Open Channel Flow

Hydraulics

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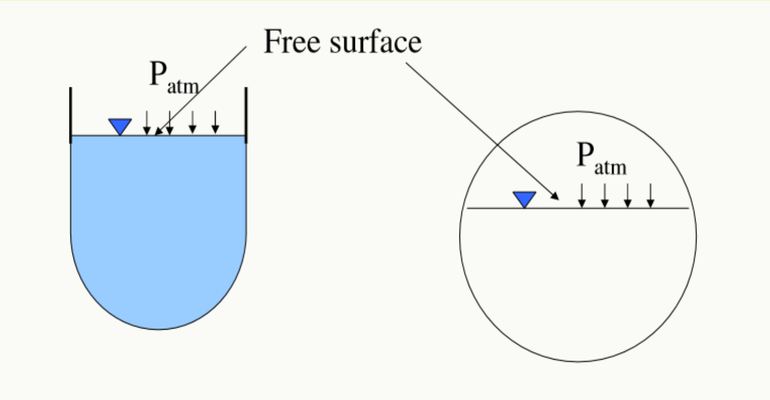
Open Channel Flow

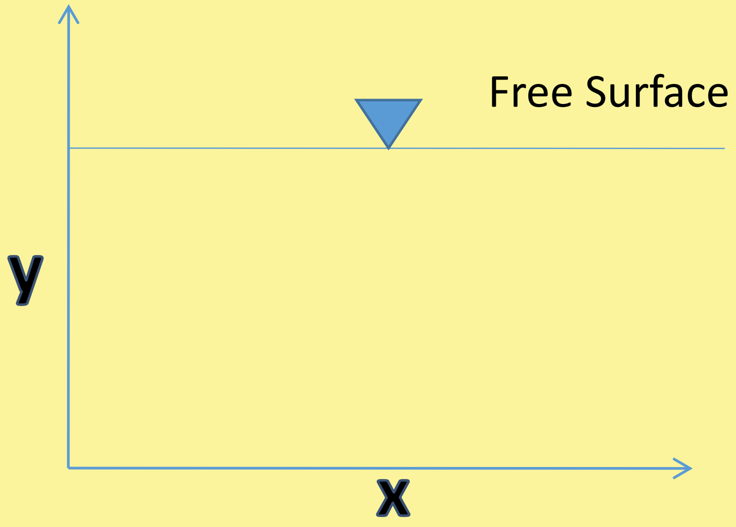
- **Flow of liquid in channel or conduit that is not completely filled**



- **Liquid (water) flow with a free surface (interface between water and air) that can distort**
- **relevant for**
- **natural channels: rivers, streams**
- **engineered channels: canals, sewer lines or culverts (partially full), storm drains**

Notations

- **Fluid depth : y**
- **Time : t**
- **Distance along channel : x**



Classification: Open Channel Flow

Time based

- **Unsteady**: $$\frac{dy}{dt} \neq 0$$
- **Steady**: $$\frac{dy}{dt} = 0$$

Classification:: Open Channel Flow

Space based

- **Uniform Flow**: $$\frac{dy}{dx} = 0$$
- **Non-uniform/varied Flow**: $$\frac{dy}{dx} \neq 0$$
  - **Gradually**: $$\frac{dy}{dx} << 1$$
  - **Rapidly**: $$\frac{dy}{dx} \approx 1$$

Classification: Open Channel Flow

Reynolds number based

- **Laminar**: $Re < 500$
- **Transitional**: $500 < Re < 12500$
- **Turbulent**: $Re > 12500$

$$\mathrm{Re} = \frac{\rho V R_h}{\mu}$$

1) **The dividing Reynolds number are approximate**
2) **Since water has very low viscosity and large characteristic length (hydraulic radius) it is difficult to have laminar flow**

ρ: Density of water

V: Average velocity of fluid

R_h: Hydraulic Radius of channel

μ : Dynamic Viscosity of water

Classification: Open Channel Flow

Froude number based

- **Subcritical**: Fr < 1
- **Critical**: Fr = 1
- **Supercritical**: Fr > 1

$$Fr = \frac{V}{\sqrt{gl}}$$

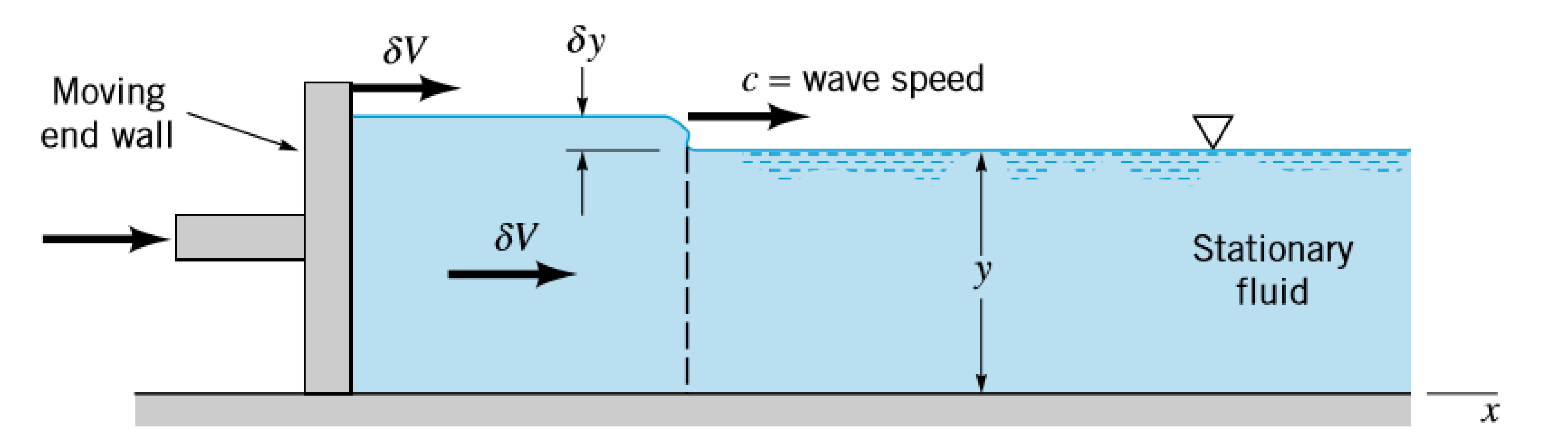
V: Average velocity of fluid

l: Characteristic length of flow

g : Acceleration due to gravity

Surface Solitary Waves

- **Open Channel flow – free surface- can distort- waves generated**

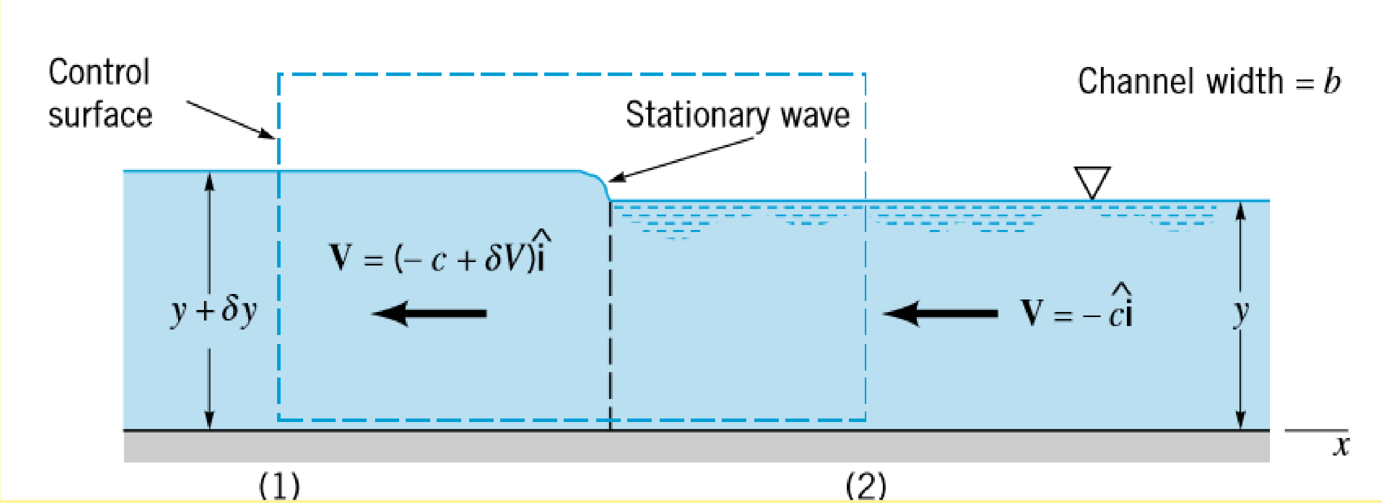


Production of single wave in a channel

Surface Solitary Waves

- Water was stationary at time t = 0
- Wall starts moving with speed δV
- Stationary observer observes single wave move down the channel with wave speed c
  - He sees no motion ahead of the wave
  - Notices fluid with velocity δV behind the wave
  - The motion is therefore unsteady for such observer
- For an observer moving along the channel with wave speed c, flow will be steady

Surface Solitary Waves



Wave as seen by observer that moves with *wave speed c*

- **To such observer**
  - **Fluid velocity shall be** $V = -c\hat{\imath}$ **to the right of observer**
  - **Fluid velocity shall be** $V = (-c + \delta V)\hat{\imath}$ **to the left of the observer**

Surface Solitary Waves

- **Assuming uniform 1D Flow**
  - **Equation of continuity** $-cyb = (-c + \delta V)(y + \delta y)b$

$$c = \frac{(y + \delta y)\delta V}{\delta y}$$

  - **Under assumption of small-amplitude waves with δy<<y**

$$c = y\frac{\delta V}{\delta y}$$ **Eq. 1**

Surface Solitary Waves

- **Equation of momentum**
  - **Mass flow rate m=ρbcy**
  - **Pressure is hydrostatic within fluid**
  - **Pressure force on channel cross section 1**

    $$F1 = \Upsilon y_{c1} A_1 = \Upsilon (y+\delta y)^2 b/2$$

  - **Pressure force on channel cross section 2**

    $$F2 = \Upsilon y_{c2} A_2 = \Upsilon y^2 b/2$$

Surface Solitary Waves

$$\frac{1}{2}\gamma y^2 b - \frac{1}{2}\gamma(y+\delta y)^2 b = \rho b c y[(c-\delta V)-c]$$

- **Assumption of small-amplitude waves $(\delta y)^2 << y\delta y$**

$$\frac{\delta V}{\delta y} = \frac{g}{c} \qquad \textbf{Eq. 2}$$

Surface Solitary Waves

- Substitute Eq. 2 into Eq. 1

$$c = y\frac{g}{c}, c^2 = gy$$

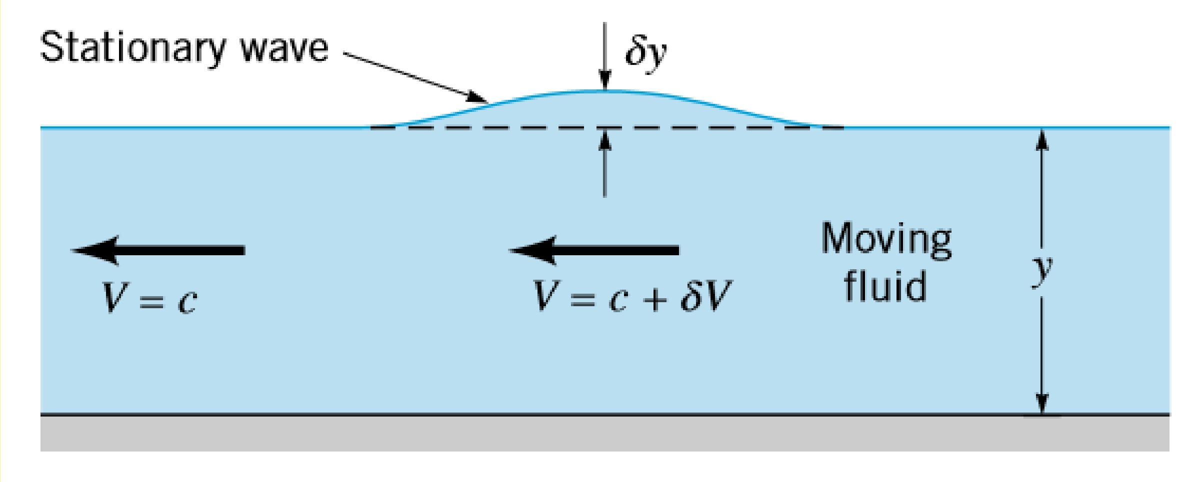
$$c = \sqrt{gy} \qquad \text{Eq. 3}$$

- Wave speed c of a small amplitude solitary wave is
  - Independent of wave amplitude δy
  - Proportional to square root of fluid depth y

- **Fluid density (ρ) is not an important parameter (why ?)**
  - **Wave motion is balance between inertial effects (proportional to ρ) and hydrostatic pressure effects (proportional to ρg)**

Surface Waves: Energy Approach

- Eq. 3 can also be obtained using energy and continuity equations



Stationary simple wave

- The flow is steady for an observer travelling with wave speed c
- The pressure is constant at any point on free surface

Surface Waves: Energy Approach

- **Bernoulli equation for the flow is**

$$\frac{V^2}{2g} + y = C$$

- **On differentiating above equation**

$$\frac{V\delta V}{g} + \delta y = 0$$

Eq. 4a

- **Differentiating Continuity Equation *Vy= constant***

$$y\delta V + V\delta y = 0$$

Eq. 4b

Surface Waves: Energy Approach

- **Combine Eq. 4a and Eq. 4b to get**

$$V = \sqrt{gy}$$

- **Since observer moves with speed c, V = c, We obtain**

$$c = \sqrt{gy}$$

Froude Number Effect: Solitary waves

$$Fr = \frac{V}{\sqrt{gy}} = \frac{V}{c}$$

- Consider Fluid flowing to left with speed V, Waves moves with speed c to the right.
  - Wave will travel to right (upstream) with speed of c-V
  - If V=c , stationary waves, If V>c, waves will be washed to left with speed V-c
  - If c > V ; waves travel upstream: Fr <1; subcritical flow
  - If c < V ; waves do not travel upstream: Fr >1; supercritical flow

Solitary Waves of finite amplitude

- Previous results are restricted to waves of small amplitude.
- For waves of finite sized amplitude δy, the wave speed is given by

$$c \approx \sqrt{gy}\left(1+\frac{\delta y}{y}\right)^{\frac{1}{2}}$$ **Eq. 5**

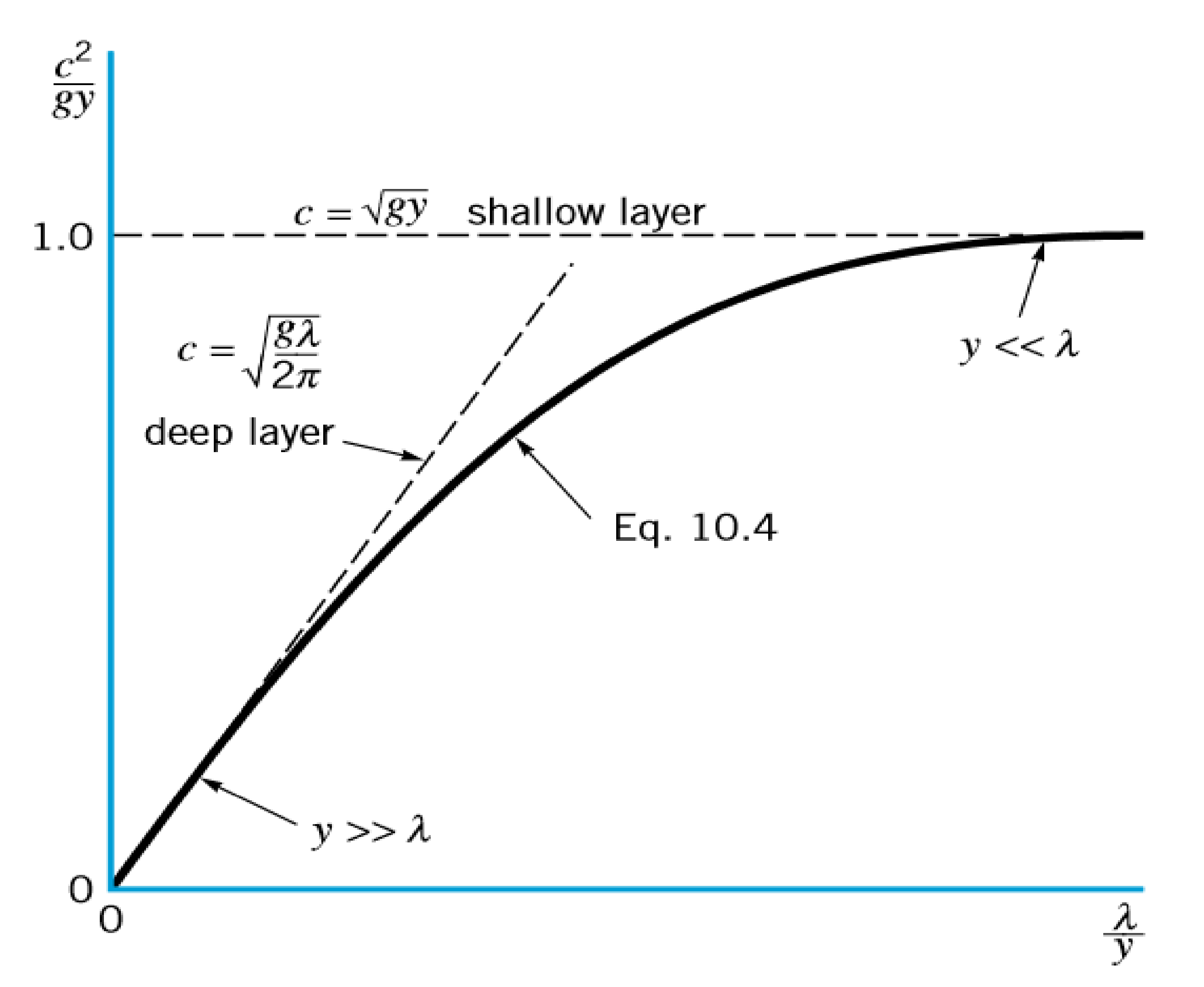
- This implies larger the amplitude, faster the wave travels

Sinusoidal Surface waves

- **Linear wave theory is used to describe waves of small amplitude**
- **Mathematical derivation is outside scope of the course.**

$$c = [\frac{g\lambda}{2\pi}\tanh(\frac{2\pi y}{\lambda})]^{\frac{1}{2}} \qquad \textbf{Eq. 6}$$

  - **λ is the wave length of waves**
  - **Derive shallow water ??**
  - **Derive Deep water equation for waves ??**



Wave speed as a function of wavelength

Questions

1) Determine acceleration due to gravity of a planet where small amplitude waves travel across a 2 m deep pond with speed of 4 m/s. Is the planet more dense than Earth ?

2) A rectangular channel 3 m wide carries 10 m^3/s at depth of 2m. Is the flow sub or supercritical. What shall be critical depth.

3) A trout jumps producing waves on surface of a 0.8 m deep mountain stream. What is the minimum velocity of current if the waves do not travel upstream. [Hint $c = \sqrt{gy}$]

Answers:

1) $V = \sqrt{gy}$ $\quad V^2 = gy \quad g = \frac{V^2}{y} = \frac{4^2}{2} = \frac{16}{2} = 8\ m/s^2$

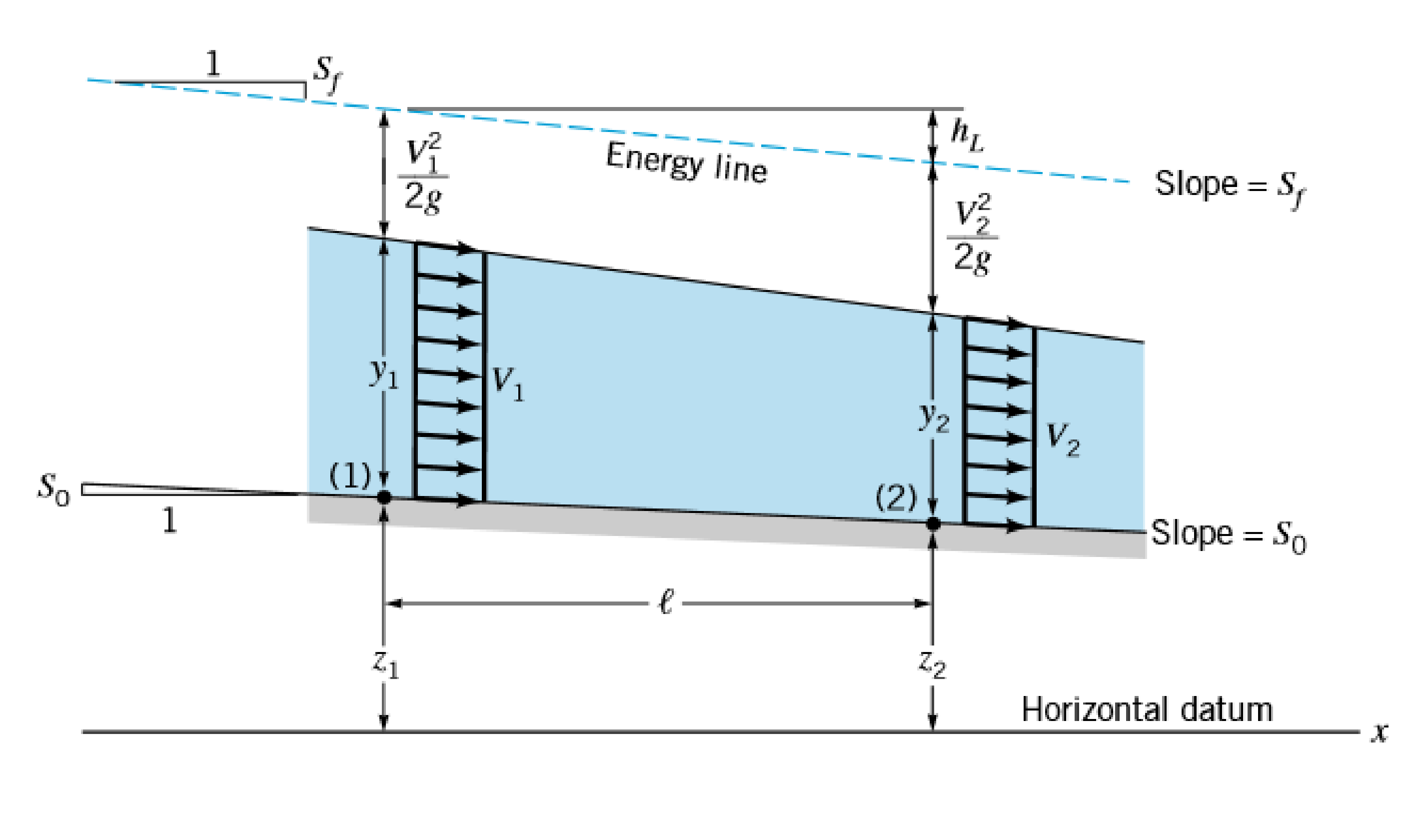
2) $Q = AV$ $\quad V = \frac{Q}{A} = \frac{10}{3 \times 2} = 1.66$ $\quad F_r = \frac{V}{\sqrt{gy}} = \frac{1.66}{\sqrt{9.8 \times 2}} = 0.376$

$F_r < 1$ Flow is sub critical

For critical depth $F_r = \frac{V}{\sqrt{gy}} = 1$ $\quad \frac{1.66}{\sqrt{9.8 \times y}} = 1$ $\quad y = 0.281$ m

3) $c = \sqrt{gy} = \sqrt{9.8 \times 0.8} = 2.8\ m/s$

Energy in Open Channel Flow



Representative Open Channel Geometry

Energy in Open Channel Flow

1) The slope of the channel Bottom $S_0=(z_1-z_2)/l$ is assumed constant over length l. Quite small

2) Fluid depths at two cross section are y_1 and y_2

3) Fluid velocities are V_1 and V_2 at two cross sections

4) Under assumption of uniform velocity profile across any cross section , 1 D energy equation for this flow is

$$\frac{p_1}{\gamma}+\frac{V_1^{\,2}}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^{\,2}}{2g}+z_2+h_L$$

Energy in Open Channel Flow

1) Slope of energy line can be written as $S_f = h_L / l$, referred as friction slope.

2) Under assumption of hydrostatic pressure at any cross section $p_1/\gamma = y_1$ and $p_2/\gamma = y_2$

$$y_1 - y_2 = \frac{(V_2^2 - V_1^2)}{2g} + (S_f - S_0)l$$ **Eq. 7**

or

$$E_1 = E_2 + (S_f - S_0)l$$ **Eq. 8**

where

Specific Energy

$$E = y + \frac{V^2}{2g}$$ **Eq. 9**

Specific Energy

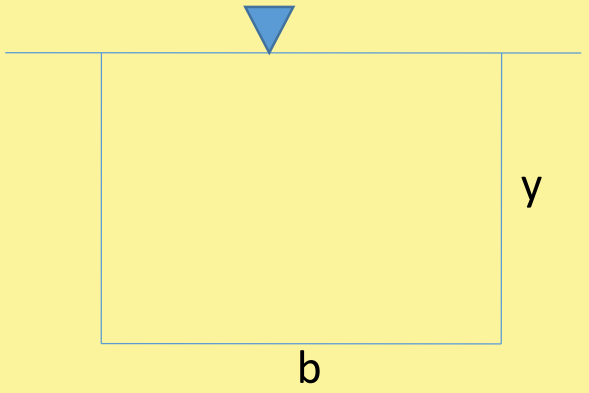
1) **Starting from Eq. 8, if head losses are negligible then $S_f = 0$. Then, $(S_f - S_0)l = -S_0 l = z_2 - z_1$. Eq. 8 reduces to**

$$E_1 + z_1 = E_2 + z_2$$

The sum of specific energy and elevation of channel bottom remains constant

Specific Energy

1) **Consider a rectangular channel with constant width b**

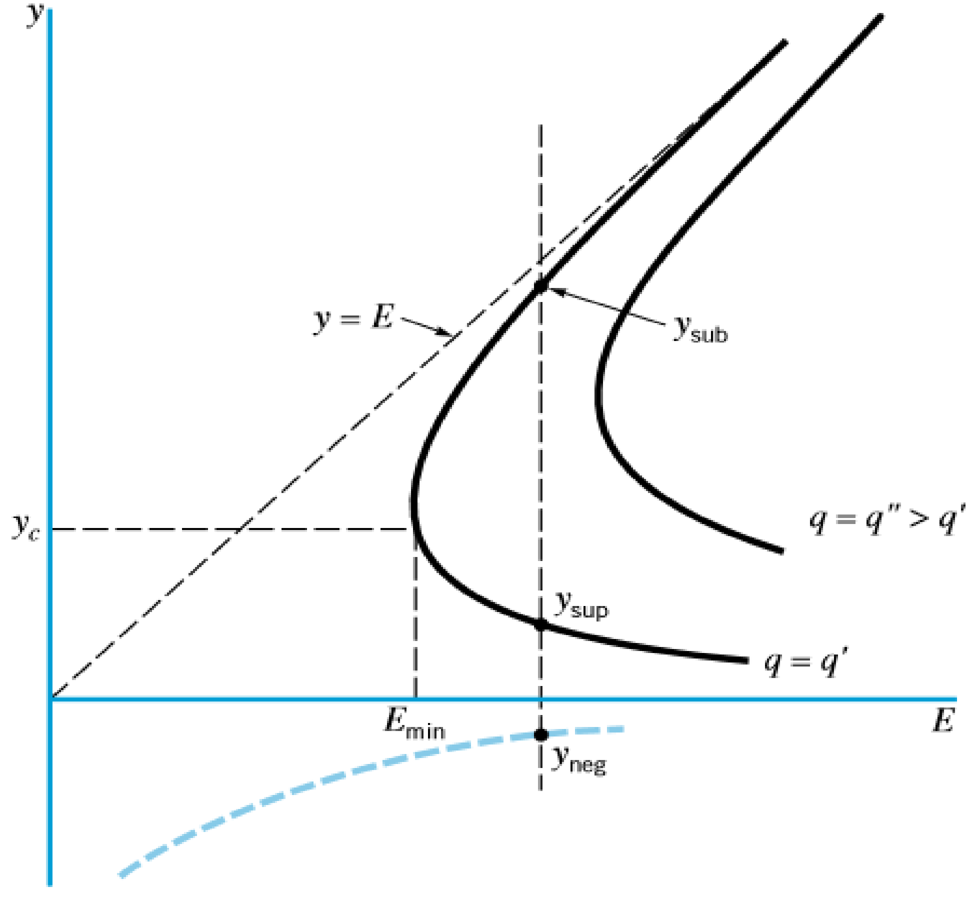


$$q = Q / b = Vyb / b = Vy$$

- **Eq. 9 can be written as**

$$E = y + \frac{q^2}{2gy^2} \qquad \textbf{Eq. 10}$$

Specific Energy



- For a given q and E, Eq. 10 is a cubic equation
- 3 Solutions: y_{sup} , y_{sub}, y_{neg}
- Y_{neg} has no physical meaning
- Y_{sup} and Y_{sub} are termed alternate depths

MCQ

- What does Y=E represent ?
- What does y=0 represent ?
- Prove $y_{sup} < y_{sub}$

MCQ Answers

1) $y=E$ corresponds to very deep channel flowing very slowly as $E=y+V^2/2g \sim y$ as y goes to infinity with $q=Vy$.

2) $y=0$ corresponds to a very high speed flow in a shallow channel as $E=y+V^2/2g \sim V^2/2g$ as y goes to 0.

3) $Y_{sub} > Y_{sup}$ (see figure) implies $V_{sub} < V_{sup}$ as $q=Vy$ is constant.

Class Question

- **In Specific Energy diagram**
  - **Determine y_c in terms of q**
  - **Determine E_{min} in terms of y_c**
  - **Determine V_c**
  - **Determine Fr_c**

*** Note: subscript c denotes critical condition (at E_{min})**

Answer

$$E = y + \frac{q^2}{2gy^2}$$ **Eq. P1**

- To obtain E_{min}, set dE/dy=0

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$ **Eq. P2**

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

- subscript c denotes conditions at E_{min}

Answer

- **Substituting y_c (Eq P2) into Eq P1**

$$E_{\min} = \frac{3y_c}{2}$$

- **Since q=Vy is constant**

$$V_c = \frac{q}{y_c} = \frac{(y^{\frac{3}{2}}{}_c g^{\frac{1}{2}})}{y_c} = \sqrt{gy_c}$$

- **Froude Number Fr_c**

$$Fr_c = \frac{V_c}{\sqrt{gy_c}} = 1$$

Class Question

A rectangular channel has a width of 2 m and carries a discharge of 6 m$^3/s$ at a depth of 0.20 m. Calculate critical depth and Specific energy at critical depth

Solution:

At critical depth, $$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

For a rectangular channel $$A_c = By_c$$

Top Width $$T_c = B$$

Hence $$\frac{Q^2}{g} = \frac{y_c^3 B^3}{B}$$

Or, $$\frac{\frac{Q^2}{B^2}}{g} = \frac{q^2}{g} = y_c^3$$

Where q = discharge intensity = discharge per unit width

$$y_c = (q^2/g)^{1/3}$$

Here q = 6/2 = 3 m^3/s/m

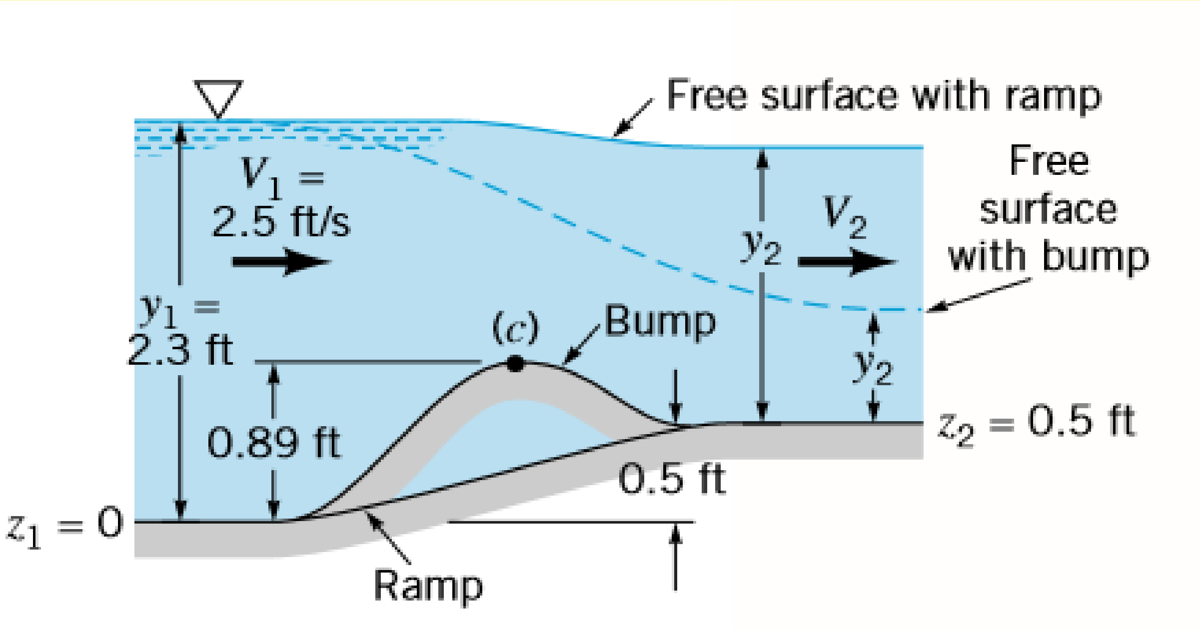
$$y_c = \left(\frac{3^2}{9.8}\right)^{1/3} = 0.972\ m$$

$$V_c = \frac{Q}{By_c} = \frac{6.0}{2 \times 0.972} = 3.087\ m/s$$

$$E_c = Specific\ energy\ at\ critial\ depth = y_c + \frac{{V_c}^2}{2g} = 0.972 + \frac{(3.087)^2}{2 \times 9.81} = 1.458\ \text{m}$$

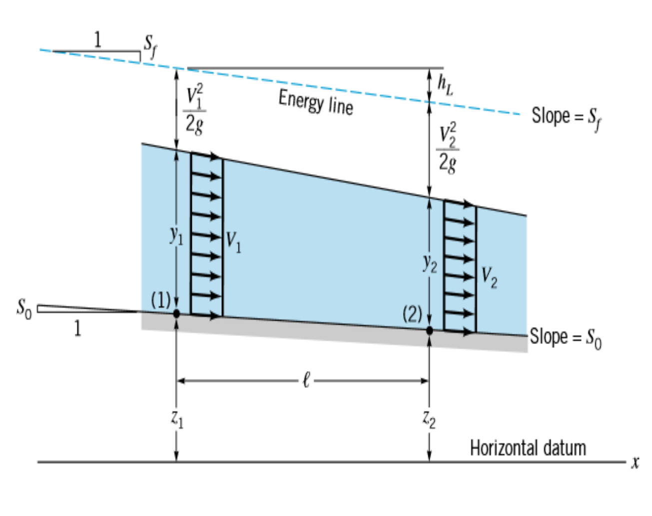
Homework Question

Water flows up a 0.5 ft tall ramp in a constant width rectangular channel at a rate q= 5.75 ft^2/s. If the upstream depth is 2.3 ft, determine the elevation of the water surface downstream of the ramp y_2+z_2, Neglect viscous effects.



Channel Depth variation

- Assumptions: Gradually varying flow (dy/dx <<1)
- The total head H is given by $H = \frac{V^2}{2g} + y + z$ **Eq. 11**
- The energy equation becomes $H_1 = H_2 + h_L$
  - h_L is the head loss between sections 1 and 2
- The slope of energy line is $\frac{dH}{dx} = \frac{dh_L}{dx} = S_f$
- Slope of channel bottom is $\frac{dz}{dx} = S_0$



Channel Depth variation

- **Differentiating Eq. 11 w.r.t x**

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g} + y + z\right) = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

- **Using slope of energy line and bottom slope we obtain**

$$\frac{dh_L}{dx} = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + S_0$$

$$\frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} = S_f - S_0 \qquad \textbf{Eq. 12}$$

Channel Depth variation

- The velocity of flow in rectangular channel of constant width b is given by V=q/y

- Differentiating it wrt x we obtain

$$\frac{dV}{dx} = -\frac{q}{y^2}\frac{dy}{dx} = -\frac{V}{y}\frac{dy}{dx}$$

- Multiplying above equation with V/g we obtain

$$\frac{V}{g}\frac{dV}{dx} = -\frac{V^2}{gy}\frac{dy}{dx} = -F_r{}^2\frac{dy}{dx} \qquad \text{Eq. 13}$$

- Here F_r is the local Froude number of the flow

Channel Depth variation

- **Substituting Eq 13 into Eq 12 we obtain**

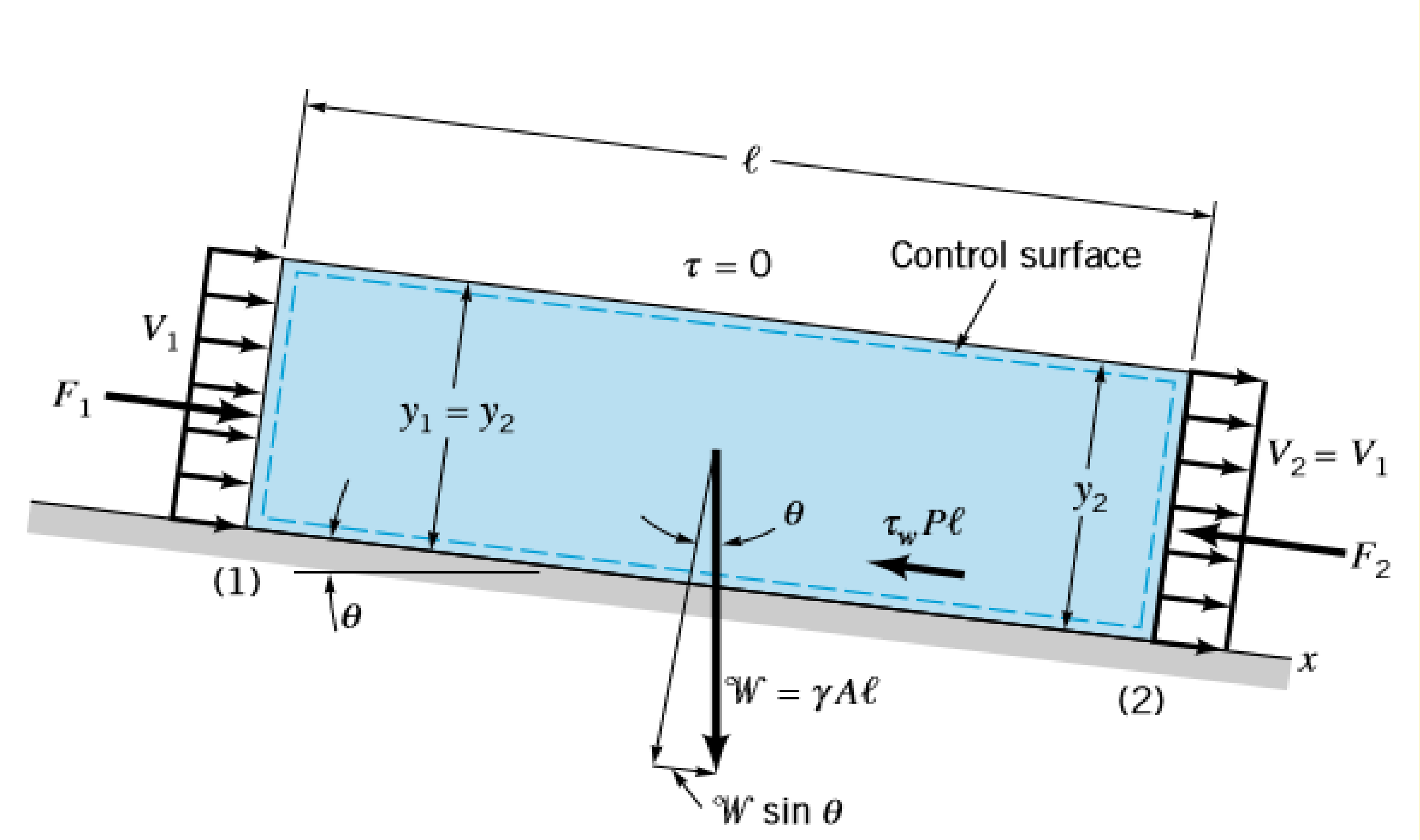
$$\frac{dy}{dx} = \frac{(S_f - S_0)}{(1 - F_r^{\ 2})} \qquad \textbf{Eq. 14}$$

- **Rate of change of fluid depth (dy/dx) depends**
  - **Local slope of channel bottom S_0**
  - **Slope of energy line S_f**
  - **Froude number F_r**
- **The equation is also valid for channels with any constant cross sectional shape**

Uniform Depth Flow

- Several channels are designed to carry fluid at uniform depth along all their length
  - Irrigation Canals ?
  - Rivers ?
  - Creeks ?
- Uniform depth flow means dy/dx =0. Can be made by adjusting bottom slope such that it equals the slope of energy line.
- y corresponding to uniform depth flow is called ‘normal depth’ denoted by y_0

Uniform Depth Flow



Control Volume for uniform flow in an open channel

Uniform Depth Flow

- Applying the x component of momentum equation on the control volume

$$\sum F_x = \rho Q(V_2 - V_1) = 0 \qquad \text{since } V_1 = V_2$$

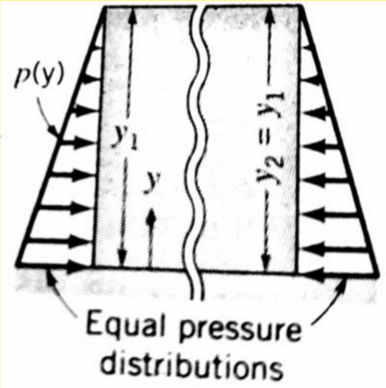
- There is no acceleration of fluid and momentum flux across section 1 is equal to that across section 2.
  - Implies horizontal force balance

$$F_1 - F_2 - \tau_w P l + W \sin\theta = 0 \qquad \text{Eq. 15}$$

Uniform Depth Flow

$$F_1 - F_2 - \tau_w Pl + W\sin\theta = 0 \qquad \textbf{Eq. 15}$$

- Here
  - F_1 and F_2 are hydrostatic pressure forces
  - Wsinϴ is component of fluid weight acting down the slope
  - τ_wPl is the shear force on fluid. This acts up the slope trying to slow down the flow (viscous force)
  - Since y_1=y_2 i.e. flow is at uniform depth F_1=F_2



$$\tau_w = \frac{W\sin\theta}{Pl}$$

Uniform Depth Flow

- Here
  - ϴ is very small. Bottom slope is very small.
    - Therefore sin ϴ ~ tan ϴ ~ S_0

$$\tau_w = \frac{WS_0}{Pl}$$

    - Putting W=ϒAl and Hydraulic Radius R_h=A/P

$$\tau_w = \frac{\gamma A l S_0}{Pl} = \gamma R_h S_0 \qquad \text{Eq. 16}$$

Uniform Depth Flow

- **Open channels flows are mostly Turbulent**
  - **Reynolds number lies fully in turbulent regime**
- **Here, we draw analogy from Pipe flow for turbulent flow**
  - **For very large R_e, friction factor *f* for pipe flows is independent of R_e and dependent only upon relative roughness, ϵ/D**
  - **The wall shear stress is proportional to dynamic pressure $\rho V^2/2$ and independent of the viscosity.**

$$\tau_w = K\rho \frac{V^2}{2}$$

K is a constant
Depends upon pipe roughness

Uniform Depth Flow

- **Assuming similar dependence for high R_e open-channel flows, Eq. 16 can be written as**

$$K\rho \frac{V^2}{2} = \gamma R_h S_0$$

$$V = C\sqrt{R_h S_0} \qquad \textbf{Eq. 17}$$

- ***Constant C is Chezy Coefficient and Eq. 17 is called Chezy Equation***
  - **Developed by French Engineer while designing canal**
  - **C is determined from experiments**
  - **Find the dimension ??**

$$\frac{L^{\frac{1}{2}}}{T}$$

Manning Equation

- From series of experiments it was found by R. Manning that dependence on hydraulic radius R_h is not proportional to $R_h^{0.5}$ but V~ $R_h^{2/3}$
  - He proposed a modified equation for open channel flow

$$V = \frac{{R_h}^{2/3}{S_0}^{1/2}}{n} \qquad \text{Eq. 18}$$

- Eq. 18 is called Manning Equation and parameter *n* is called Manning resistance parameter
  - *n* is obtained from Tables. Precise values are difficult to obtain
  - Rougher the perimeter, larger the value of n

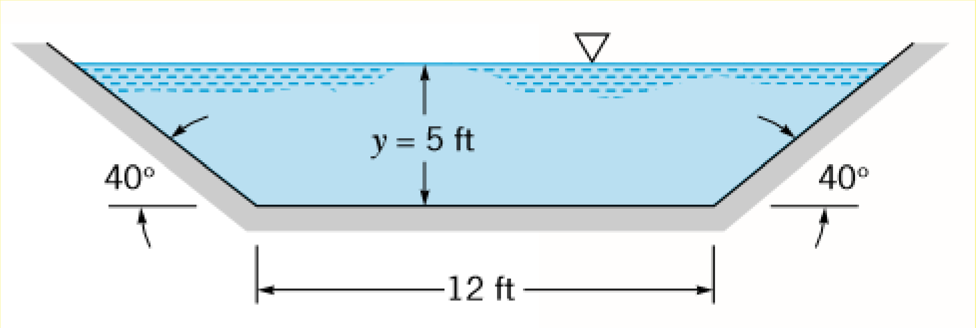
Manning's n Table

Values of the Manning Coefficient, *n* (Ref. 6)

Wetted Perimeter	*n*	Wetted Perimeter	*n*
A. *Natural channels*		D. *Artificially lined channels*	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. *Floodplains*		Steel, painted	0.014
		Steel, riveted	0.015
Pasture, farmland	0.035	Cast iron	0.013
Light brush	0.050	Concrete, finished	0.012
Heavy brush	0.075	Concrete, unfinished	0.014
Trees	0.15	Planed wood	0.012
C. *Excavated earth channels*		Clay tile	0.014
		Brickwork	0.015
Clean	0.022	Asphalt	0.016
Gravelly	0.025	Corrugated metal	0.022
Weedy	0.030	Rubble masonry	0.025
Stony, cobbles	0.035		

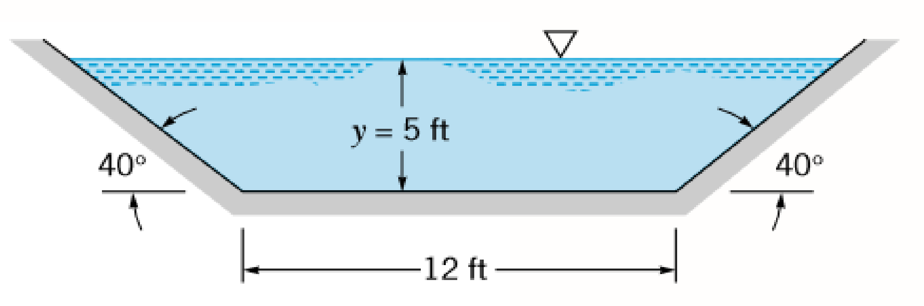
Class Question

- Water flows in the canal of trapezoidal cross section shown in Fig. below. The bottom drops 0.42 m per 304 m of length. The canal is lined with new smooth concrete. Find
  - Area A
  - Wetted Perimeter P
  - Flow rate Q
  - Reynolds number Re
  - Froude Number Fr



Take 5ft = 1.5 m and 12 ft= 3.6 m

Class Question solution

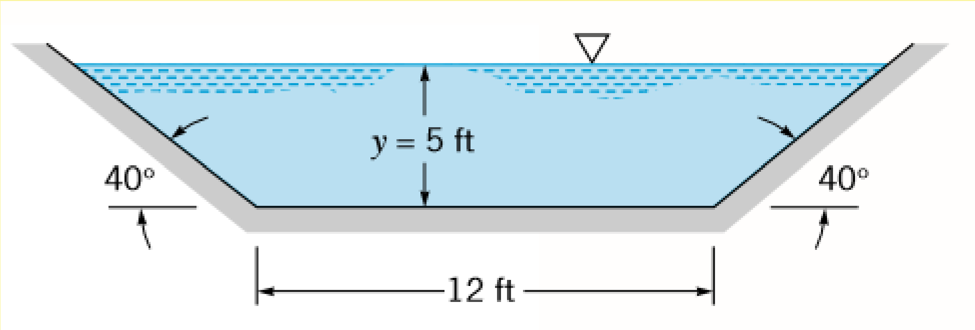


$$A = 3.6 * 1.5 + 1.5 * \left(\frac{1.5}{\tan 40^o}\right) = 8.08 \quad m^2$$

$$P = 3.6 + 2 * (1.5 / \sin 40^o) = 8.26 \quad m$$

$$R = \frac{A}{P} = 0.99 \quad m$$

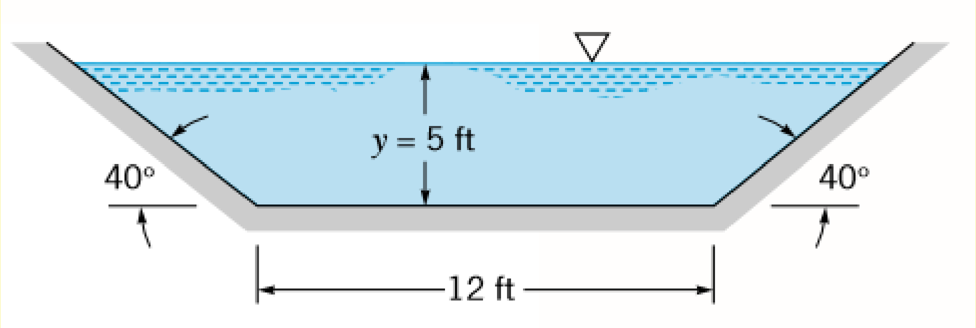
Class Question solution



$$S_o = \frac{0.42}{304} = 0.0014$$

$$Q = \frac{A R_h^{2/3} S_0^{1/2}}{n} = \frac{1}{n}(8.08)(0.99)^{\frac{2}{3}}(0.0014)^{\frac{1}{2}} = \frac{0.30}{n} \quad \text{m}^3/\text{s}$$

Class Question solution

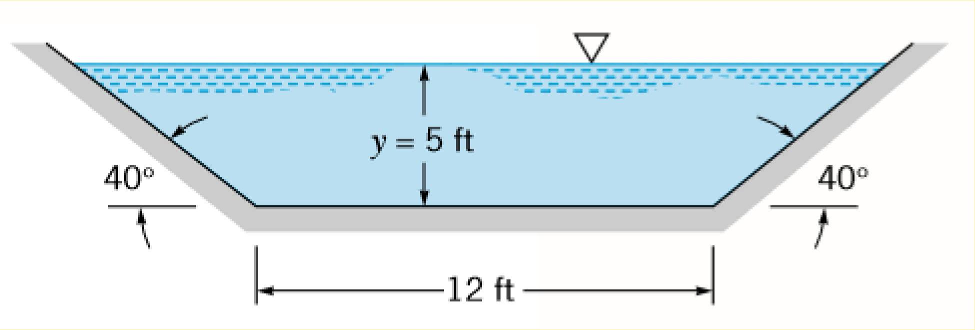


$$n = 0.012$$

For finished concrete, See Table

$$Q = \frac{0.30}{0.012} = 25$$ m³/s

Class Question solution



$$R_e = \frac{R_h V}{\nu} \qquad R_e = \frac{0.99*(25/8.08)}{0.13*10^{-5}} = 2.36*10^6$$

$$F_r = \frac{V}{\sqrt{gy}} \qquad F_r = \frac{3.1}{\sqrt{9.8*1.5}} = 0.808$$

Homework Question

A triangular duct resting on a side carries water with free surface as shown in the fig. Obtain the condition for maximum discharge in this channel when (a) m=0.5 and (m) = 1.0.