

Class Test 2: Optimal Control

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Q 2.1: Linear Quadratic Regulator Problem

Consider the discrete-time system $x_{k+1} = 5x_k + 2u_k$ with performance index $J = 2x_N^2 + \frac{1}{2} \sum_{k=0}^{N-1} x_k^2$. Let $N = 3$. Answer the following questions.

1. Determine the optimal control gain K_k for $k = 0, 1, 2$.
2. If $x_0 = 1$, then determine x_1, x_2, x_3 that arise under optimal input.

Q 2.2: Discrete-time Tracking

Consider the optimal tracking problem for the discrete-time system $x_{k+1} = f(x_k, u_k)$ with performance index

$$J = \frac{1}{2}(Cx_N - r_N)^\top S_N(Cx_N - r_N) + \frac{1}{2} \sum_{k=0}^{N-1} [(Cx_k - r_k)^\top Q(Cx_k - r_k) + u_k^\top Ru_k],$$

where $r_k, k \in \{1, 2, \dots, N\}$ being the reference signal that is given. The initial state x_0 is also given. From the first order necessary condition of optimality, write the state and costate equation that the optimal trajectory needs to satisfy, the optimal input signal and the necessary boundary conditions.

Q 2.3: Dynamic Programming with Finite States and Actions

Consider the discrete-time system $x_{k+1} = -x_k + u_k$ with the state variable takes integer values over the interval $[-3, 3]$, and the input $u_k \in \{-1, 0, 1\}$. The performance measure to be minimized is $J = \sum_{k=0}^2 (x_k^2 + |u_k|)$. Using dynamic programming, find the optimal control sequence $\{u_0^*, u_1^*, u_2^*\}$ and the minimum cost if the initial state is $x_0 = -3$.