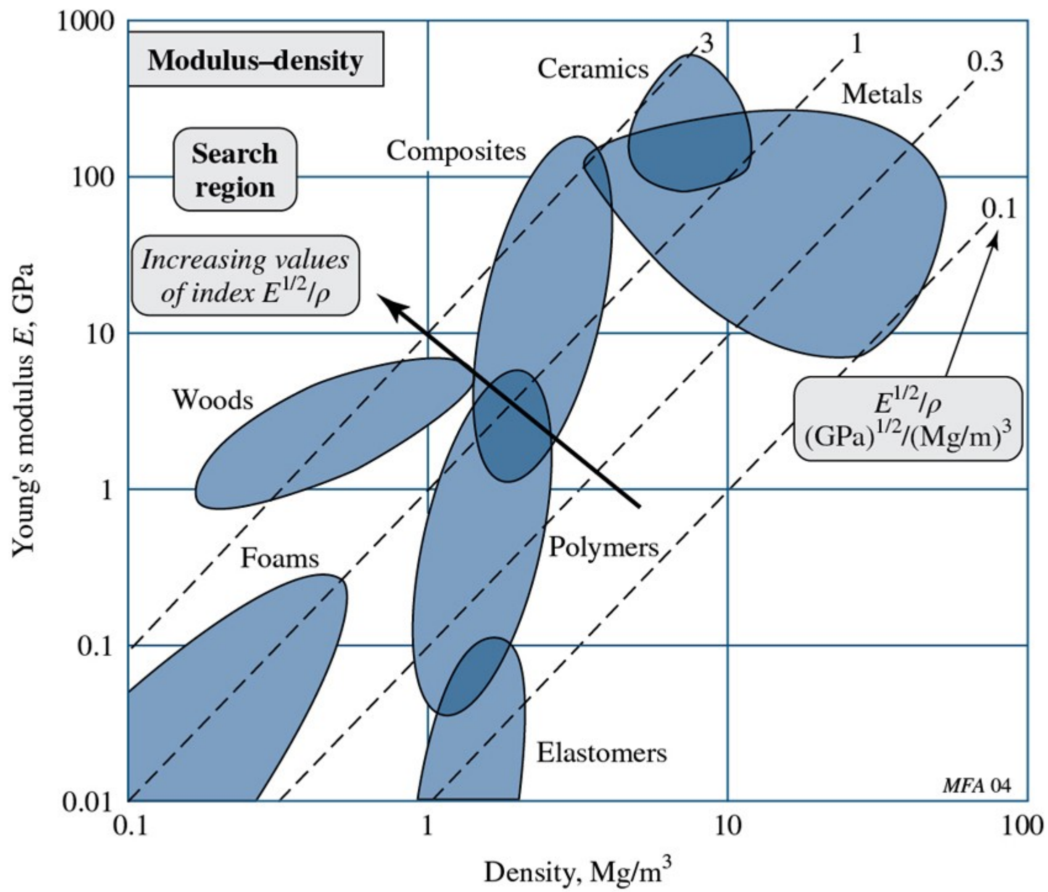


Fig. 2–25

Source: Adapted from Ashby, M. F., Materials Selection in Mechanical Design, *3rd ed., Oxford: Elsevier Butterworth- Heinemann, 2005.*

Performance Metric Example (continued) 3

- Additional constraints can be added as needed
- For example, if it is desired that $E > 50$ GPa, add horizontal line to limit the solution space
- Wood is eliminated as a viable option

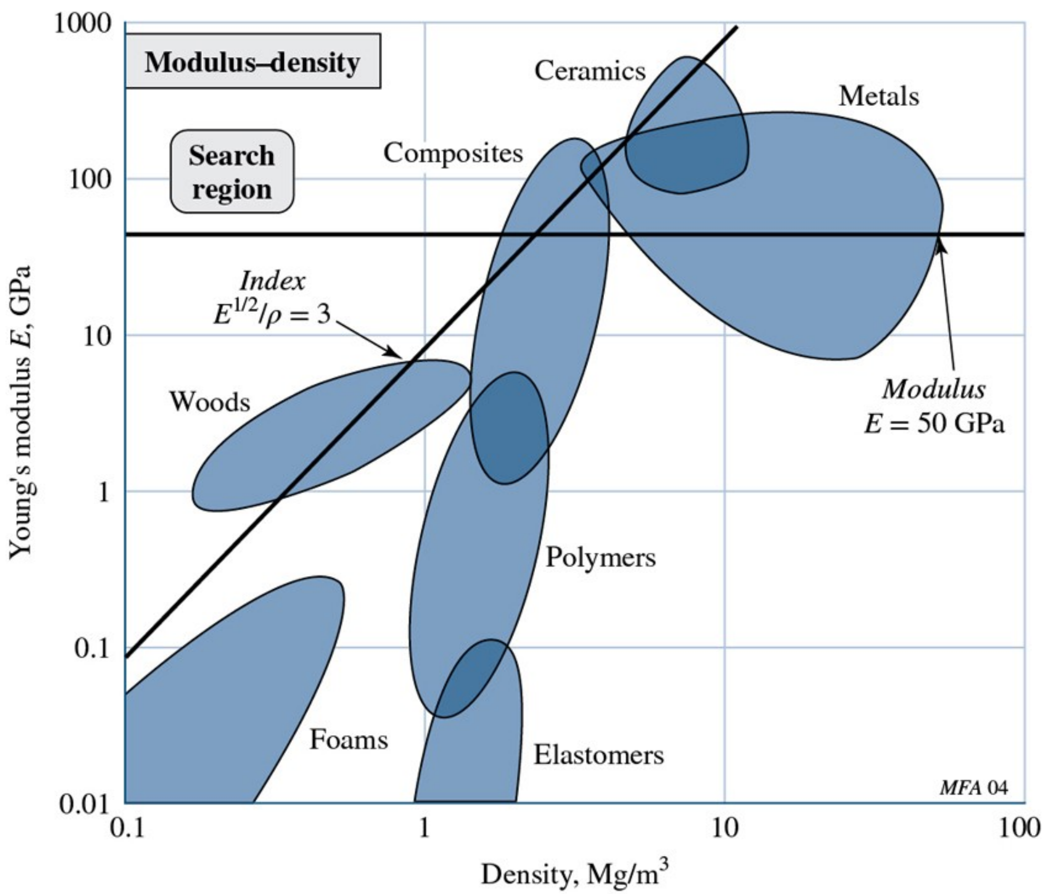


Fig. 2–26

Source: Adapted from Ashby, M. F., Materials Selection in Mechanical Design, *3rd ed., Oxford: Elsevier Butterworth- Heinemann, 2005.*

Strength vs. Density

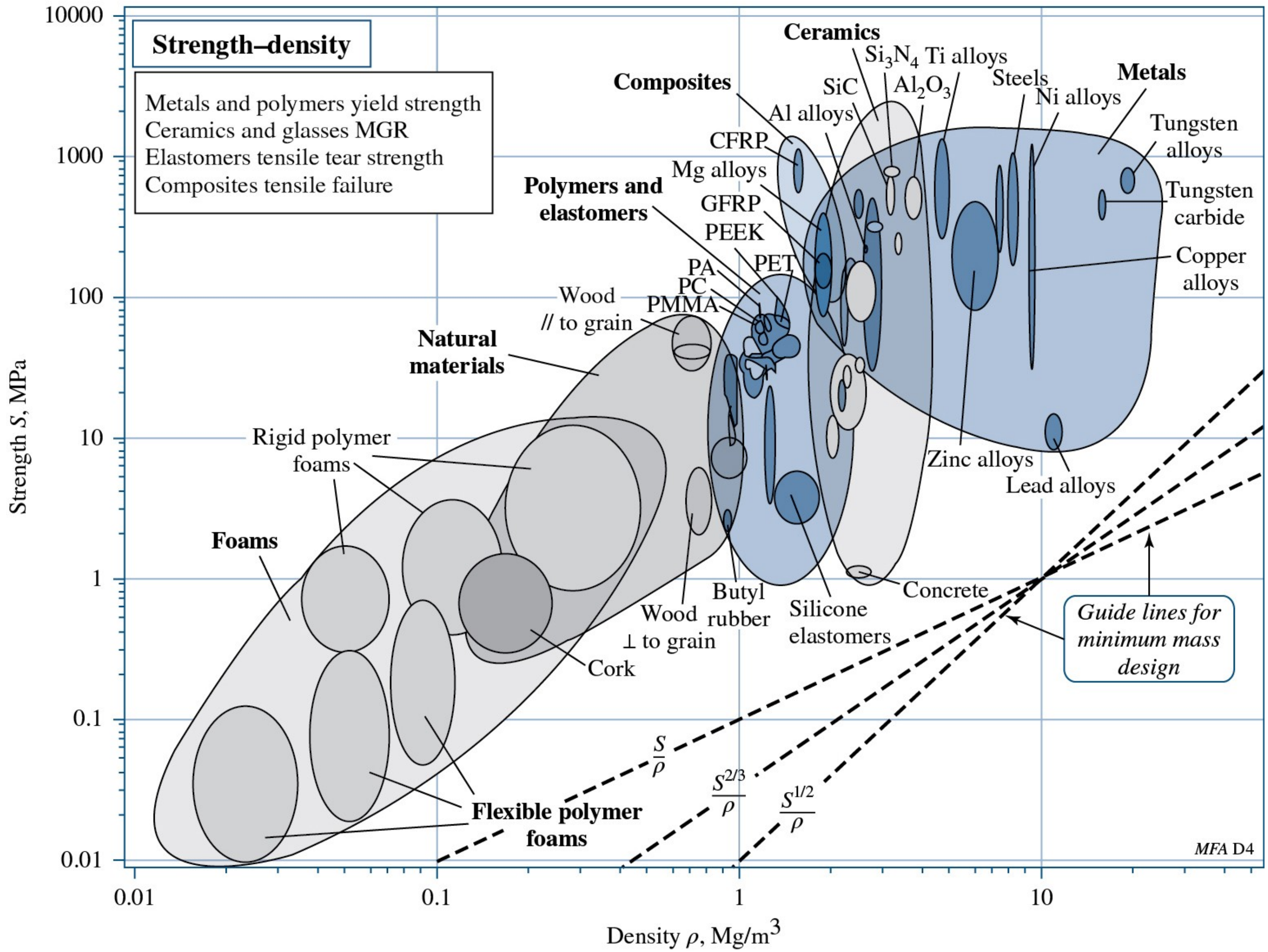


Fig. 2–27

Source: Adapted from figure by Prof. Mike Ashby, Granta Design, Cambridge, U.K.

Specific Modulus (continued)

- *Specific Strength* – ratio of strength to density, *S/ρ*
- Useful to minimize weight with primary design limitation of strength
- Parallel lines representing different values of *S/ρ* allow comparison of specific strength between materials

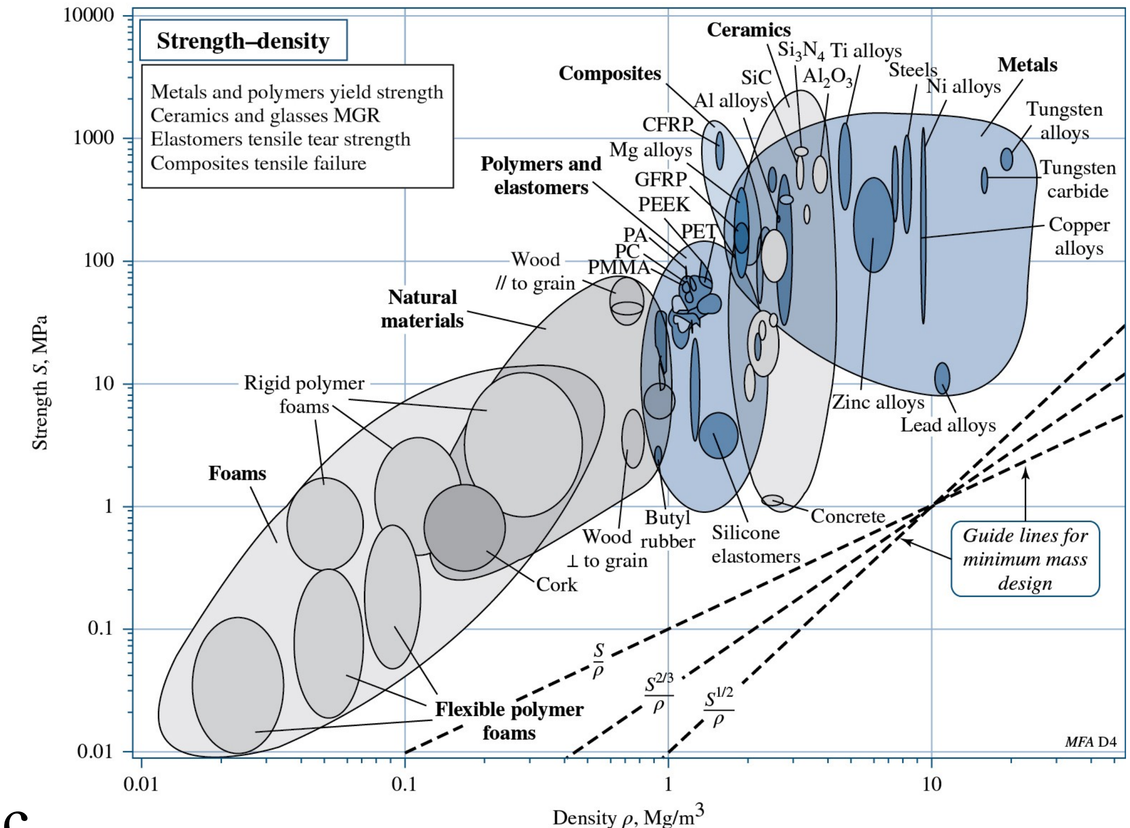


Fig. 2–27

Source: Adapted from figure by Prof. Mike Ashby, Granta Design, Cambridge, U.K.

Minimum Mass Guidelines for Strength-Density Plot

- Guidelines plot constant values of S^{β}/ρ
- β depends on type of loading
- $\beta = 1$ for axial
- $\beta = 2/3$ for bending

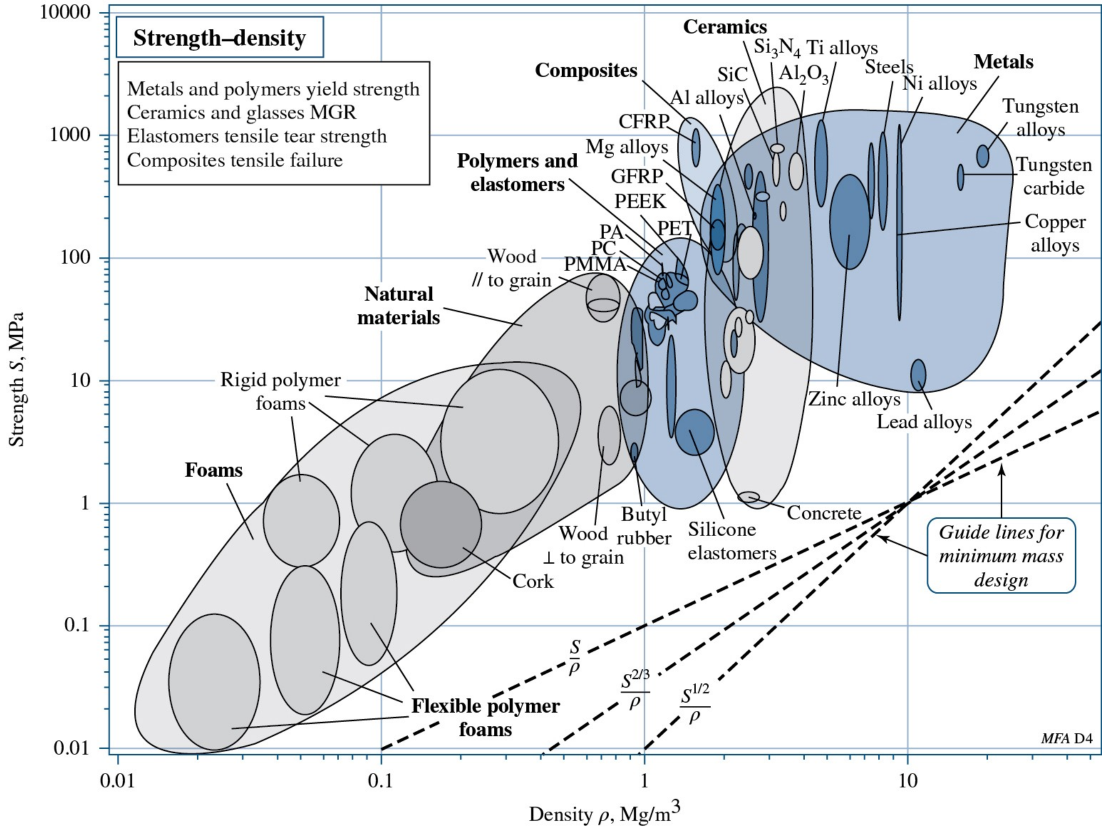


Fig. 2–27

Example, for axial loading,

$\sigma = F/A = S \quad \Rightarrow \quad A = F/S$

$m = Al\rho = (F/S)\, l\rho$

Thus, to minimize m, maximize S/ρ $\;(\beta = 1)$

Source: Adapted from figure by Prof. Mike Ashby, Granta Design, Cambridge, U.K.