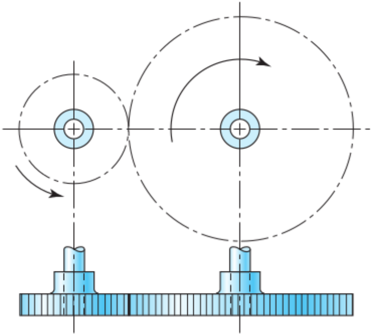
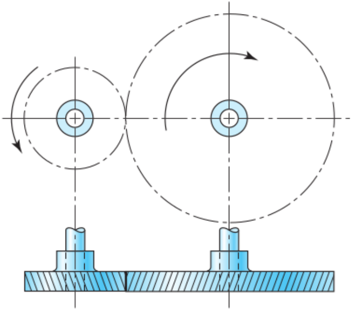
Chapter 8: Gears

Types of Gears

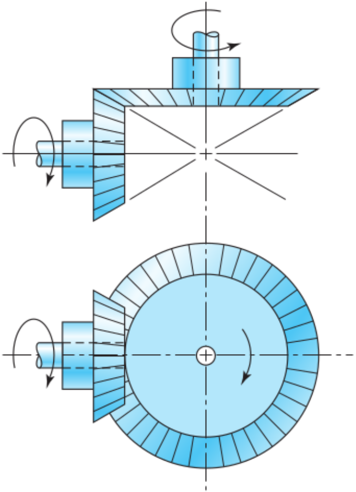


Spur gears are used to transmit rotary motion between parallel shafts.

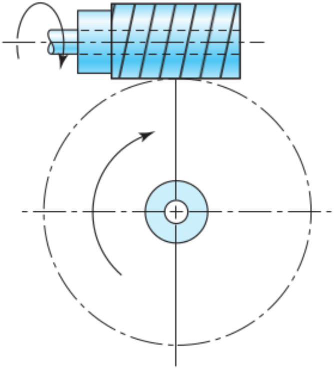


Helical gears are used to transmit motion between parallel or nonparallel shafts.

Types of Gears

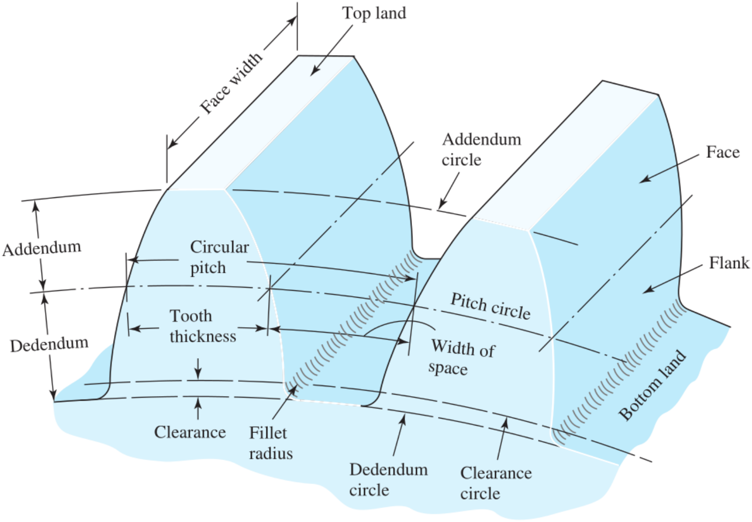


Bevel gears are used to transmit rotary motion between intersecting shafts.



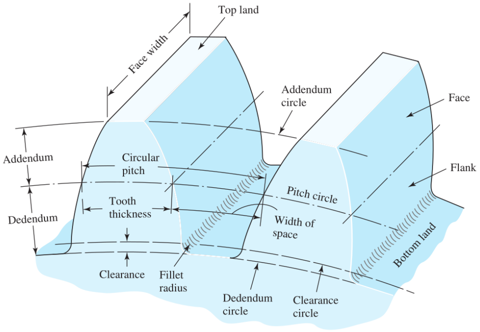
Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.

Nomenclature of Spur Gear Teeth



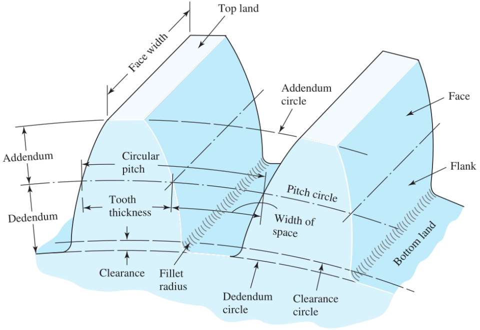
Nomenclature of Spur Gear Teeth

- **Pitch circle**: It is a theoretical circle upon which all calculations are usually based. The pitch circles of a pair of mating gears are tangent to each other.
- **Pitch diameter** d: Diameter of the pitch circle.
- **Pinion**: It is the smaller of two mating gears. The larger one is just called the gear.



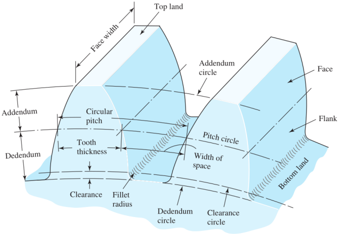
Nomenclature of Spur Gear Teeth

- **Circular pitch** p: Distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. Thus, p is equal to the sum of the *tooth thickness* and the *width of space*.
- **Module** m: Ratio of the pitch diameter to the number of teeth. Customary unit of length used is mm. Module is the index of tooth size in SI system.
- **Diametral pitch** P: Ratio of the number of teeth on the gear to the pitch diameter. It is the reciprocal of the module. Since diametral pitch is used only with US units, it is expressed as teeth per inch.



Nomenclature of Spur Gear Teeth

- **Addendum** a: Radial distance between the *top land* and the pitch circle
- **Dedendum** b: Radial distance from the *bottom land* to the pitch circle
- **Face width** F: Thickness of the tooth. In general, gear manufacturers will provide a few options for face width in the range of three to five times the circular pitch.
- **Whole depth** h_t: Sum of the addendum and the dedendum.
- **Clearance circle**: Circle that is tangent to the addendum circle of the mating gear.
- **Clearance** c: Amount by which the dedendum in a given gear exceeds the addendum of its mating gear.
- **Backlash**: Amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.



Tooth Size

$$P = \frac{N}{d}$$

$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

where

P : diametral pitch, teeth per inch

N : number of teeth

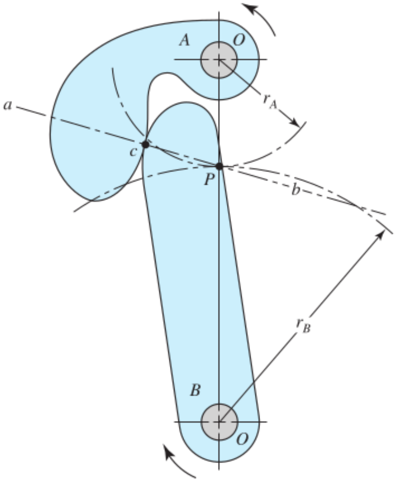
d : pitch diameter, in or mm

m : module, mm

p : circular pitch, in or mm

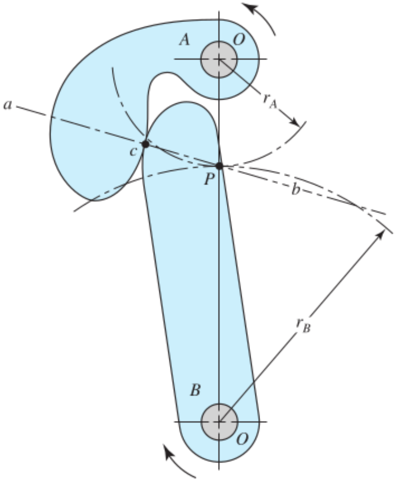
Conjugate Action

- When surfaces roll/slide against each other and produce constant angular velocity ratio, they are said to have conjugate action.
- When one curved surface pushes against another, the point of contact occurs where the two surfaces are tangent to each other.



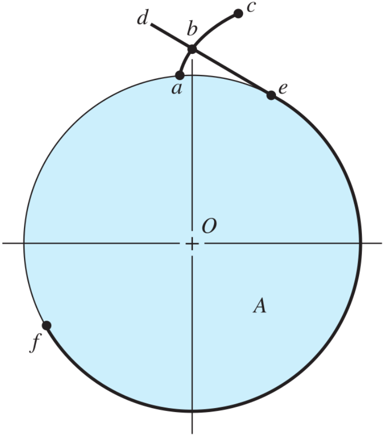
Conjugate Action

- The forces at any instant are directed along the common normal *ab* to the two curves.
- The line *ab*, representing the direction of action of the forces, is called the line of action.
- The line of action intersects the line of centres $O - O$ at some point P, called the pitch point.
- The angular velocity ratio is inversely proportional to their radii to the point P, i.e.
  $\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$
- Circles drawn through P from each centre are called pitch circles, each with a pitch radius.



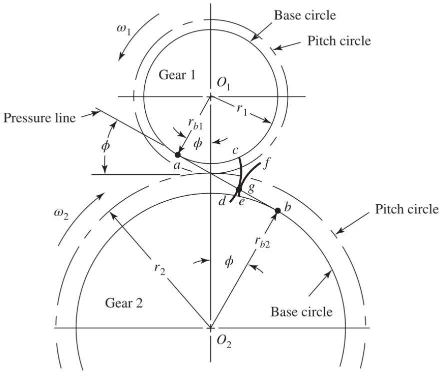
Involute Profile

- The most common conjugate profile is the involute profile.
- Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.
- Circle is called base circle.



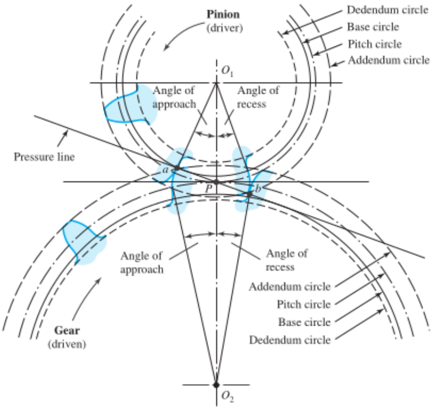
Involute Gears Meshing

- When two gears are in mesh, their pitch circles roll on one another without slipping.
- The line *ab* is called the line of action, or the generating line, or the pressure line.
- The angle between the pressure line and the perpendicular to the line of the gear centres, $O_1 O_2$, is called the pressure angle, ϕ.
- The pressure line is tangent to each gear at their respective base circle.
- The base circle radius of gear i is: $r_{bi} = r_i \cos\phi$, where r_i is the pitch circle radius.



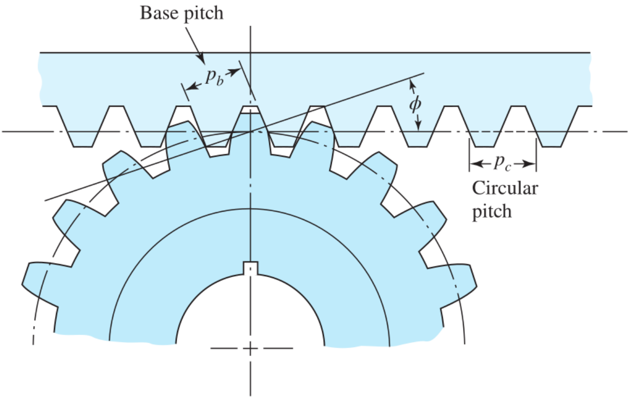
Involute Gears - Tooth Action

- The initial contact takes place when the flank of the driver comes into contact with the tip of the driven tooth.
- Occurs at point *a* where the addendum circle of the driven gear crosses the pressure line → Defines the *angle of approach* for each gear.
- As meshing proceeds, the point of contact slides up the side of the driving tooth until the tip is in contact just before contact ends.
- The final point of contact is where the addendum circle of the driver crosses the pressure line. → Defines the *angle of recess* for each gear.
- Angle of action = Angle of approach + Angle of recess

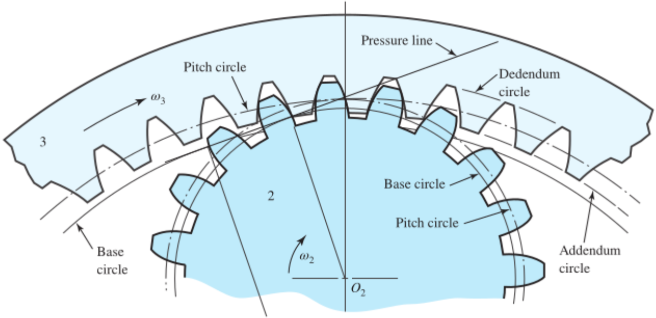


Rack

- A rack is a spur gear having an infinitely large pitch diameter.
- The sides of the teeth are straight lines making an angle to the line of centres equal to the pressure angle.
- The base pitch and circular pitch are related by $p_b = p_c \cos\phi$



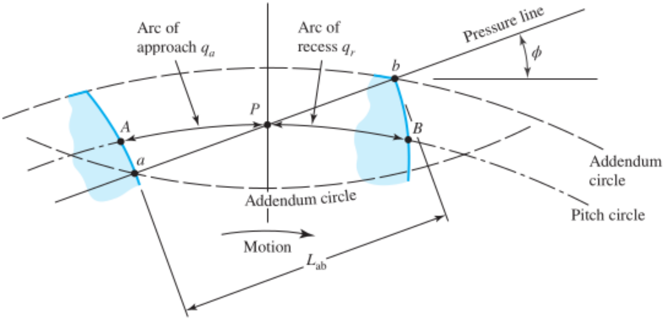
Internal or Ring Gear



Note the location of the base circle of the internal gear.

Contact Ratio

- Arc of action q_t is the sum of the arc of approach q_a and the arc of recess q_r, that is $q_t = q_a + q_r$
- The contact ratio m_c is the ratio of the arc of action and the circular pitch: $m_c = \dfrac{q_t}{p}$
- The contact ratio is the average number of pairs of teeth in contact.

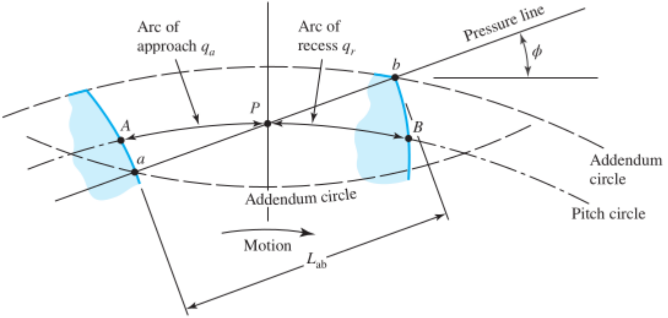


Contact Ratio

- Contact ratio can also be found from the length of the line of action

$$m_c = \frac{L_{ab}}{p \cos \phi}$$

- The contact ratio should be at least 1.2



Tooth Sizes in General Use

Diametral Pitch P (teeth/in)	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Module m (mm/tooth)	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Standardized Tooth Systems (Spur Gears)

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	$22\frac{1}{2}$	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	25	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
Stub	20	$0.8/P$ or $0.8m$	$1/P$ or m

Standardized Tooth Systems

- Common pressure angle ϕ: 20° and 25°
- Old pressure angle: $14\ \frac{1}{2}^{\circ}$
- Common face width:

$$3p < F < 5p$$

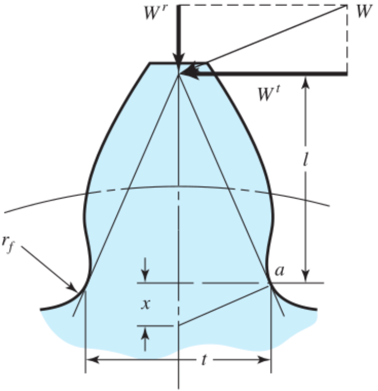
$$\text{or,} \quad \frac{3\pi}{P} < F < \frac{5\pi}{P}$$

Cantilever Beam Model of Bending Stress in Gear Tooth

$$\frac{t/2}{x} = \frac{l}{t/2}$$

$$\Longrightarrow \quad x = \frac{t^2}{4l} \quad \text{or,} \quad l = \frac{t^2}{4x}$$

$$\begin{aligned}\sigma = \frac{M}{I/c} &= \frac{6W^t l}{Ft^2} \\ &= \frac{W^t}{F}\frac{1}{t^2/(6l)} \\ &= \frac{W^t}{F}\frac{1}{t^2/(4l)}\frac{1}{\frac{4}{6}} \\ &= \frac{W^t p}{F\left(\frac{2}{3}\right)xp} \\ &= \frac{W^t}{Fpy},\end{aligned}$$



where $\quad y = 2x/(3p)$

Lewis Equation

$$\sigma = \frac{W^t}{Fpy}$$
$$p = \pi / P$$

Define: Lewis Form Factor $Y = \frac{2xP}{3}$, where $P = \pi / p$.

Lewis Equation: $\sigma = \frac{W^t P}{FY}$

Lewis Form Factor: $Y = \frac{2xP}{3} \rightarrow Y = \pi y$

Values of Lewis Form Factor Y

Number of Teeth	*Y*	Number of Teeth	*Y*
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Values of the Lewis Form Factor Y for a normal pressure Angle of 20°, full-depth teeth, and a diametral pitch of unity in the plane of rotation

Dynamic Effects

- Effective load increases as velocity increases.
- Velocity factor K_v accounts for this increase.

Table: Expressions for K_v

Profile type	V in m/s	V in feet/min
cast iron, cast profile	$\frac{3.05+V}{3.05}$	$\frac{600+V}{600}$
cut or milled profile	$\frac{6.1+V}{6.1}$	$\frac{1200+V}{1200}$
hobbed or shaped profile	$\frac{3.56+\sqrt{V}}{3.56}$	$\frac{50+\sqrt{V}}{50}$
shaved or ground profile	$\sqrt{\frac{5.56+\sqrt{V}}{5.56}}$	$\sqrt{\frac{78+\sqrt{V}}{78}}$

Lewis Equation including Velocity Vactor

Metric version: $\sigma = \dfrac{K_v W^t}{FmY}$

US customary version: $\sigma = \dfrac{K_v W^t P}{FY}$

Surface Durability

Another failure mode is wear due to contact stress.
The gear teeth mesh is modelled with contact stress between two cylinders:

$$p_{\max} = \frac{2F}{\pi b l},$$

where:

$$\begin{aligned} p_{\max} &= \text{largest surface pressure} \\ F &= \text{force pressing the two cylinders together} \\ l &= \text{length of cylinders} \\ b &= \left[\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2} \right]^{1/2} \end{aligned}$$

Surface Durability

In terms of the gear tooth, the surface contact stress (Hertzian stress) is given by:

$$\sigma_c = -\left[\frac{W^t}{\pi F \cos\phi}\frac{1/r_1 + 1/r_2}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}\right]^{1/2}$$

Critical location is usually at the pitch line, where:

$$r_1 = \frac{d_P \sin\phi}{2} \quad \text{and} \quad r_2 = \frac{d_G \sin\phi}{2}$$

Define elastic coefficient from the denominator of σ_c:

$$C_P = \left[\frac{1}{\pi\left(\frac{1-\nu_P^2}{E_P} + \frac{1-\nu_G^2}{E_G}\right)}\right]^{1/2}$$

Surface Durability

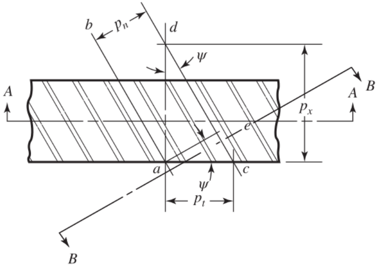
Incorporating elastic coefficient and velocity factor, the contact stress equation is:

$$\sigma_c = -C_P \left[\frac{K_v W^t}{F \cos \phi} \right]^{1/2}$$

Helical Gears - Geometry

ψ: helix angle

Distance *ac* is the transverse circular pitch p_t, usually called the circular pitch in the plane of rotation.



The distance *ae* is the normal circular pitch p_n:

$$p_n = p_t \cos\psi$$

$$\text{or,} \quad m_n = m_t \cos\psi$$

Helical Gears - Force

W : total force

W^r : radial component

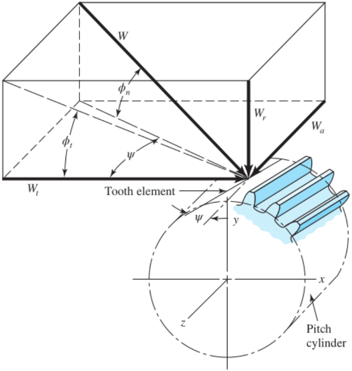
W^t : tangential component;
also called the transmitted load

W^a : axial component;
also called the thrust load

$$W^r = W \sin \phi_n,$$
$$W^t = W \cos \phi_n \cos \psi,$$
$$W^a = W \cos \phi_n \sin \psi$$

Usually, from the power and rpm, W^t is known. Other forces in terms of W^t:

$$W^r = W^t \frac{\tan \phi_n}{\cos \psi},$$
$$W^a = W^t \tan \psi,$$
$$W = \frac{W^t}{\cos \phi_n \cos \psi}$$



ϕ_n : normal pressure angle

ψ : helix angle