Pipe Flow

Hydraulics

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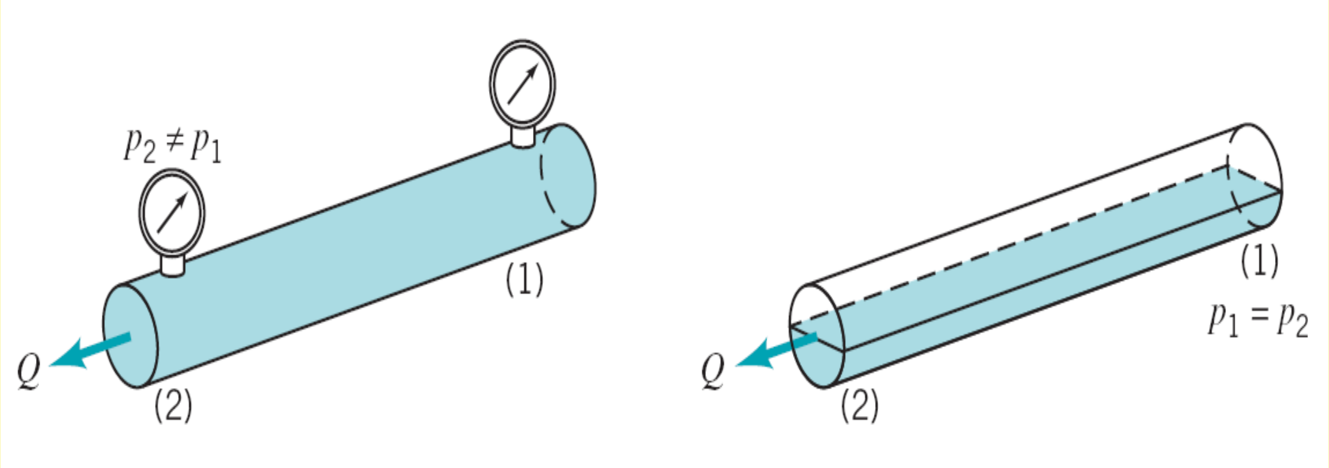
Viscous Flow in pipes

- Pipe is completely filled with water
- Main driving force is usually a pressure gradient along the pipe, though gravity might be important as well



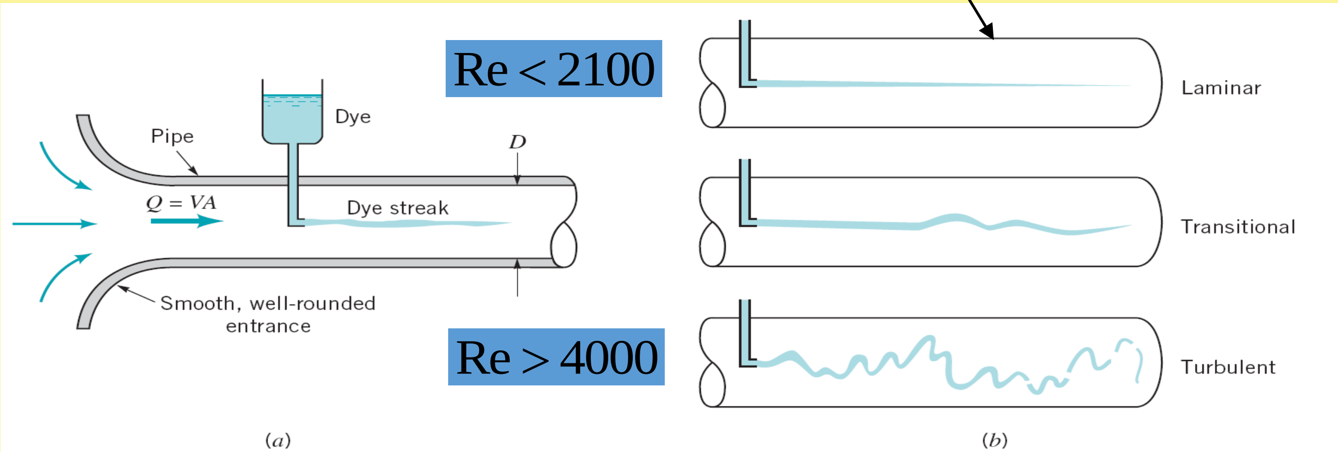
Pipe flow

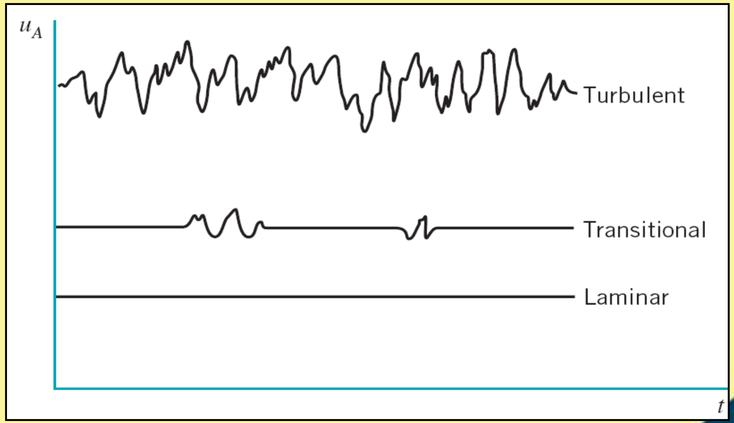
open-channel flow

Laminar or turbulent flow

well defined streakline, one velocity component

$$\mathbf{V} = u\hat{\mathbf{i}}$$





velocity along the pipe is unsteady and accompanied by random component normal to pipe axis

$$\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$$

Laminar or turbulent flow

https://www.youtube.com/watch?v=XOLl2KeDiOg

Laminar or turbulent flow

- In this experiment water flows through a clear pipe with increasing speed. Dye is injected through a small diameter tube at the left portion of the screen. Initially, at low speed (Re <2100) the flow is laminar and the dye stream is stationary. As the speed (Re) increases, the transitional regime occurs and the dye stream becomes wavy (unsteady, oscillatory laminar flow). At still higher speeds (Re>4000) the flow becomes turbulent and the dye stream is dispersed randomly throughout the flow.

Class Question

- **Water at a temperature of 10°C flows through a pipe of diameter D=1.85 cm. Determine the minimum time taken to fill a 0.355 L glass with water if the flow in the pipe is to be laminar. Determine the maximum time taken to fill the glass if the flow is to be turbulent. Repeat the calculations if the water temperature is 60°C.**

Hint :

$$Re = \frac{\rho V D}{\mu}$$

At 10°C:

$\rho = 1000$ Kg/m³

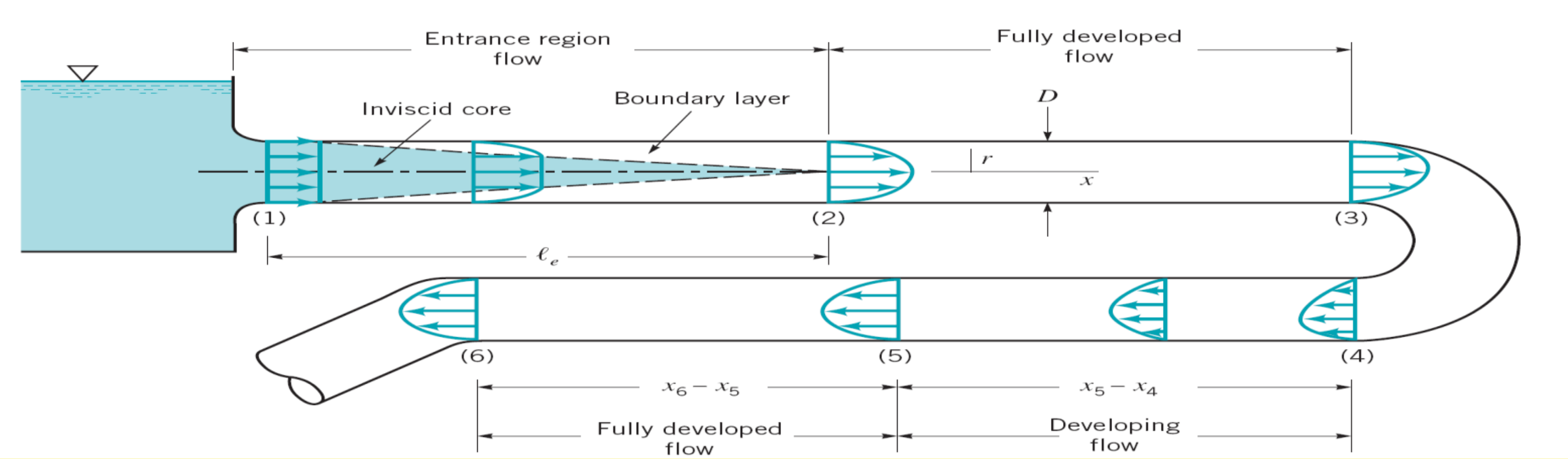
$\mu = 1.307 * 10^{-3}$ N.s/m²

At 60°C:

$\rho = 983.2$ Kg/m³

$\mu = 4.665 * 10^{-4}$ N.s/m²

Entrance region and fully developed flow



- **fluid typically enters pipe with nearly uniform velocity**
- **the length of entrance region depends on the Reynolds number**

dimensionless entrance length

$$\frac{l_e}{D} = 0.06\,\text{Re} \qquad \text{for laminar flow}$$

$$\frac{l_e}{D} = 4.4(\text{Re})^{1/6} \qquad \text{for turbulent flow}$$

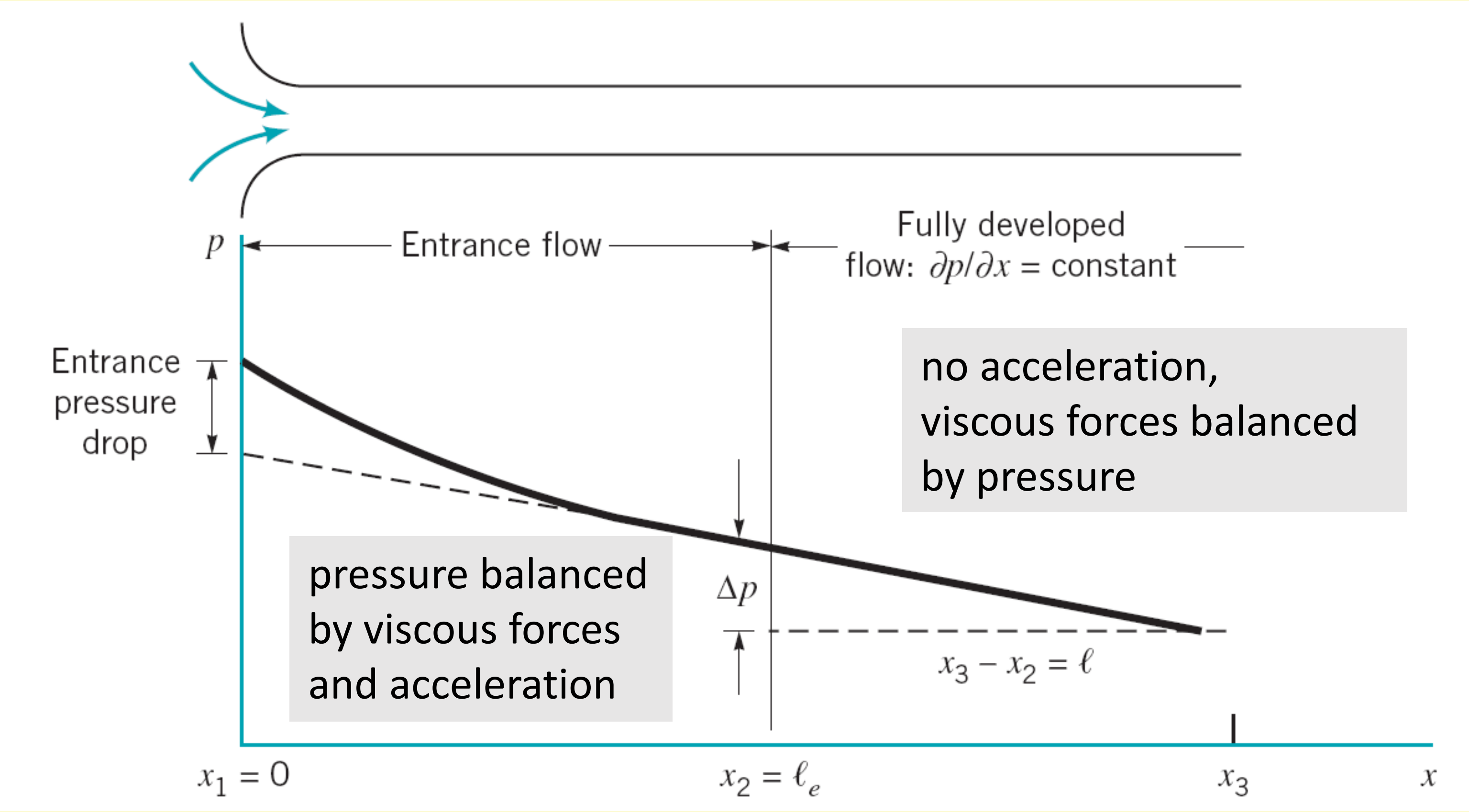
Entrance region and fully developed flow

- As the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall (the no-slip boundary condition).
- The boundary layer grows in thickness to completely fill the pipe.
- Viscous effects are of considerable importance within the boundary layer.
- For fluid outside the boundary layer [within the inviscid core surrounding the centerline from 1 to 2], viscous effects are negligible.
- Calculation of velocity profile and pressure distribution within entrance region is very complex.

Entrance region and fully developed flow

- As soon as the flow reaches the end of entrance region
  - Flow is simpler
  - Velocity dependent upon radial distance r
  - Velocity independent of x
  - Flow between section 2 and 3 is called *fully developed flow*

Pressure and shear stress



Pressure and shear stress

- The need of the pressure drop can be seen as
  - Force Balance; Pressure force is needed to overcome the viscous forces generated
  - Energy Balance; Work done by pressure forces is needed to overcome the viscous dissipation throughout the fluid

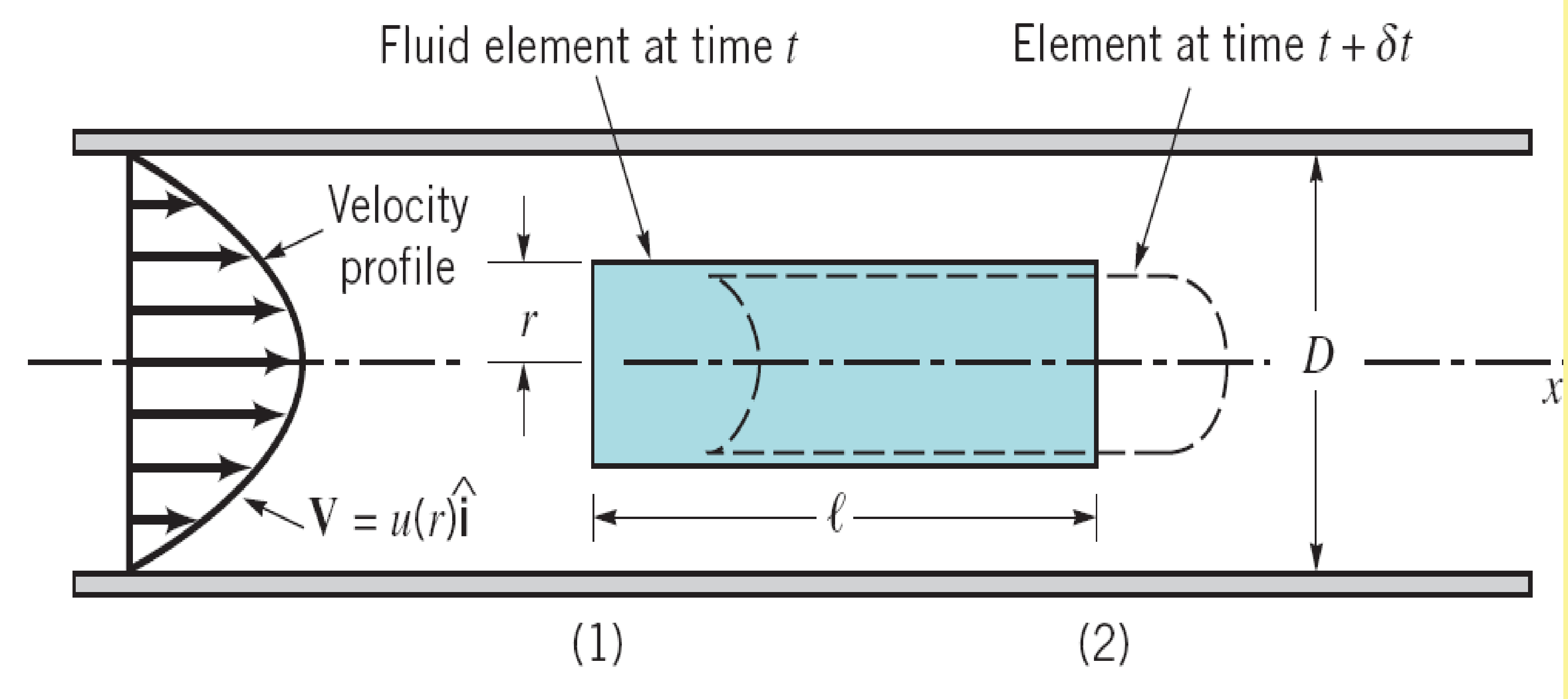
Fully developed laminar flow

- Problems
  - Most flows are turbulent
    - Theoretical analysis is yet not possible
  - Many pipes are not long enough to allow attainment of fully developed flow
- Importance
  - One of the very few theoretical viscous analysis that can be carried out ‘exactly’
  - Provides a foundation for further complex analysis
  - There are many practical situations involving the use of fully developed laminar pipe flow

Fully developed laminar flow

- Equation for fully developed laminar flow in pipe can be derived using 3 approaches:
  - from 2nd Newton law directly applied
  - from Navier-Stokes equation
  - from dimensional analysis

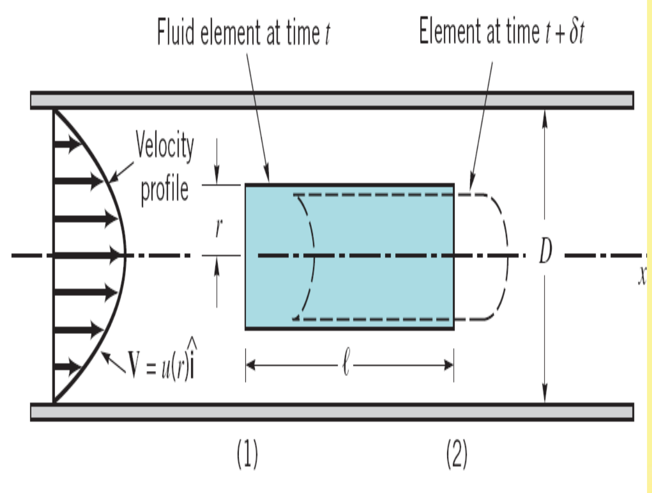
Newton's 2nd law



fluid element at time t

Newton's 2^{nd} law

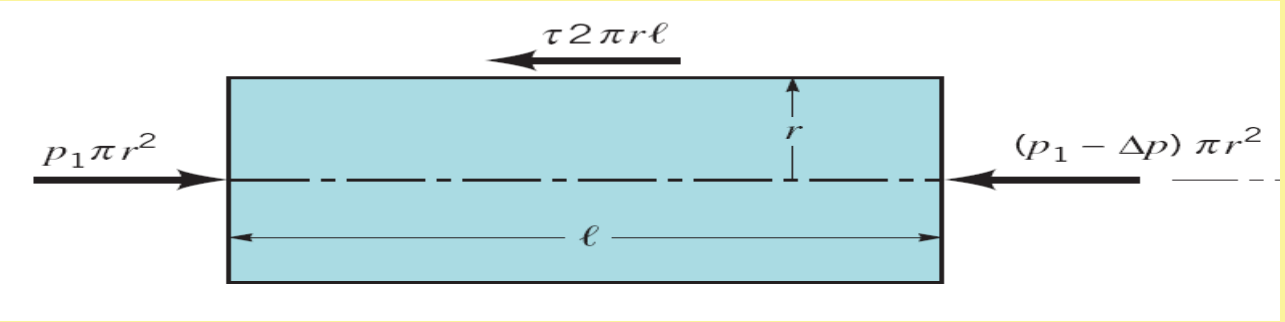
- We consider the fluid element at time t as is shown in Fig above. It is a circular cylinder of fluid of length and radius r centered on the axis of a horizontal pipe of diameter D. Because the velocity is not uniform across the pipe, the initially flat ends of the cylinder of fluid at time t become distorted at time when the fluid element has moved to its new location along the pipe as shown in the figure. If the flow is fully developed and steady, the distortion on each end of the fluid element is the same, and no part of the fluid experiences any acceleration as it flows.



Newton's 2nd law

- Assumptions
  - Local acceleration is zero since the flow is steady
  - Convective acceleration is zero since the flow is fully developed
  - Every fluid particle flows along streamline with constant velocity. The neighboring particle have slightly different velocities
  - Gravitational effects are neglected for now
  - Pressure is constant across any vertical cross section of the pipe
  - Pressure drop $\Delta p>0 \rightarrow$ pressure decreases in direction of flow

Newton's 2nd law



$$p_1\pi r^2-(p_1-\Delta p)\pi r^2-\tau 2\pi r l=0$$

$$\frac{\Delta p}{l}=\frac{2\tau}{r}$$

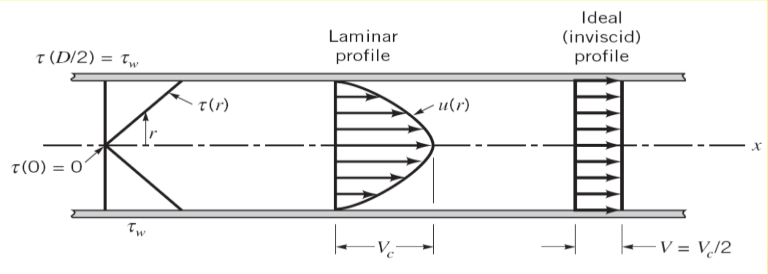
$\tau = Cr$, at $r = D/2$ stress is maximum τ_w **wall shear stress**

$$\tau=\frac{2\tau_w r}{D} \text{ and } \Delta p=\frac{4l\tau_w}{D}$$ ← **Eq. 3**

doesn't depend on radius

Eq. 1

Eq. 2



Newton's 2nd law

- Discussions
  - Shear stress varies linearly with r (Why ??)
  - If viscosity was zero → no shear stress and pressure constant throughout channel
  - A small shear stress can produce large Δp if pipe is relatively long (l/D>>1) (See Equation)
  - Analysis till now is valid for both laminar and turbulent flow (assumptions are common)
  - From here onward we assume shear stress distribution for laminar flow

Newton's 2nd law

for Newtonian liquid: $\tau = -\mu \dfrac{du}{dr}$ $\tau = \left(\dfrac{\Delta p}{2l}\right) r$

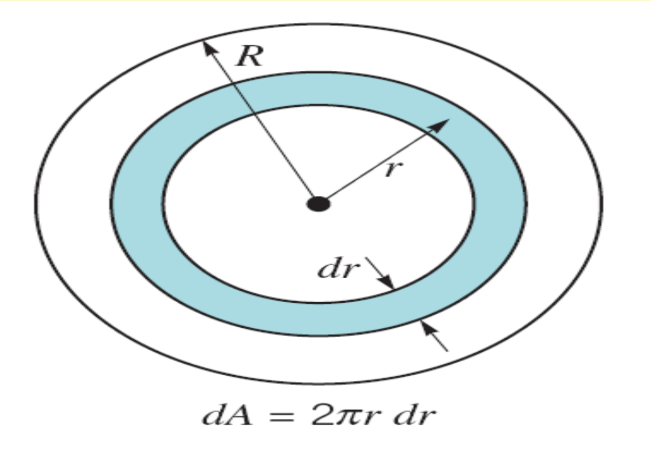
$$\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu l}\right) r$$

$$u = -\left(\frac{\Delta p}{4\mu l}\right) r^2 + C_1$$

boundary condition: $u = 0$ at $r = D/2 \Rightarrow C_1 = \left(\dfrac{\Delta p}{16\mu l}\right) D^2$

$$u(r) = \left(\frac{\Delta p D^2}{16\mu l}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

Newton's 2nd law

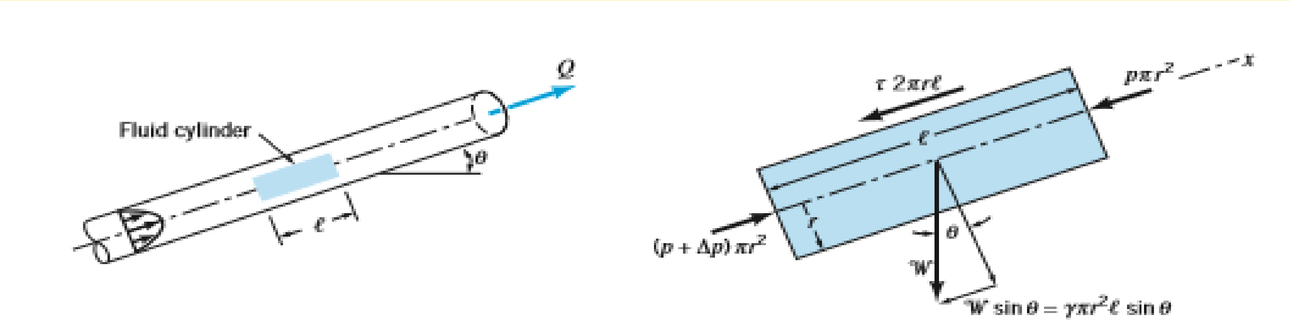


Flow rate: $$Q = \int u dA = \int_0^{D/2} u(r) 2\pi r dr = \frac{\pi D^4 \Delta p}{128 \mu l}$$ **Eq. 4**

Poiseuille's Law

Newton's 2nd law

- if gravity is present, it can be added to the pressure:



$$\frac{\Delta p - \rho g l \sin\theta}{l} = \frac{2\tau}{r}$$ **Eq. 5**

$$V = \frac{(\Delta p - \rho g l \sin\theta) D^2}{32 \mu l}$$ **Eq. 6**

$$Q = \frac{\pi (\Delta p - \rho g l \sin\theta) D^4}{128 \mu l}$$ **Eq. 7**

Basic mathematics

- **Del Operator:**

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- **Laplacian Operator:**

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- **Gradient:**

$$\nabla p = \left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right)$$

- Vector Gradient:

$$\nabla \mathbf{u} = (\nabla u, \nabla v, \nabla w)$$

- Divergence:

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

- Directional Derivative:

$$\mathbf{u} \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Navier-Stokes equation applied

- **General motion of an incompressible Newtonian fluid is governed by the continuity equation conservation of mass, is written as :**

$$\nabla \cdot \mathbf{u} = 0$$

- **Momentum equation:** $\frac{\partial V}{\partial t} + V\,.\nabla V = -\frac{\nabla p}{\rho} + g + \nu\,\nabla^2 V$ **Eq. 8**

For steady, fully developed flow in a pipe, the velocity contains only an axial component, which is a function of only the radial coordinate [V=u(r)Î] For such conditions, the left-hand side of the momentum Eqn. becomes zero. This is equivalent to saying that the fluid experiences no acceleration as it flows along. The same constraint was used in the previous section when considering F = ma for the fluid cylinder.

Thus, with g = -g$\widehat{k}$ the Navier Stokes equations become

$$\nabla V = 0$$

$$\nabla p + \rho g\hat{k} = \mu \nabla^2 V$$

The flow is governed by a balance of pressure, weight, and viscous forces in the flow direction.

In cylindrical coordinates:

$$\frac{\partial p}{\partial x} + \rho g \sin\theta = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

The assumptions and the result are exactly the same as Navier-Stokes equation is drawn from 2nd Newton law

Dimensional analysis

$$\Delta p = F\left(V, l, D, \mu\right)$$

$$\frac{D\Delta p}{\mu V} = \phi\left(\frac{l}{D}\right)$$

assuming pressure drop proportional to the length:

$$\frac{D\Delta p}{\mu V} = \frac{Cl}{D} \qquad \Rightarrow \qquad \frac{\Delta p}{l} = \frac{C\mu V}{D^2}$$

$$Q = AV = \frac{(\pi / 4C)\Delta p D^4}{\mu l}$$

Eq. 9

Darcy friction factor (f)

- **Rewriting Poiseuille's law**

$$Q = \int u\,dA = \int_0^{D/2} u(r)2\pi r\,dr = \frac{\pi D^4 \Delta p}{128\mu l}$$

$$\Delta p = \frac{32\mu l V}{D^2}$$

Dividing both sides by dynamic pressure

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\frac{32\mu l V}{D^2}}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{l}{D}\right) = \frac{64}{\mathrm{Re}}\left(\frac{l}{D}\right)$$

Darcy friction factor (f)

- Often written as

$$\Delta p = f\left(\frac{l}{D}\right)\frac{\rho V^2}{2} \qquad \text{Eq. 10}$$

Where f is called Darcy friction factor.

f for laminar fully developed pipe flow is given by

$$f = \frac{64}{\text{Re}} \qquad \text{Eq. 11}$$

In terms of wall shear stress Using Eq. 3

$$f = \frac{8\tau_w}{\rho V^2} \qquad \text{Eq. 12}$$

Energy in fully developed Laminar flow

- Consider energy flow between two locations

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^{\,2}}{2g}+z_1=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^{\,2}}{2g}+z_2+h_L \qquad \textbf{Eq. 13}$$

- For uniform velocity profile α=1 (α>1 for non-uniform profile)
  - For fully developed flow $\alpha_1 = \alpha_2 = 1$
- h_L accounts for energy loss associated with the flow
  - Viscous dissipation here. For inviscid flow h_L = ?

Energy in fully developed Laminar flow

- **Rewriting Eq. 13**

$$\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)=h_L$$ **Eq. 14**

$$\frac{p_1-p_2}{\gamma}=\frac{\Delta p}{\gamma}$$

$$z_2-z_1=l\sin\theta$$

$$\frac{\Delta p}{\gamma}-l\sin\theta=\frac{\Delta p-\gamma l\sin\theta}{\gamma}=h_L$$

Use Eq. 5

$$\frac{\frac{2l\tau}{r}}{\gamma}=h_L$$

$$\frac{2l\tau}{\gamma r}=h_L$$ **Eq. 15**

Use Eq. 2

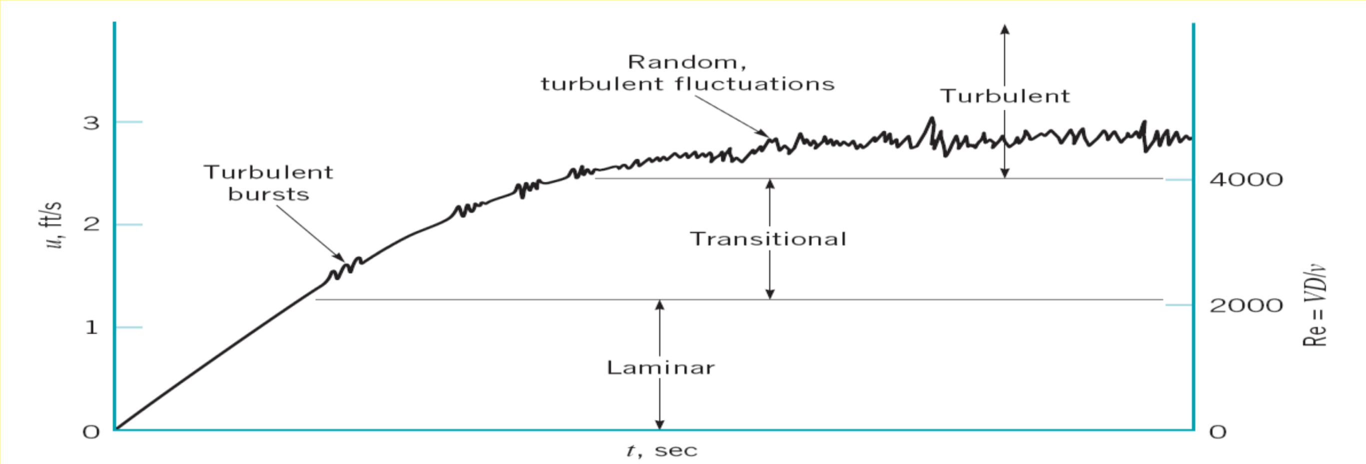
$$h_L=\frac{4l\tau_w}{\gamma D}$$ **Eq. 16**

Energy in fully developed Laminar flow

$$h_L = \frac{4l\tau_w}{\gamma D} \qquad \text{Eq. 16}$$

- **The above equation is valid for both laminar and turbulent fluid flow**

Turbulent flow

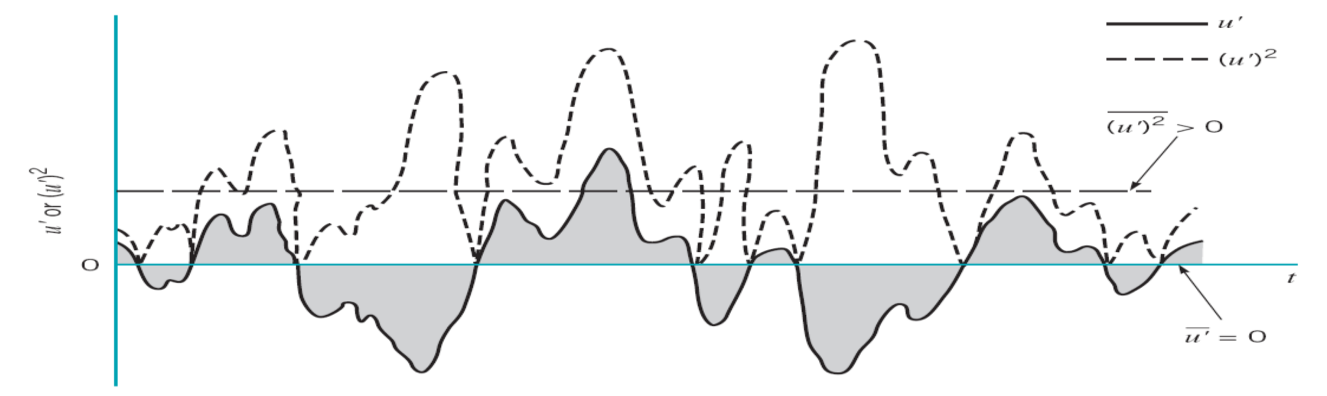


- **In turbulent flow the axial component of velocity fluctuates randomly, components perpendicular to the flow axis appear**
- **heat and mass transfer are enhanced in turbulent flow**
- **In many cases reasonable results on turbulent flow can be obtained using Bernoulli equation (Re=inf).**

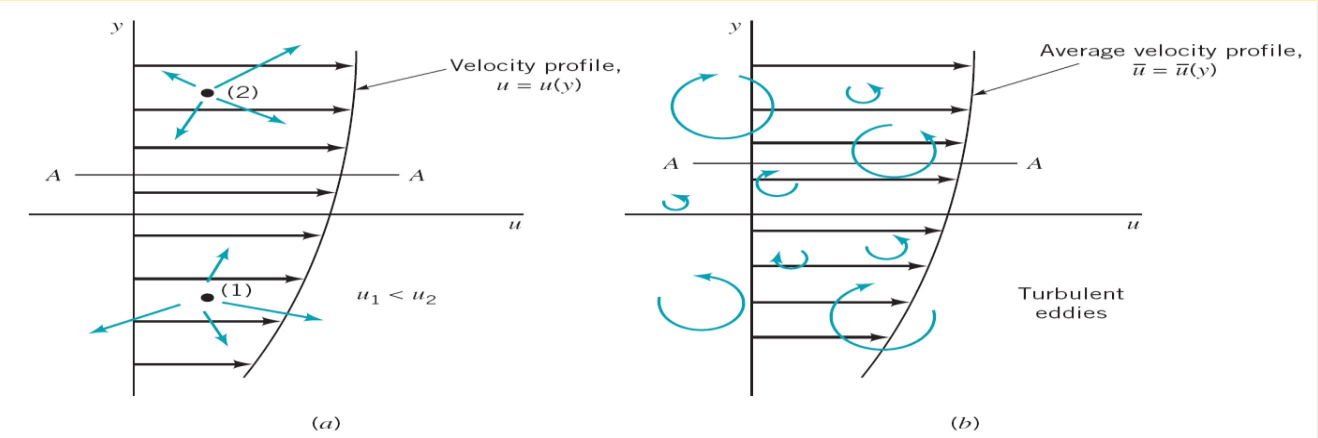
Fluctuation in turbulent flow

- All parameters fluctuate in turbulent flow (velocity, pressure, shear stress, temperature etc.) behave chaotically
- flow parameters can be described as an average value + fluctuations (random vortices)
- can be characterized by turbulence intensity and time scale of fluctuation

turbulence intensity $$T = \frac{\sqrt{\overline{(u')}^2}}{\overline{u}} = \frac{\left(\frac{1}{T}\int_{t_0}^{t_0+T}(u')^2\,dt\right)^2}{\overline{u}}$$



Shear stress in turbulent flow



- **Turbulent flow can often be thought of as a series of random, 3-dimensional eddy motions (swirls) ranging from large eddies down through very small eddies**
- **Vortices transfer momentum, so the shear force is higher compared with laminar flow:**

$$\tau = \tau_{lam} + \tau_{turb} = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'}$$

Shear stress in turbulent flow

- **Shear stress is a sum of laminar portion and a turbulent portion**

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_{lam} + \tau_{turb}, \quad u' = u - \bar{u}$$

positive

<u>shear stress is larger in turbulent flow</u>

- **Alternatively:**

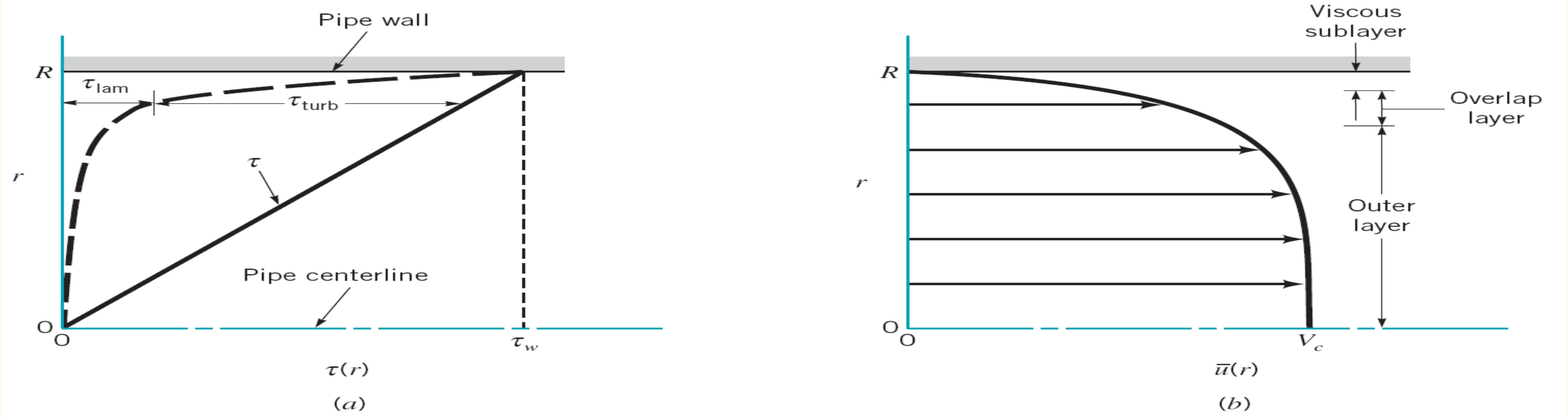
$$\tau_{turb} = \eta \frac{d\bar{u}}{dy}$$

η = eddy viscosity

Prandtl suggested that turbulent flow is characterized by random transfer over certain distance *l_m*:

$$\eta = \rho l_m^{\,2} \left| \frac{d\bar{u}}{dy} \right| \;\Rightarrow\; \tau_{turb} = \rho l_m^{\,2} \left(\frac{d\bar{u}}{dy} \right)^2$$

Turbulent velocity profile



$$\tau_{turb} = 100 - 1000\tau_{lam}$$

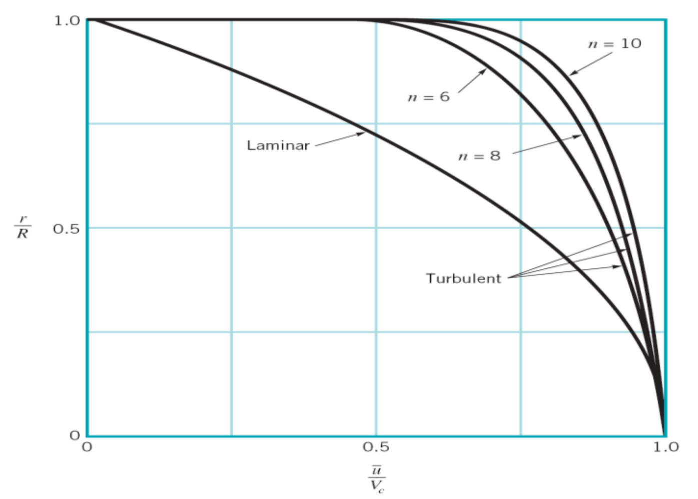
- **Viscous sub layer: Viscous shear stress dominates, viscosity is dominant and density unimportant**
- **Outer Layer: Reynolds stress in dominant viscosity is unimportant and density dominant**

Turbulent velocity profile

- **In the viscous sublayer**

Law of the wall

$$\frac{\bar{u}}{u*} = \frac{yu*}{\nu}$$



where, y=R-r, u – time averaged x component, u*=(τ/ρ)½ friction velocity

valid near smooth wall: $0 \le yu*/\nu \le 5$

$$y = \delta_{sublayer} = \frac{5\nu}{u^*}$$

Thickness of viscous sublayer

- **In the overlap layer: Log law**

$$\frac{\bar{u}}{u^*} = 2.5\ln\left(\frac{yu^*}{\nu}\right) + 5.0$$

Coefficients have been obtained experimentally

Turbulent velocity profile

- **In the turbulent layer:**

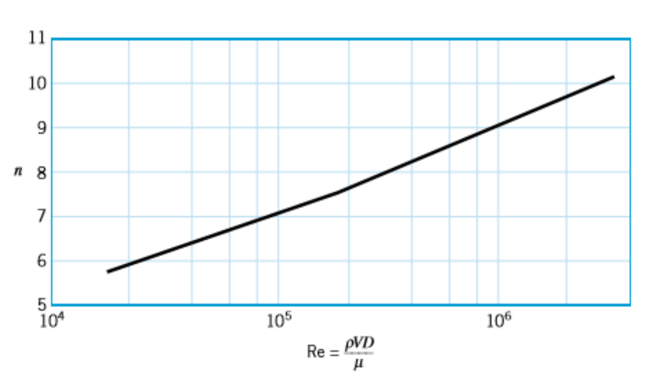
$$(V_c - \bar{u})/u^* = 2.5\ln(R/y) \qquad or \qquad \frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

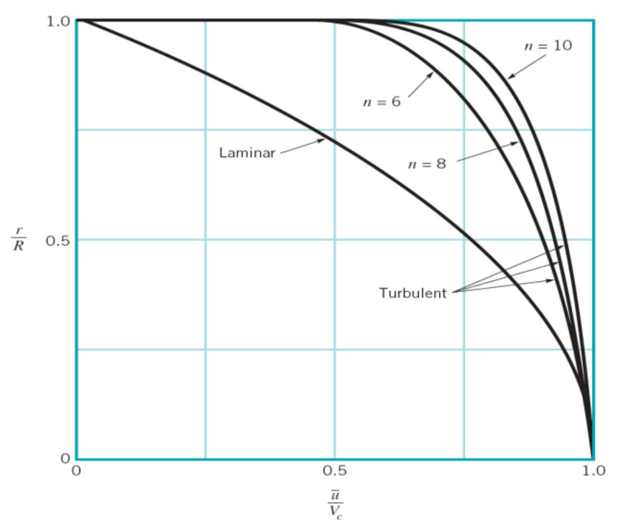
Velocity Defect Law in outer layer

function of Reynolds number

Power law velocity profile

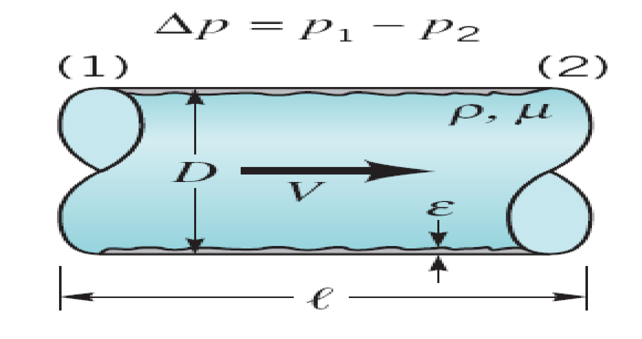
Velocity defect or retardation of the flow due to wall effects.





Dimensional analysis of pipe flow

- **major loss in pipes: due to viscous flow in the straight elements**
- **minor loss: due to other pipe components (junctions etc.)**



Major loss:

$$\Delta p = F(V, D, l, \varepsilon, \mu, \rho)$$

roughness

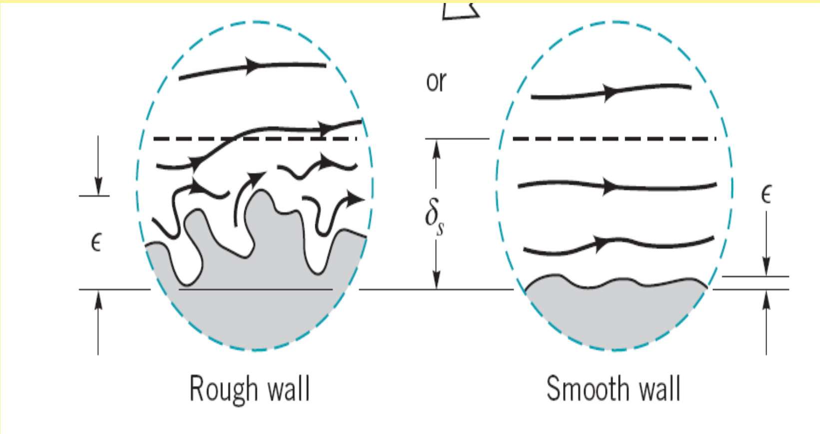
- **those 7 variables represent complete set of parameters for the problem**

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{l}{D}, \frac{\varepsilon}{D}\right)$$

Dimensional analysis of pipe flow

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{l}{D}, \frac{\varepsilon}{D}\right)$$

as pressure drop is proportional to length of the tube:



$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{l}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

Dimensional analysis of pipe flow

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{l}{D}\phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

$$f = \frac{\Delta p D}{\frac{1}{2} l \rho V^2}$$ ← **friction factor**

$$f = \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right) \quad \text{and} \quad \Delta p = f\frac{l}{D}\frac{\rho V^2}{2}$$

Valid for horizontal pipes

- **for fully developed laminar flow**

$$f = 64/\mathrm{Re}$$

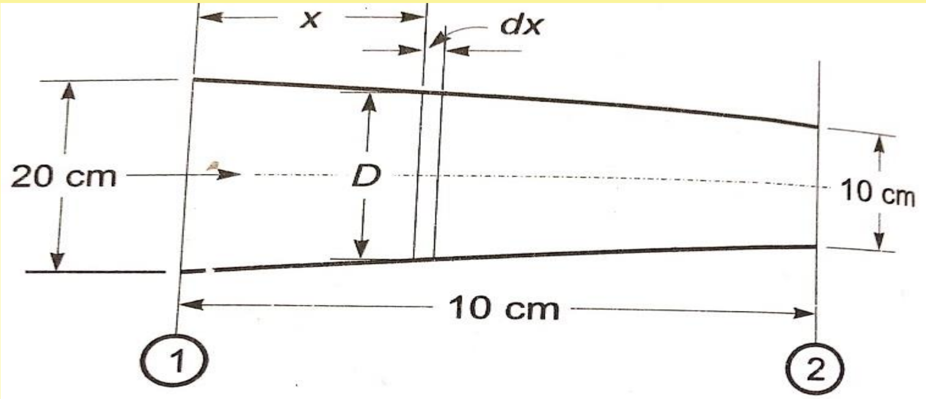
- **for fully developed steady incompressible flow (from Bernoulli eq.):**

$$h_{L\,major} = \frac{\Delta p}{\rho g} = f\frac{l}{D}\frac{V^2}{2g}$$

Darcy-Weisbach equation

Class Problem

- **Water flows through a pipeline whose diameter varies from 20 cm to 10 cm in a length of 10m. If Darcy-Weisbach friction factor is assumed to be constant at 0.02 for the whole pipe, determine the head loss in friction when the pipe is flowing full with a discharge of 50 L/s.**



Equivalent roughness for pipes

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

Pipe	Equivalent Roughness, ε Feet	Equivalent Roughness, ε Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

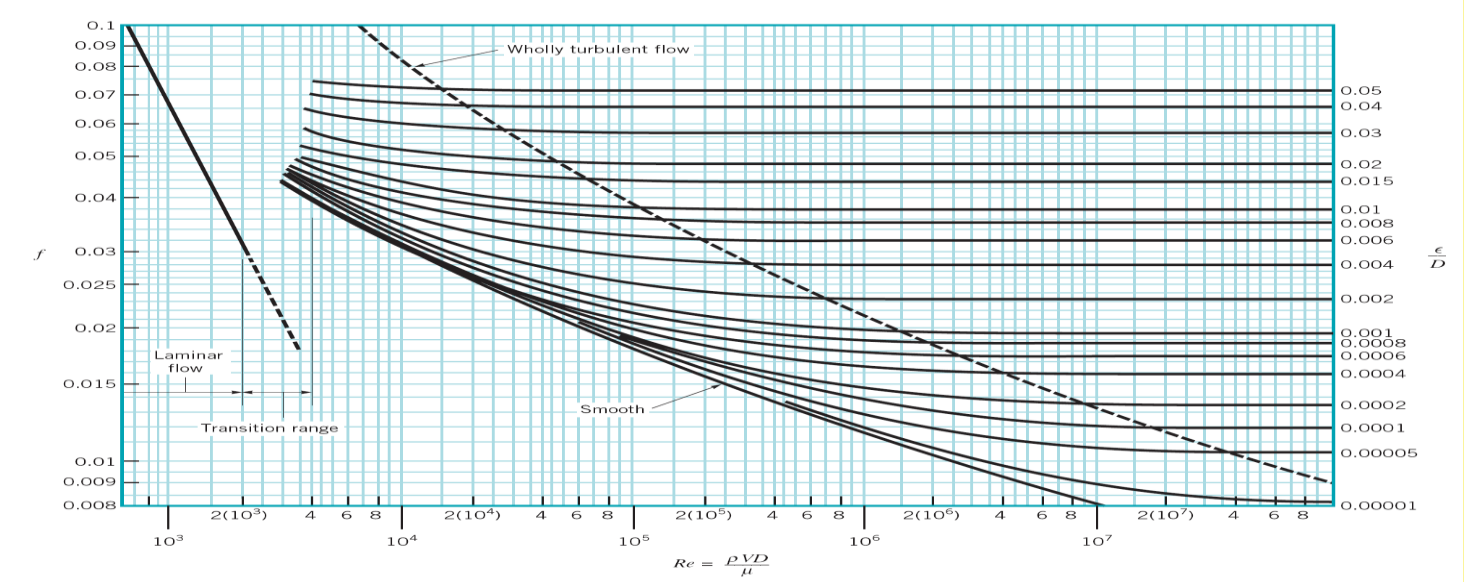
Class Problem

- A badly corroded concrete pipe of diameter 1.5 m has an equivalent sand roughness of $\epsilon_s = 15mm$. A 10 mm thick lining is proposed to reduce the roughness value to $\epsilon_s = 0.2\ mm$ .For a discharge of 4.0 m^3/s in the pipe calculate the power saved per kilometer of pipe.

  Take $v = 1x\ 10^{-6} m^2/s$

Moody chart

Friction factor as a function of Reynolds number and relative roughness for round pipes



Colebrook formula; implicit

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Haaland equation: explicit

$$\frac{1}{\sqrt{f}} = -1.8\log[(\frac{\varepsilon/D}{3.7})^{1.11} + \frac{6.9}{\mathrm{Re}}]$$

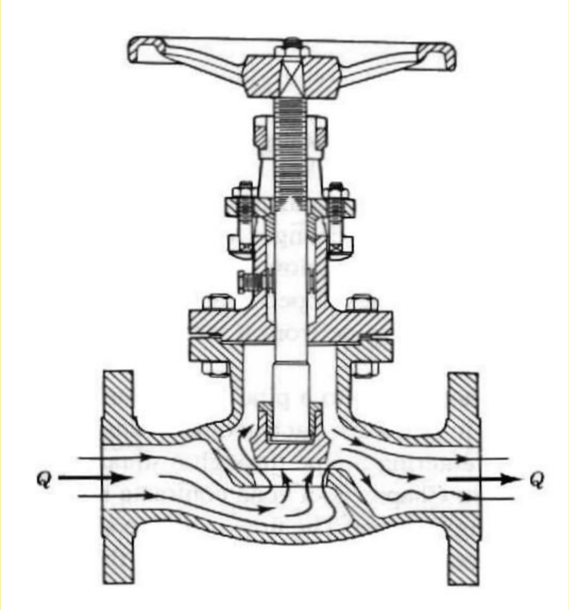
Class Problem

- Air under standard conditions flows through a 4.0 mm diameter drawn tubing with an average velocity V=50 m/s. of For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow.

  - Determine the pressure drop in a 0.1 m section of the tube if the flow is laminar.
  - Repeat the calculations if the flow is turbulent.

Minor losses

It is due to the change of the velocity of the flowing fluid in the magnitude or in direction [turbulence within bulk flow as it moves through and fitting]



Flow pattern through a valve

The minor losses occurs due to

- **Valves**
- **Tees**
- **Bends**
- **Reducers**
- **And other appurtenances**

Minor losses

- It has the common form

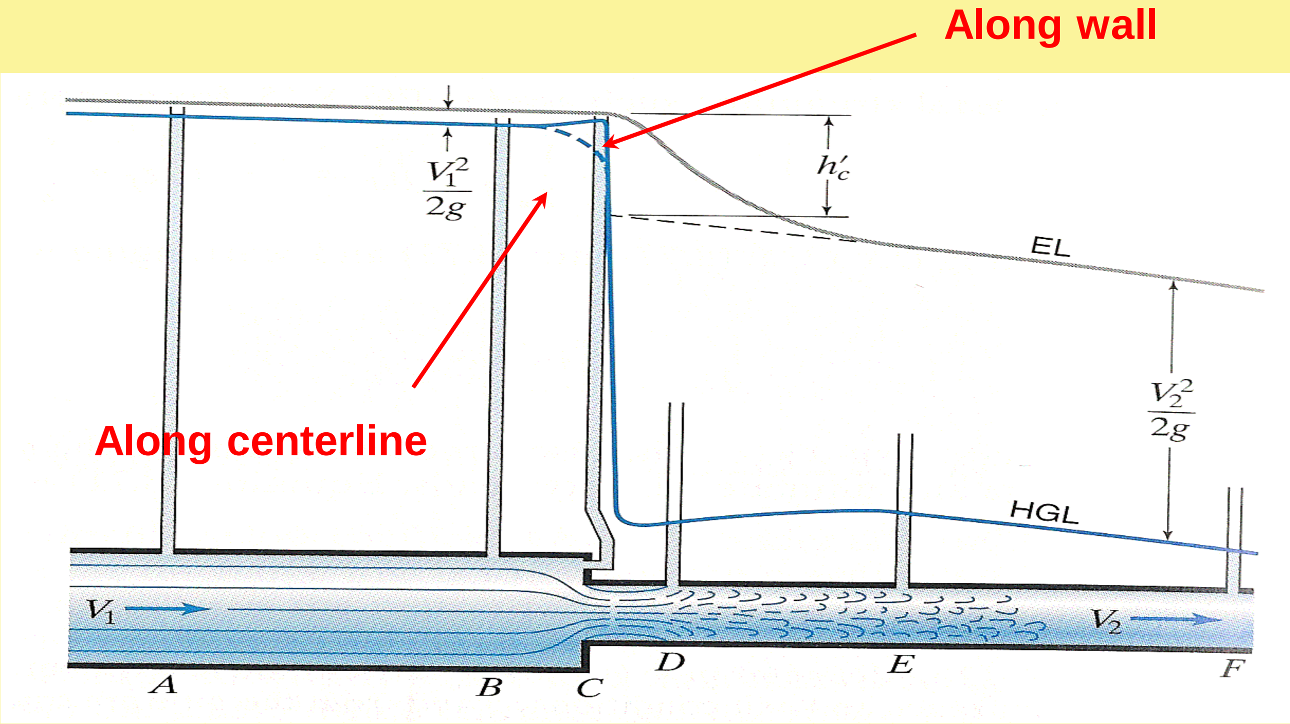
$$h_m = k_L \frac{V^2}{2g} = k_L \frac{Q^2}{2gA^2}$$

"minor" compared to friction losses in long pipelines but,

can be the dominant cause of head loss in shorter pipelines

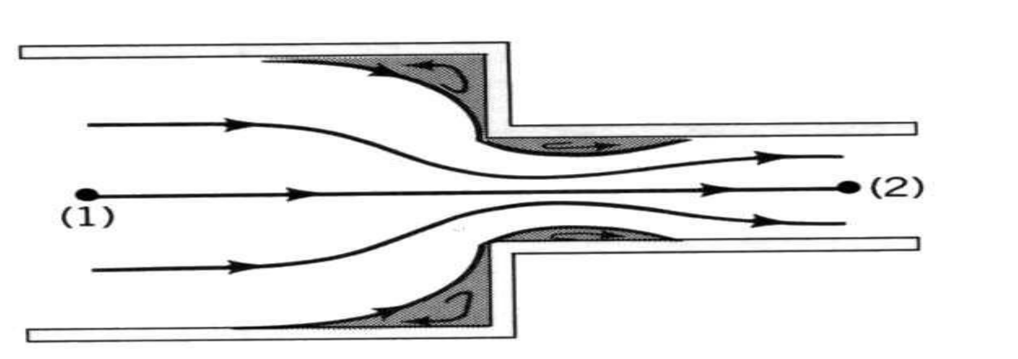
Losses due to contraction

A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe due to both the increase in velocity and the loss of energy to turbulence.

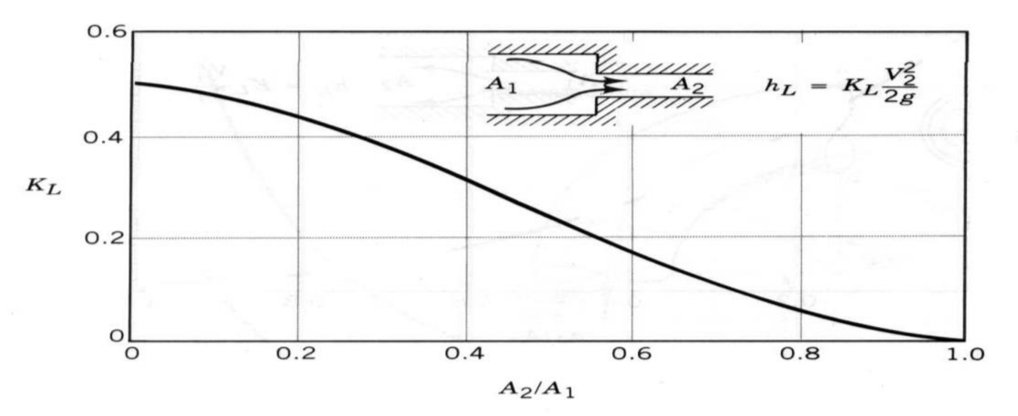


$$h_c = k_c \frac{V_2^{\,2}}{2g}$$

Head Loss Due to a Sudden Contraction

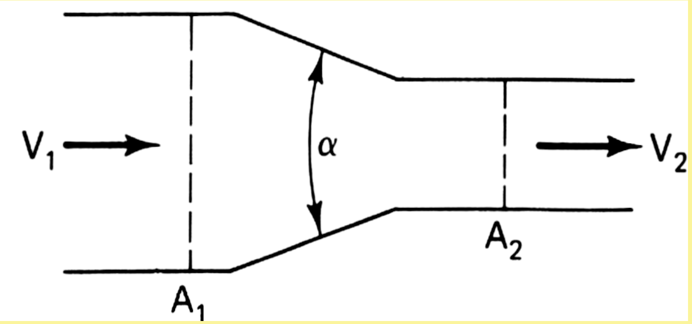


$$h_L = K_L \frac{V_2^{\,2}}{2g}$$

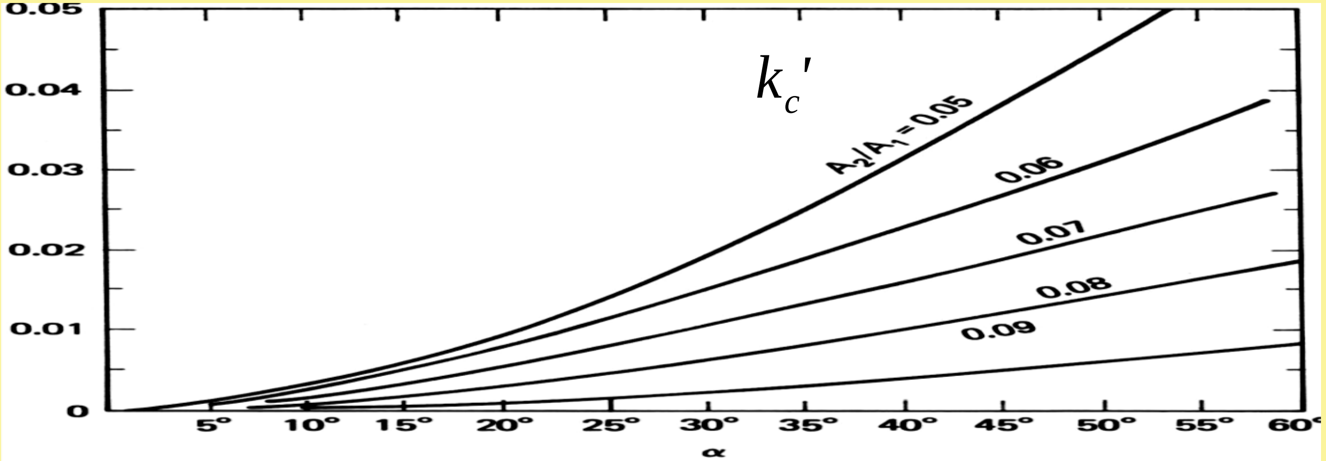


$$h_L = 0.5 \frac{V_2^{\,2}}{2g}$$

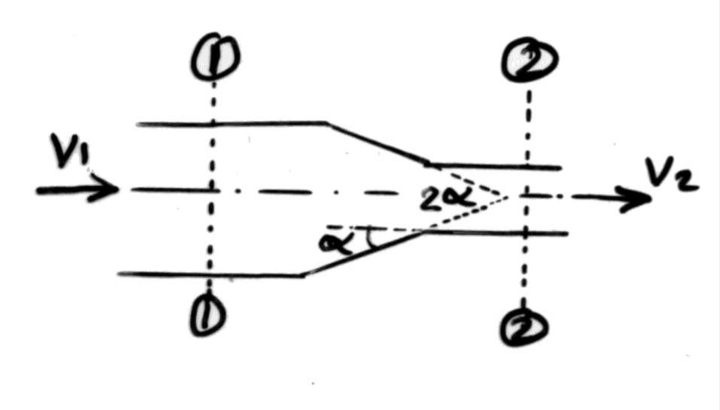
Head losses due to pipe contraction may be greatly reduced by introducing a <u>gradual pipe transition</u> known as a <u>confusor</u>



$$h_c' = k_c' \frac{V_2^{\,2}}{2g}$$



Head Loss Due to Gradual Contraction (reducer or nozzle)



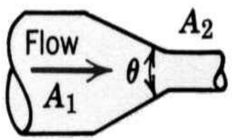
$$h_L = K_L \frac{\left(V_2^2 - V_1^2\right)}{2g}$$

α	10^0	20^0	30^0	40^0
K_L	0.2	0.28	0.32	0.35

Head Loss Due to Gradual Contraction (reducer or nozzle)

A different set of data is :

Table Loss Coefficients (K) for Gradual Contractions.

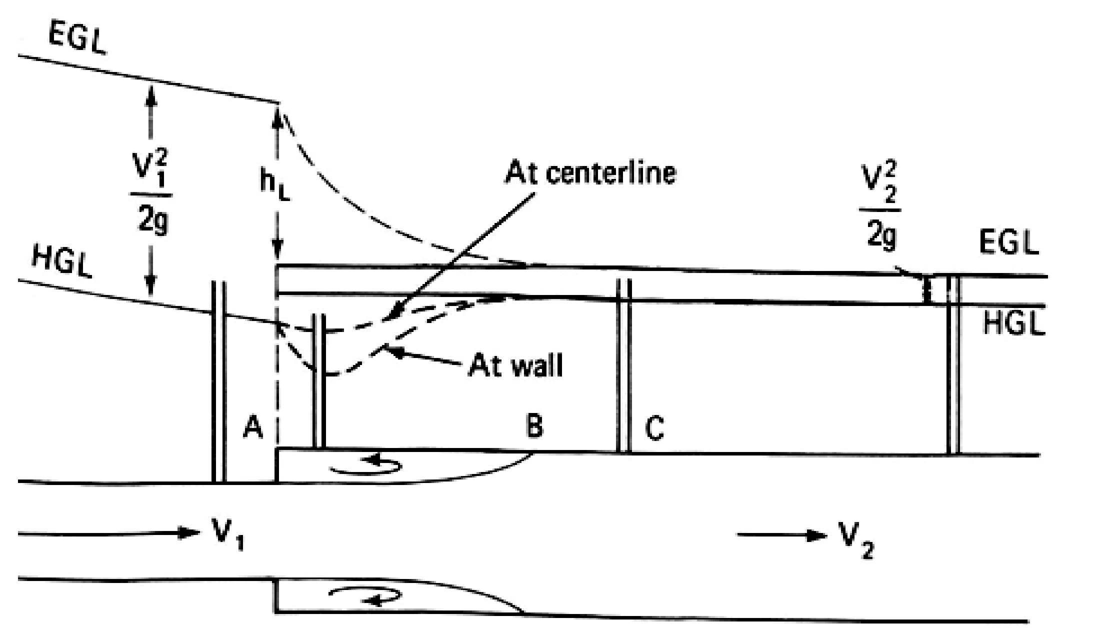


	Included Angle, θ, Degrees						
A_2/A_1	**10**	**15–40**	**50–60**	**90**	**120**	**150**	**180**
0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{l_m} = K(\bar{V}_2^2/2)$.

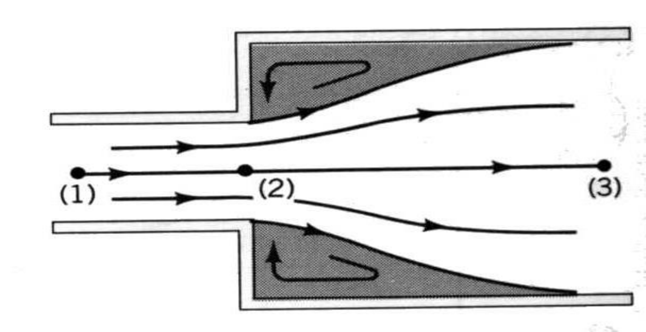
Losses due to Enlargement

<u>A sudden Enlargement</u> in a pipe

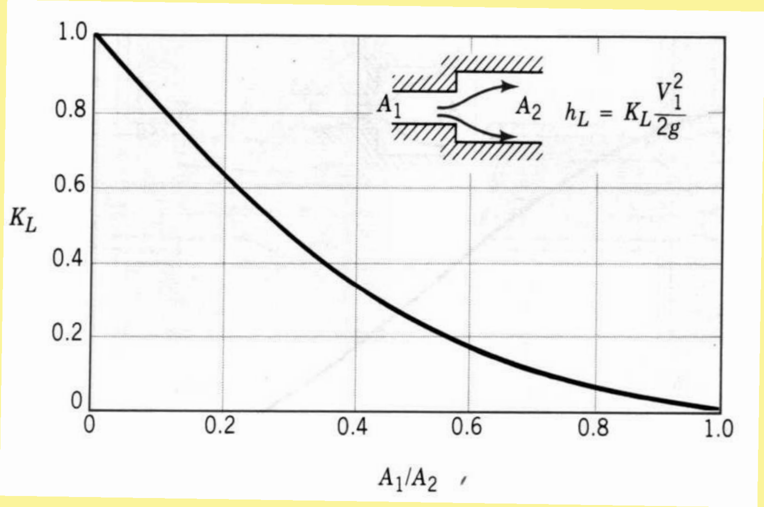


$$h_E = \frac{(V_1 - V_2)^2}{2g}$$

Head Loss Due to a Sudden Enlargement



$$h_L = K_L \frac{V_1^2}{2g}$$

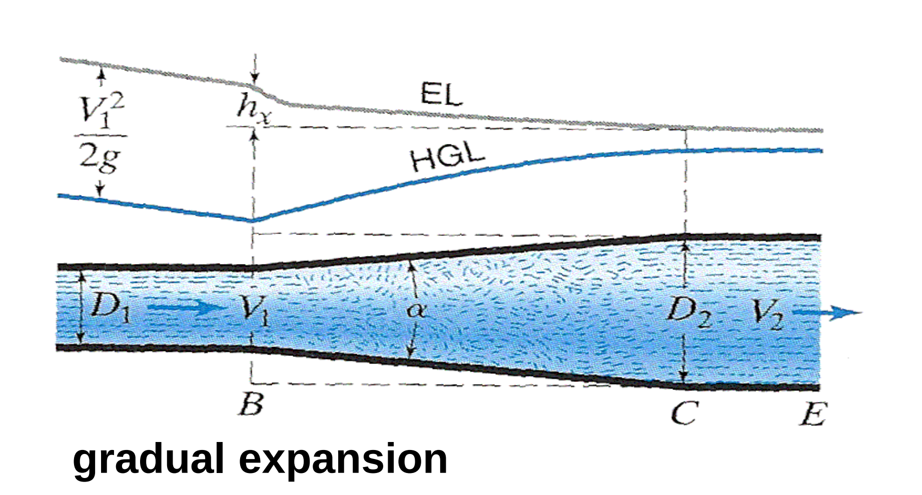


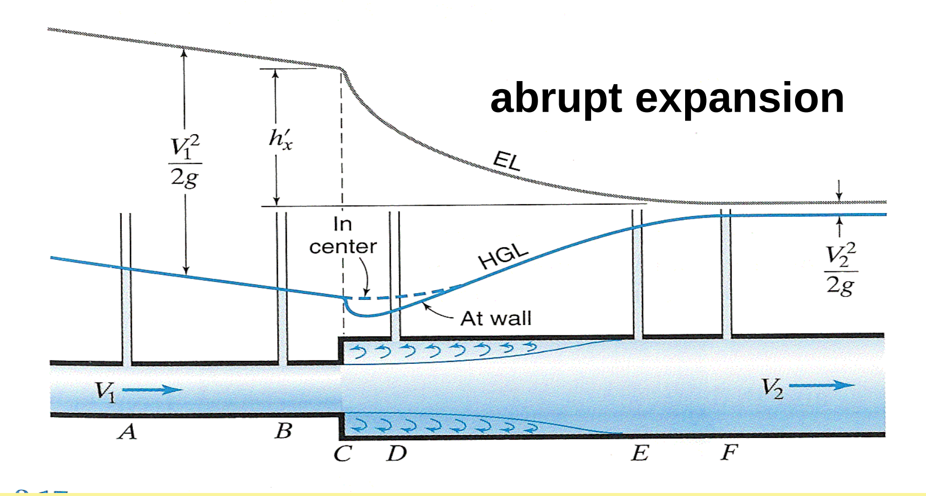
$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

or :

$$h_L = \frac{\left(V_1 - V_2\right)^2}{2g}$$

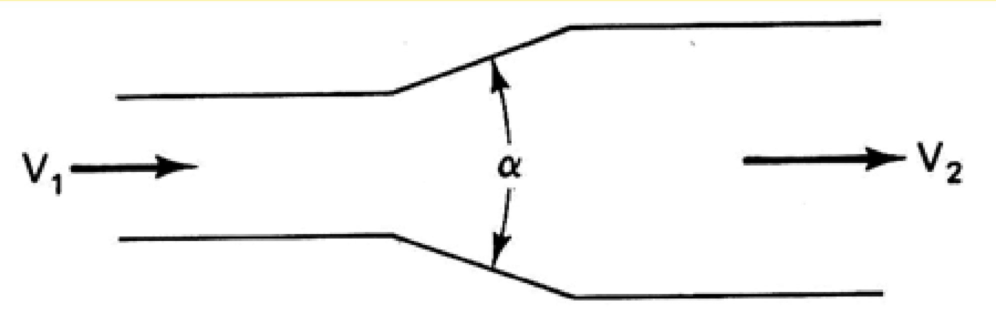
- **Note that the drop in the energy line is much larger than in the case of a contraction**





Smaller head loss than in the case of an abrupt expansion

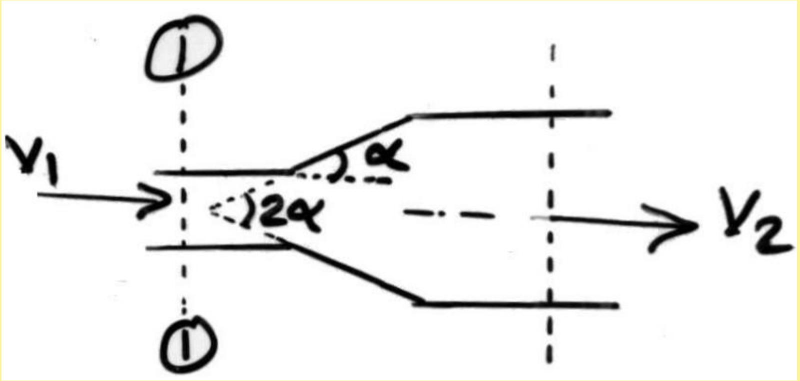
- **Head losses due to pipe enlargement may be greatly reduced by introducing a <u>gradual pipe transition</u> known as a <u>diffusor</u>**



$$h_E' = k_E' \frac{V_1^2 - V_2^2}{2g}$$

α	10°	20°	30°	40°	50°	60°	75°
K_E'	.078	.31	.49	.60	.67	.72	.72

Head Loss Due to Gradual Enlargement (conical diffuser)



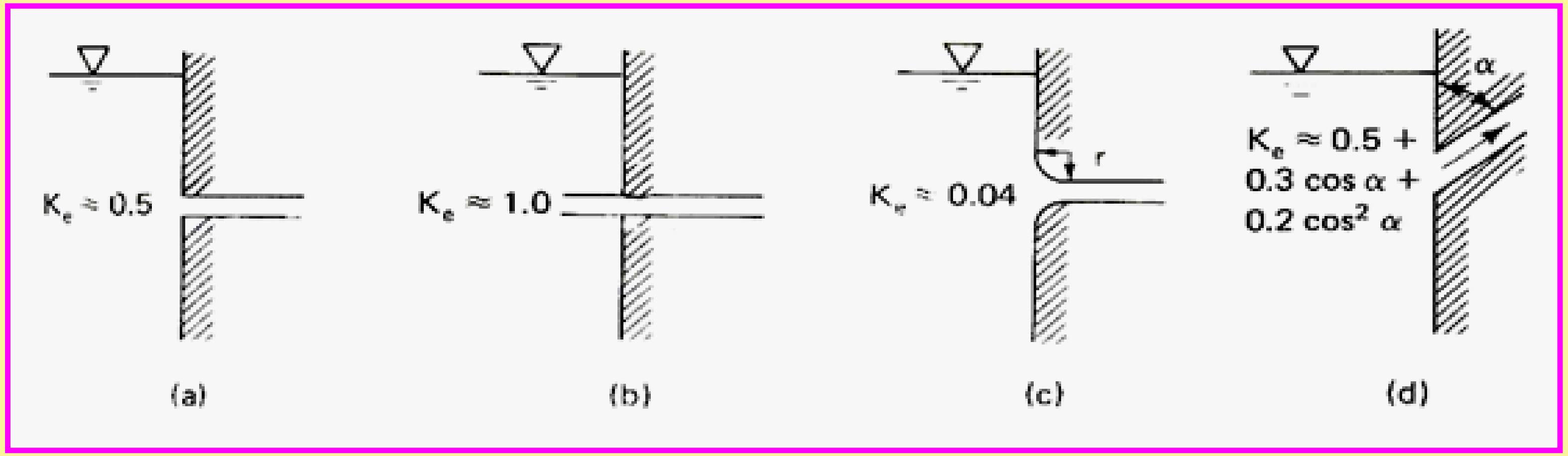
$$h_L = K_L \frac{\left(V_1^2 - V_2^2\right)}{2g}$$

α	10^0	20^0	30^0	40^0
K_L	0.39	0.80	1.00	1.06

Loss due to pipe entrance

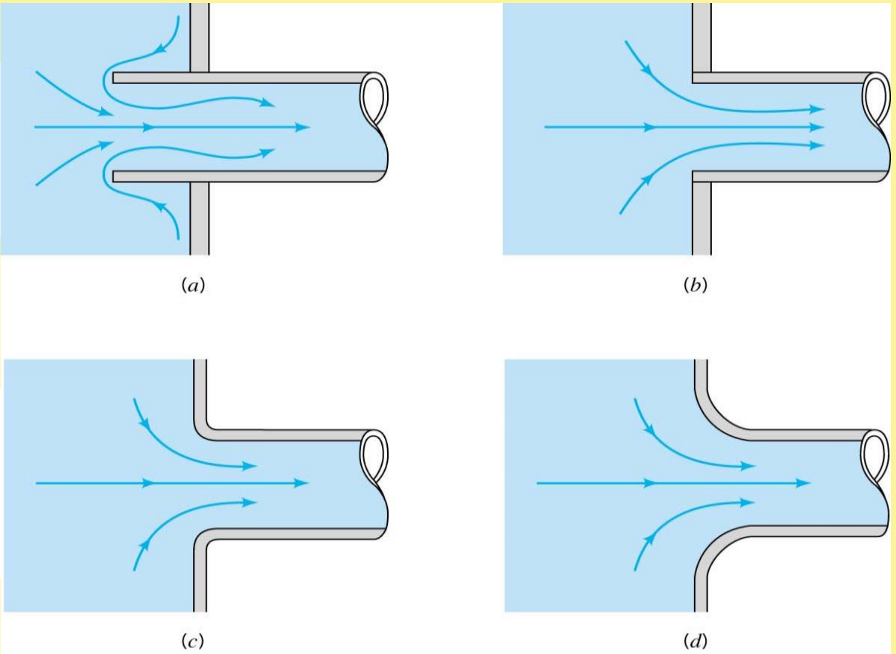
General formula for head loss at the entrance of a pipe is also expressed in term of velocity head of the pipe

$$h_{ent} = K_{ent} \frac{V^2}{2g}$$



Head Loss at the Entrance of a Pipe
(flow leaving a tank)

Reentrant
(embeded)
$K_L = 0.8$



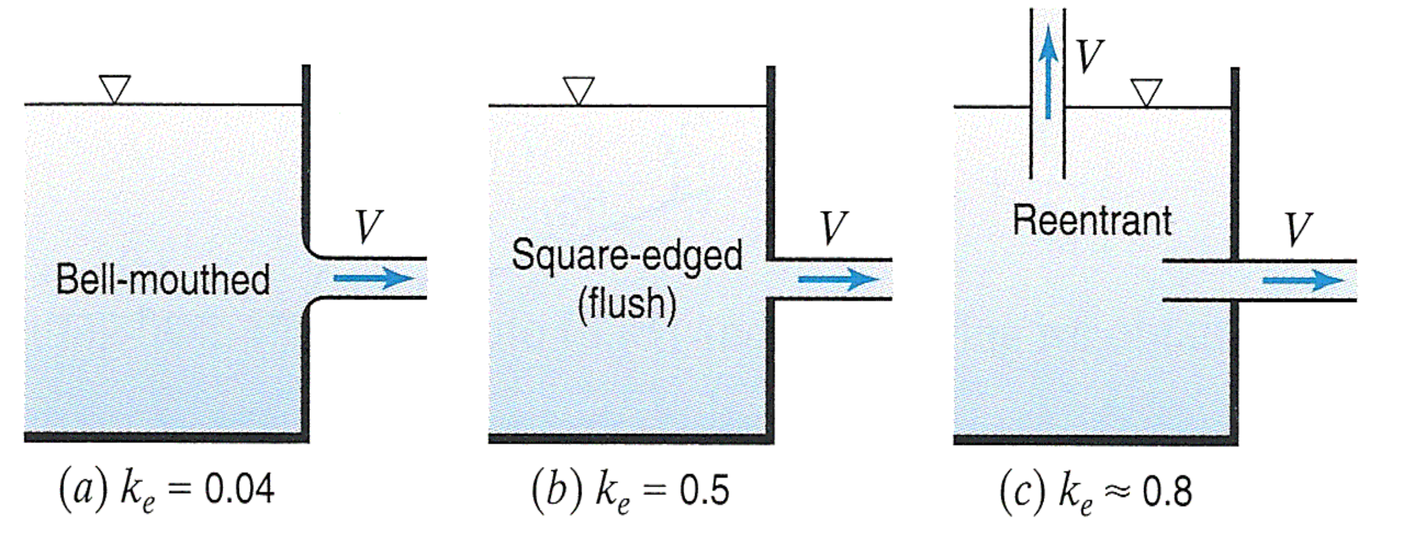
Sharp
edge
$K_L = 0.5$

Slightly
rounded
$K_L = 0.2$

Well
rounded
$K_L = 0.04$

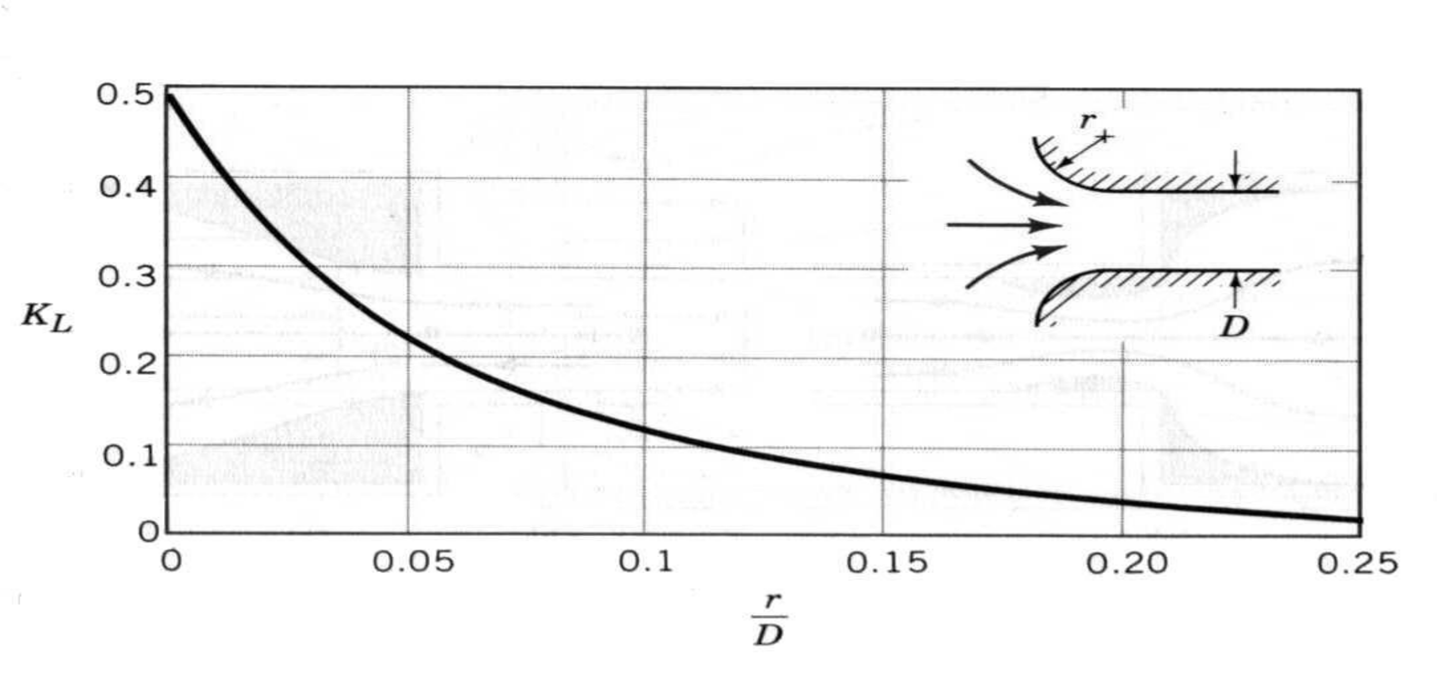
$$h_L = K_L \frac{V^2}{2g}$$

Different pipe inlets

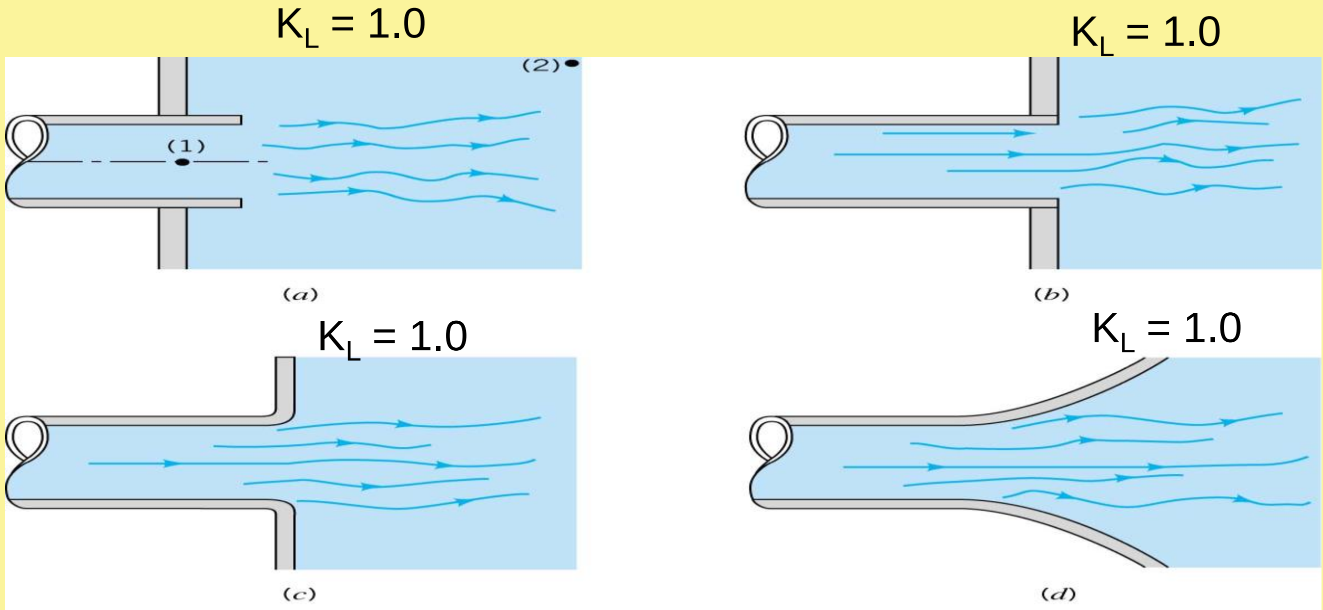


increasing loss coefficient

Another Typical values for various amount of rounding of the lip



Head Loss at the Exit of a Pipe (flow entering a tank)



$$h_L = \frac{V^2}{2g}$$

The entire kinetic energy of the exiting fluid (velocity *V1*) is dissipated through viscous effects as the stream of fluid mixes with the fluid in the tank and eventually comes to rest (V_2 = 0).

Head Loss Due to Bends in Pipes

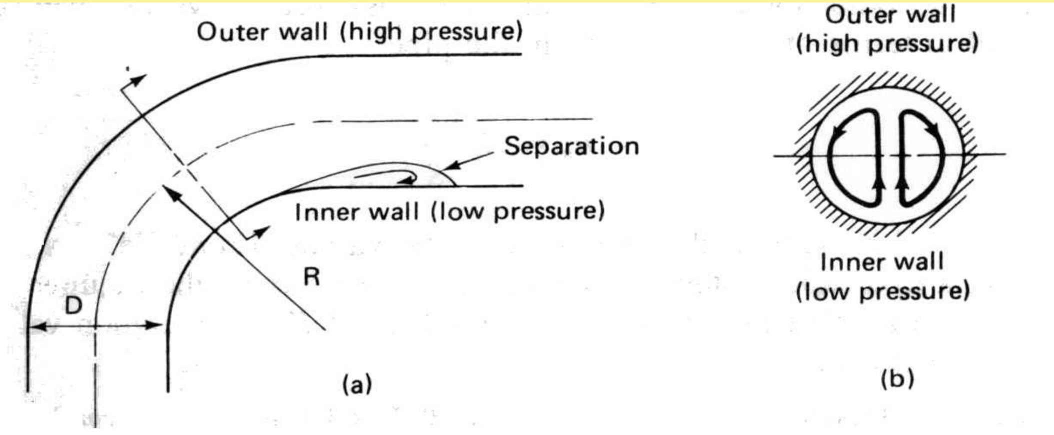


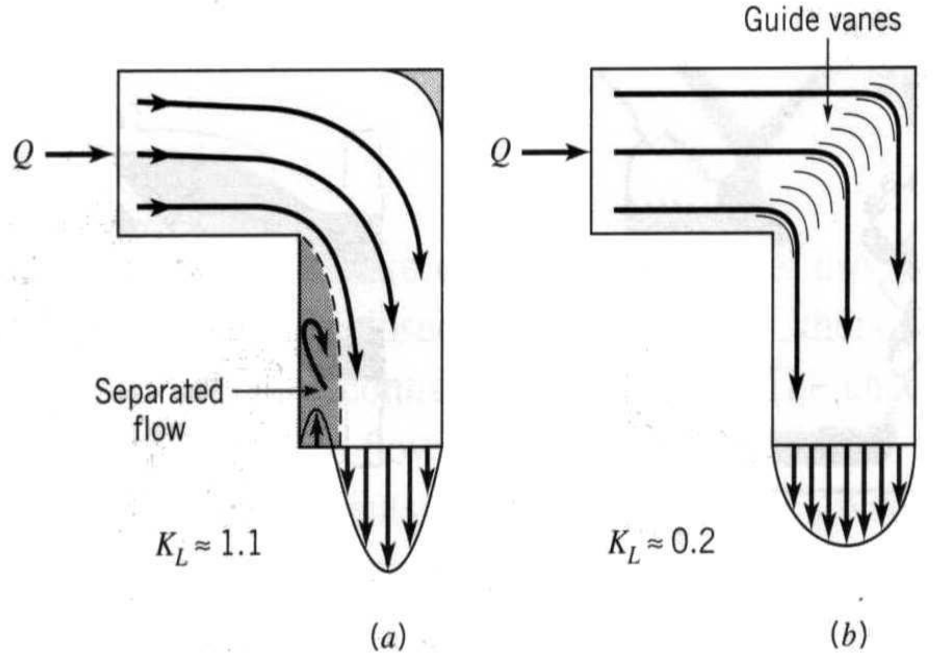
Figure Head loss at a bend: (a) flow separation in a bend; (b) secondary flow at a bend.

$$h_b = k_b \frac{V^2}{2g}$$

R/D	1	2	4	6	10	16	20
K_b	0.35	0.19	0.17	0.22	0.32	0.38	0.42

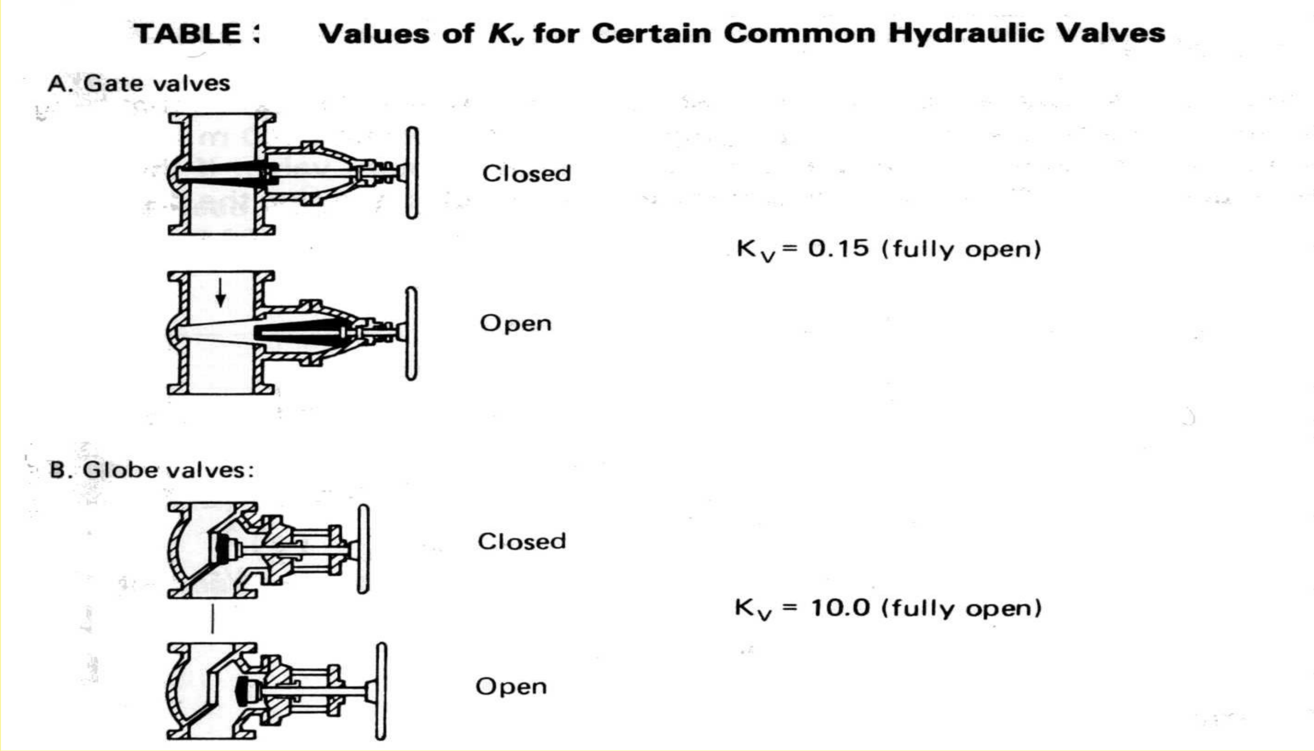
Miter bends

For situations in which space is limited,



■ FIGURE
Character of the flow in a 90° miter bend and the associated loss coefficient: (*a*) without guide vanes. (*b*) with guide vanes.

Head Loss Due to Pipe Fittings (valves, elbows, bends, and tees)



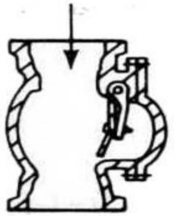
$$h_v = K_v \frac{V^2}{2g}$$

C. Check valves:

Closed

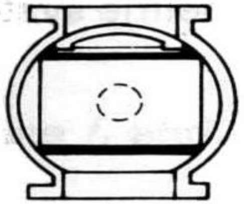
Hinge (Swing type)

Swing type K_V = 2.5 (fully open)
Ball type K_V = 70.0 (fully open)
Lift type K_V = 12.0 (fully open)



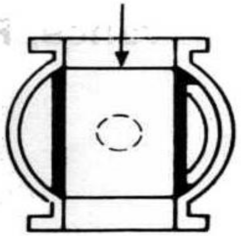
Open

D. Rotary valves:



Closed

K_V = 10.0 (fully open)

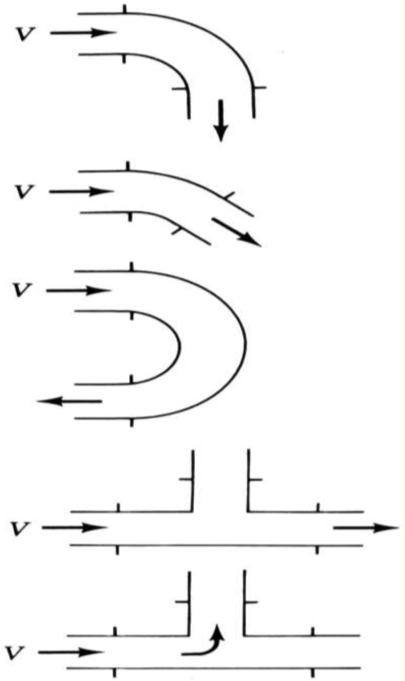


Open

The loss coefficient for elbows, bends, and tees

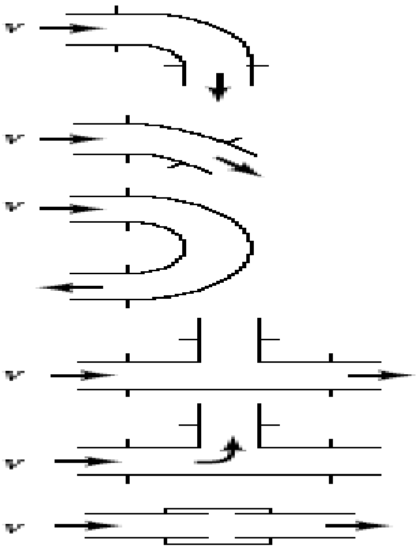
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0

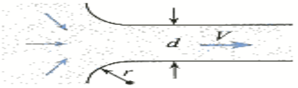
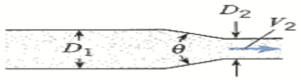
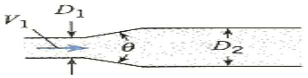
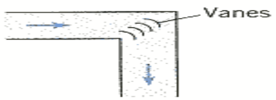
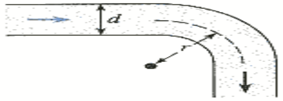


Loss coefficients for pipe components (Table)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Minor loss coefficients (Table)

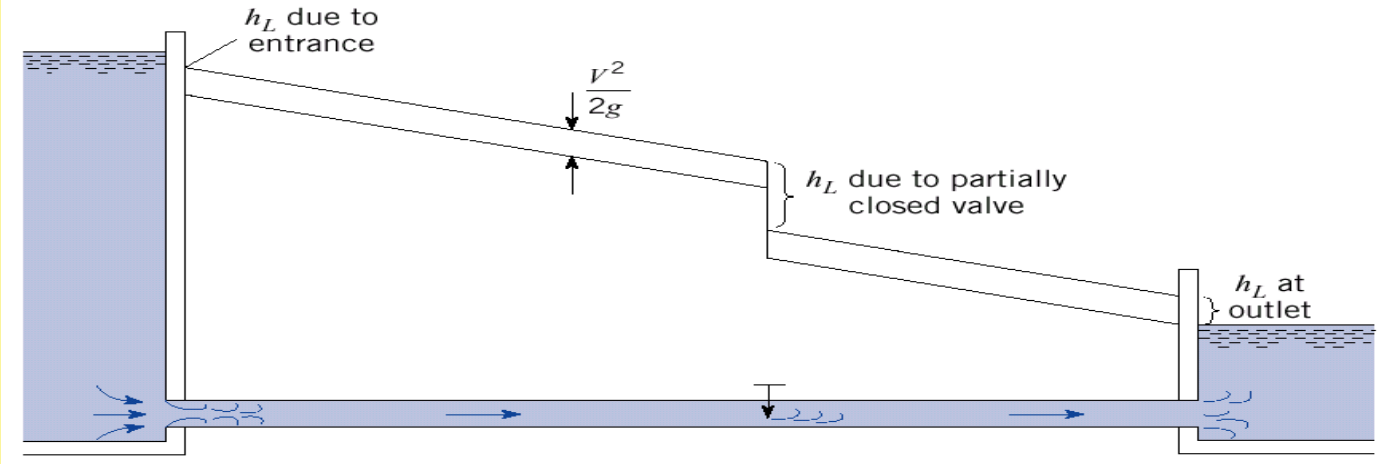
Description	Sketch	Data		K	Source
Pipe entrance	 	r/d		K_e	(2)*
		0.0		0.50	
		0.1		0.12	
$h_L = K_e V^2/2g$		>0.2		0.03	
Contraction	 	D_2/D_1	K_C $\theta = 60°$	K_C $\theta = 180°$	(2)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
$h_L = K_C V_2^2/2g$		0.90	0.06	0.10	
Expansion	 	D_1/D_2	K_E $\theta = 20°$	K_E $\theta = 180°$	(2)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend	 	Without vanes		$K_b = 1.1$	(39)
		With vanes		$K_b = 0.2$	(39)
90° smooth bend	 	r/d			(5) and (15)
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings	Globe valve—wide open	$K_v = 10.0$			(39)
	Angle valve—wide open	$K_v = 5.0$			
	Gate valve—wide open	$K_v = 0.2$			
	Gate valve—half open	$K_v = 5.6$			
	Return bend	$K_b = 2.2$			
	Tee				
	straight-through flow	$K_t = 0.4$			
	side-outlet flow	$K_t = 1.8$			
	90° elbow	$K_b = 0.9$			
	45° elbow	$K_b = 0.4$			

Minor loss calculation using equivalent pipe length

$$L_e = \frac{k_l\ D}{f}$$

L_e	Equivalent pipe length
D	Diameter of pipe
k_l	Loss coefficient for any fitting, valve…
f	Darcy-Weisbach coefficient

Energy and hydraulic grade lines



Unless local effects are of particular interests, the changes in the EGL and HGL are often shown as abrupt changes (even though the loss occurs over some distance)

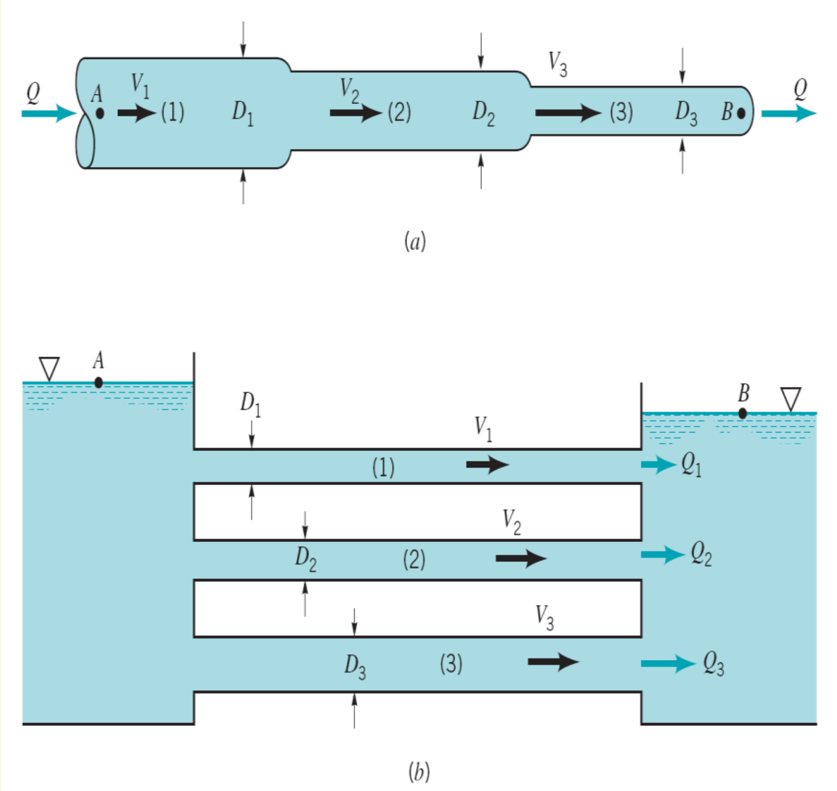
Pipe network

- **Serial connection**

$$Q_1 = Q_2 = Q_3$$
$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

- **Parallel connection**

$$Q = Q_1 + Q_2 + Q_3$$
$$h_{L_1} = h_{L_2} = h_{L_3}$$



Pipe Network

- A water distribution system consists of complex interconnected pipes, service reservoirs and/or pumps, which deliver water from the treatment plant to the consumer.
- Water demand is highly variable, whereas supply is normally constant. Thus, the distribution system must include storage elements, and must be capable of flexible operation.
- Pipe network analysis involves the determination of the pipe flow rates and pressure heads at the outflows points of the network. The flow rate and pressure heads must satisfy the continuity and energy equations.

Pipe Network

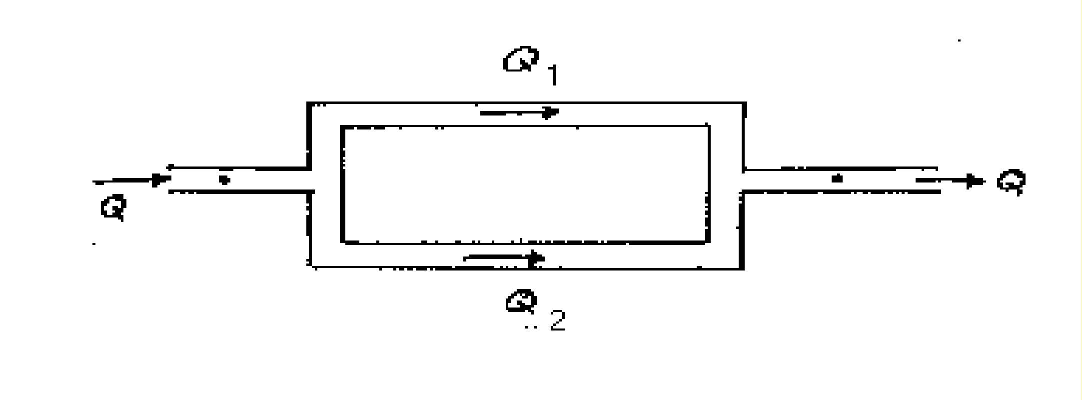
- The earliest systematic method of network analysis (Hardy-Cross Method) is known as the head balance or closed loop method.
- This method is applicable to system in which pipes form closed loops. The outflows from the system are generally assumed to occur at the nodes junction.
- For a given pipe system with known outflows, the Hardy-Cross method is an iterative procedure based on initially iterated flows in the pipes.

Pipe Network

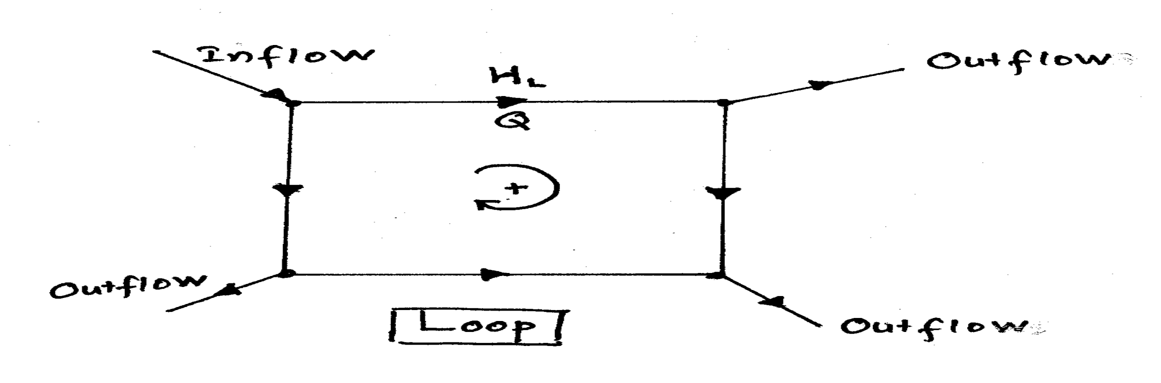
- **At each junction these flows must satisfy the continuity criterion, i.e. the algebraic sum of the flow rates in the pipe meeting at a junction, together with any external flows is zero.**
- **Algebraic sum of head losses round each loop must be zero**

Class Question

- A pipe 6-cm in diameter, 1000 m long and with λ = 0.018 is connected in parallel between two points M and N with another pipe 8-cm in diameter, 800-m long and having λ = 0.020. A total discharge of 20 L/s enters the parallel pipe through division at A and rejoins at B. Estimate the discharge in each of the pipe.



Hardy Cross Method



- Assigning clockwise flows and their associated head losses are positive, the procedure is as follows:
  - Assume values of Q to satisfy $\sum Q = 0$.
  - Calculate H_L from Q using $H_L = K^1 Q^2$
  - If $\sum H_L = 0$, then the solution is correct.
  - If $\sum H_L \neq 0$, then apply a correction factor, ΔQ, to all Q and repeat from step (2).

Hardy Cross Method contd.

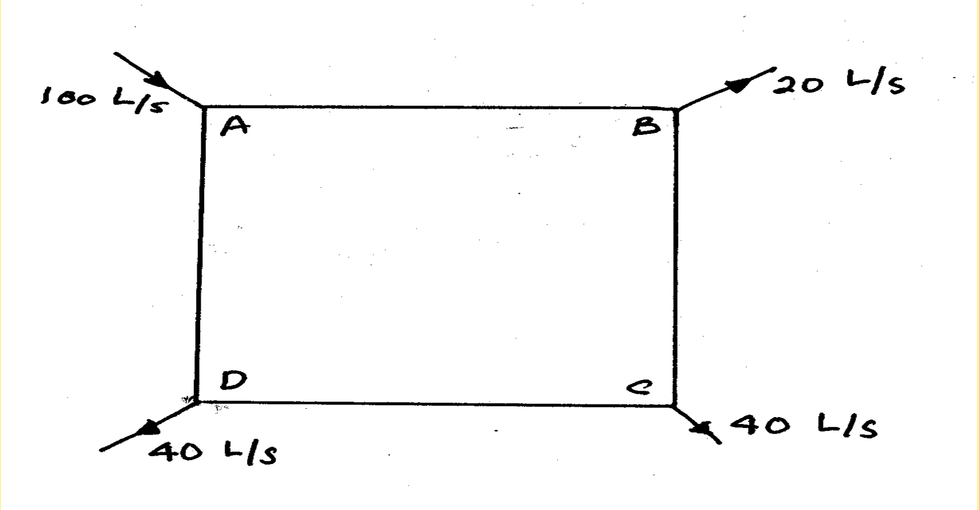
- For practical purposes, the calculation is usually terminated when $\sum H_L < 0.01$ m or $\Delta Q < 1$ L/s.

- A reasonably efficient value of ΔQ for rapid convergence is given by;

$$\Delta Q = -\frac{\sum H_L}{2\sum {H_L}/{Q}}$$

Class Problem (Hardy Cross)

For the square loop shown, find the discharge in all the pipes. All pipes are 1 km long and 300 mm in diameter, with a friction factor of 0.0163. Assume that minor losses can be neglected.



Class problem

- For the network shown in the figure the head loss is given by . The values of r for each pipe, and the discharge into or out of various nodes are shown in the sketch. The discharges are in an arbitrary unit. Obtain the distribution of discharge in the network.