

Some combinatorial games on rooted multi-type Galton-Watson trees

Moumanti Podder *

November 22, 2021

Abstract

In a rooted multi-type Galton-Watson branching process, the root is assigned a colour from a finite set Σ of colours according to some probability distribution \mathbf{p} , and a vertex of the tree, conditioned on its colour $\sigma \in \Sigma$, gives birth to offspring according to some probability distribution χ_σ on \mathbb{N}_0^Σ . In particular, one may consider $\Sigma = \{\text{red}, \text{blue}\}$ and the resulting random tree, denoted \mathcal{T} , can be viewed as a directed random graph if each edge is attributed a direction from parent to child. I consider the *normal*, *misère* and *escape* games on \mathcal{T} , each played between P1 and P2, with P1 being allowed to move the token *only* along monochromatic directed edges and P2 being allowed to move the token *only* along non-monochromatic directed edges. I then investigate the probabilities of win, loss and (where pertinent) draw of each player as fixed points of distributional recursions, establish inequalities between win / loss / draw probabilities of the players across different games, seek possible phase transitions in win / loss / draw probabilities as the parameters involved in the offspring distributions are made to vary etc.

*Department of Mathematics, IISER Pune.