Blowup-polynomials of graphs

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Abstract

Given a finite simple connected graph $G = (V, E)$, we introduce a novel invariant which we call its blowup-polynomial $p_G(n_v : vinV)$. To do so, we compute the determinant of the distance matrix of the graph blowup, obtained by taking n_v copies of the vertex v, and remove an exponential factor. First: we show that as a function of the sizes n_v, p_G is a polynomial, is multi-affine, and is real-stable. Second: we show that the multivariate polynomial p_G is intimately related to the characteristic polynomial q_G of the distance matrix D_G, and that it fully recovers G whereas q_G does not. Third: we obtain a novel characterization of the complete multi-partite graphs, as precisely those whose "homogenized" blowup-polynomials are Lorentzian/strongly Rayleigh. Finally: we show how to obtain from p_G a novel delta-matroid for every graph; we also provide a second delta-matroid for every tree, which too is hitherto unexplored, but whose construction does not extend to all graphs. (Joint with Apoorva Khare.)

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