CLASS TEST-3 : 17th march

① Let $\dot{x}(t) = -x^3(t) + u(t)$, $x(0) = 0.5$

cost function $J = \frac{1}{2}x(2)^2 + \frac{1}{2}\int_0^2 (x^2+u^2)\,dt$

write costate equation, optimality conditions & boundary conditions. Eliminate $u^*(t)$ from state & costate equations.

② Let $x \in \mathbb{R}^n$, $u \in \mathbb{R}$. $\dot{x} = Ax + Dxu + bu$

cost function $J = \frac{1}{2}\, x(T)^T S(T)\, x(t) + \frac{1}{2}\int_0^T [x(t)^T Q x(t) + r u^2(t)]\,dt$

Clearly state the optimality conditions, boundary conditions, and the expression of optimal input $u^*(t)$

Eliminate $u^*(t)$ and write state and costate equations.

③ Consider minimum energy optimal control of LTI single-input system.

$$\min_{u(t),\,x(t)} \quad \frac{1}{2}\int_{t_0}^{t_f} \left(u(t)^T R\, u(t)\right) dt$$

s.t. $\quad \dot{x}(t) = Ax(t) + Bu(t)$

$\quad |u(t)| \leq 1.$

$\quad x(t_0),\ t_0,\ t_f$: given

clearly state optimality conditions, and the expression of optimal input. Sketch $u^*(t)$ vs. "switching function".

Is singularity possible ?