Gradually Varied Flow (GVF)

Hydraulics

Prof. Mohammad Saud Afzal

Department of Civil Engineering

Gradually Varied Flow (GVF)

- The flow in a channel is termed GRADUALLY VARIED, if the flow depth changes gradually over a large length of the channel.

The cross- sectional shape, size and bed slope are constant

- Assumptions
  - The channel is prismatic.
  - The flow in the channel is steady and and non-uniform.

- The channel bed- slope is small.
- The pressure distribution at any section is hydrostatic.
- The resistance to flow at any depth is given by the corresponding uniform flow equation. Example: Manning's equation

Remember: In the uniform flow equations, energy slope S_f is used in place of bed slope S_0 . When Manning's formula is used we get

$$S_f = \frac{n^2 V^2}{R^{4/3}}$$

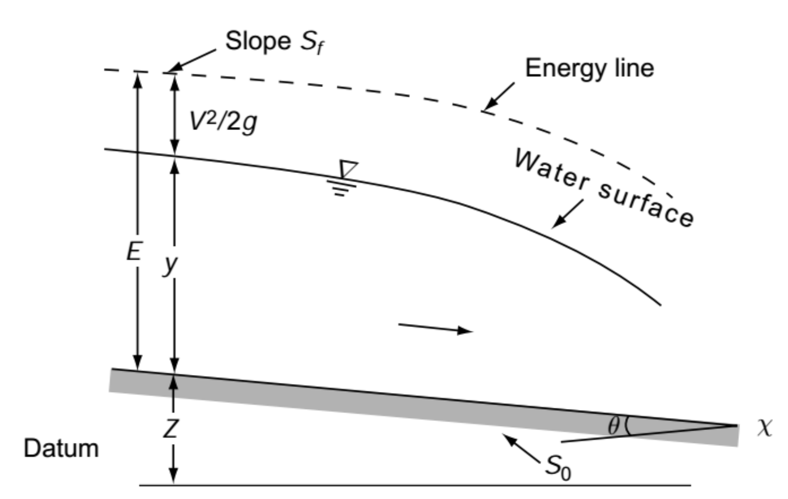
Differential Equation of GVF

- The total energy *H* of a GVF can be expressed as:

$$H = z + y + \frac{\alpha V^2}{2g}$$

- Assuming $\alpha = 1$, we get

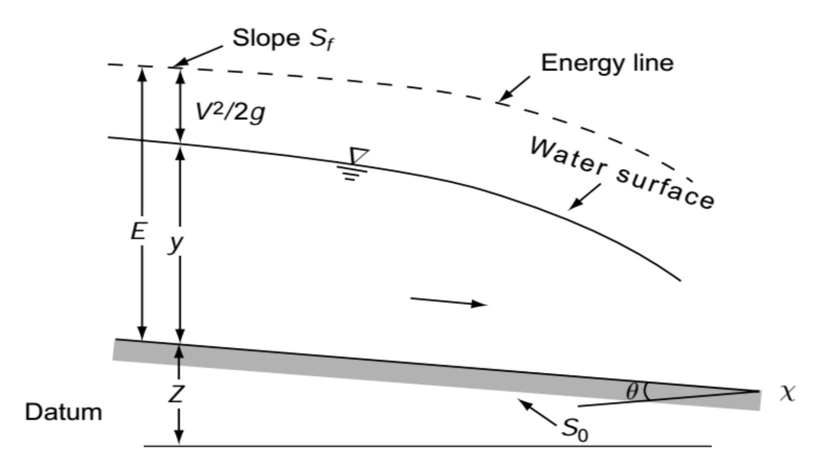
$$H = z + y + \frac{V^2}{2g}$$



Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

- **Differentiating both the sides of the above equation w.r.t *x***

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad \textbf{(Eq. 1)}$$



Represents the water surface slope w.r.t the channel bed

Represents energy slope

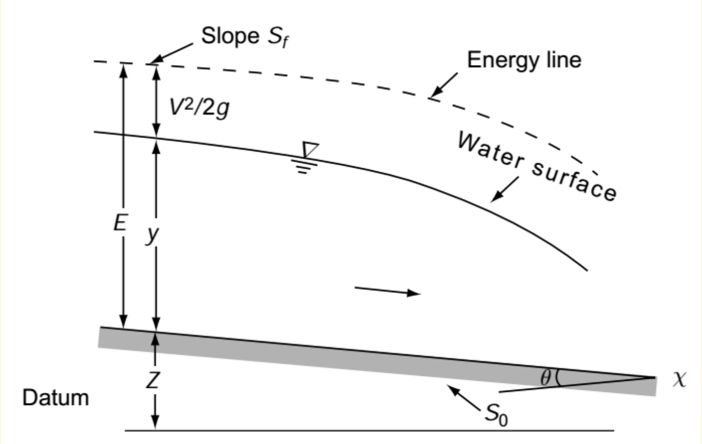
$$\frac{dH}{dx} = -S_f$$

Represents bottom slope

$$\frac{dz}{dx} = -S_0$$

- **Further,**

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)\frac{dy}{dx}$$



or

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{-Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx}$$

$\frac{dA}{dy} = T$, **where *T* is the top-width of the channel**

- So we can rewrite Eq. 1 as

$$-S_f = -S_0 + \frac{dy}{dx} - \left(\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$$

or

<u>NOTE:</u> $\frac{Q^2T}{gA^3} = {F_r}^2$, where F_r is Froude Number

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}$$

Differential Equation of GVF

Classification of Flow Profiles

- If Q, n and S_0 are fixed, then the normal depth y_0 and the critical depth y_c are fixed.

Depth obtained from uniform flow equations

- Three possible relationships that may exist between y_0 and y_c are:
  - $y_0 > y_c$
  - $y_0 < y_c$
  - $y_0 = y_c$

- Further, y_0 does not exist when:
  - The channel bed is horizontal. $S_0 = 0$
  - The channel has an adverse slope. $S_0 < 0$
- Based on these, the channels are classified into 5 categories as:

1. **Mild Slope (*M*) -** $y_0 > y_c$ Subcritical flow at normal depth
2. **Steep Slope (*S*) -** $y_0 < y_c$ Supercritical flow at normal depth
3. **Critical Slope (*C*) -** $y_0 = y_c$ Critical flow at normal depth
4. **Horizontal Bed (*H*) -** $S_0 = 0$
5. **Adverse Slope (*A*) -** $S_0 < 0$

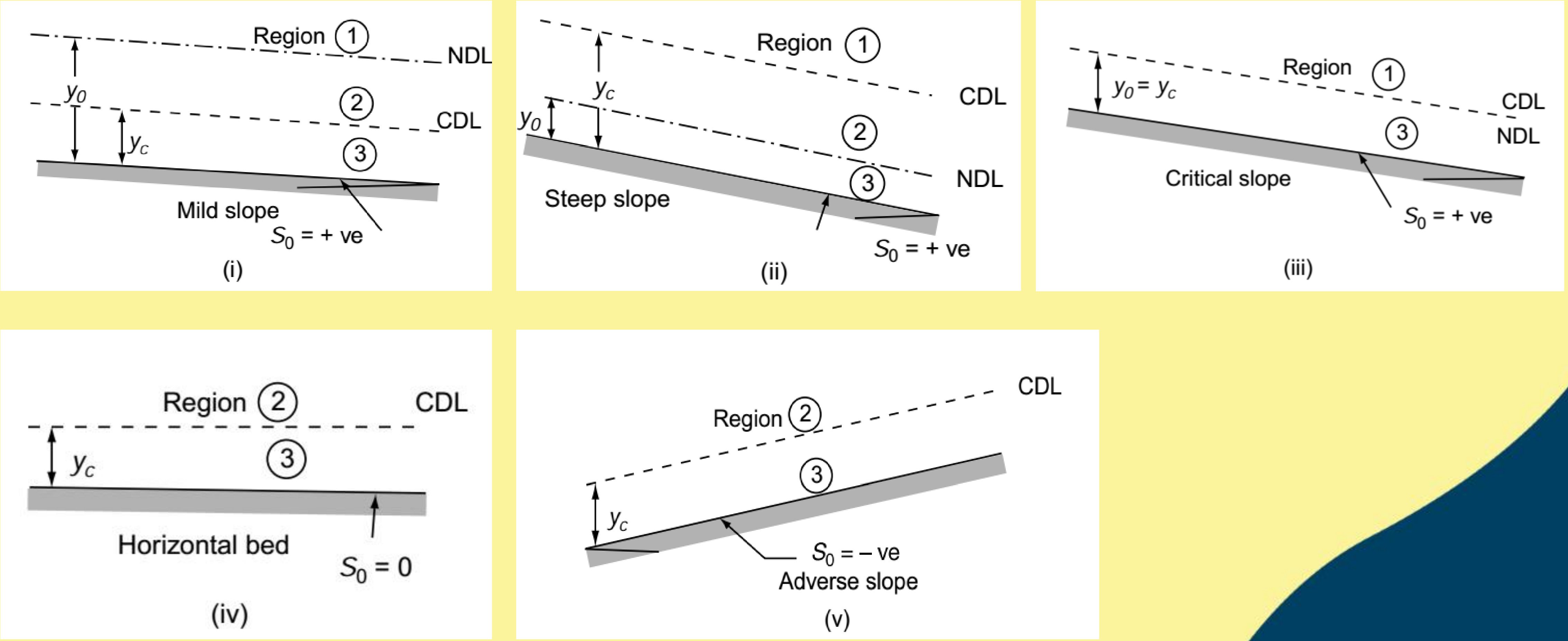
(4 and 5) Cannot sustain uniform flow

- Lines representing the critical depth (CDL) and the normal depth (NDL), when drawn in the longitudinal section, divide the flow space into the following 3 regions:

  - Region 1 – Space above the topmost line.

  - Region 2 – Space between the top line and the next lower line.

  - Region 3 – Space between the second line and the bed.

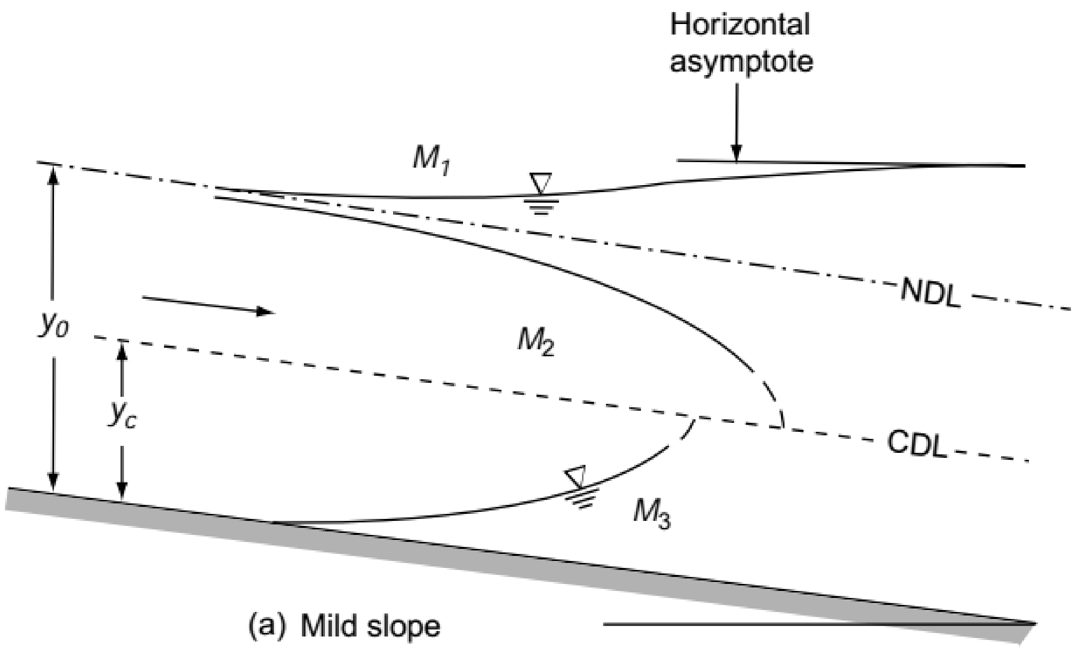


Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw-Hill Publishing Co. Ltd.

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed	2	$y > y_c$	H_2
	3	$y < y_c$	H_3
Adverse slope	2	$y > y_c$	A_2
	3	$y < y_c$	A_3

Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

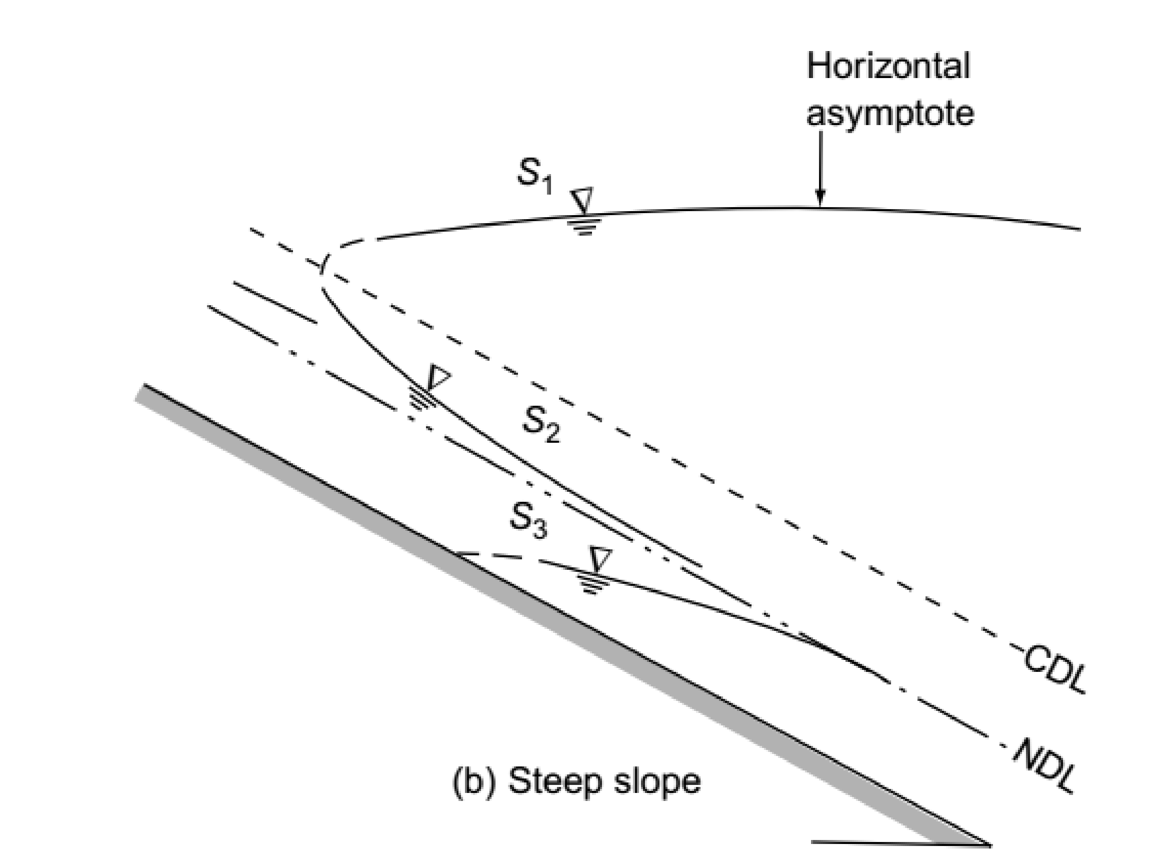
Mild Slope



(a) Mild slope

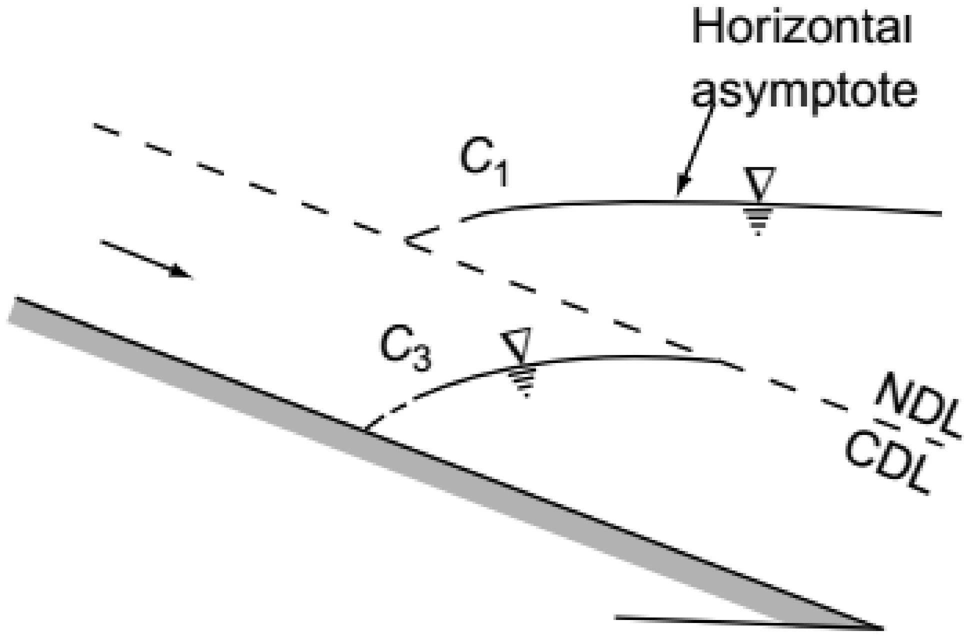
Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

Steep Slope



Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

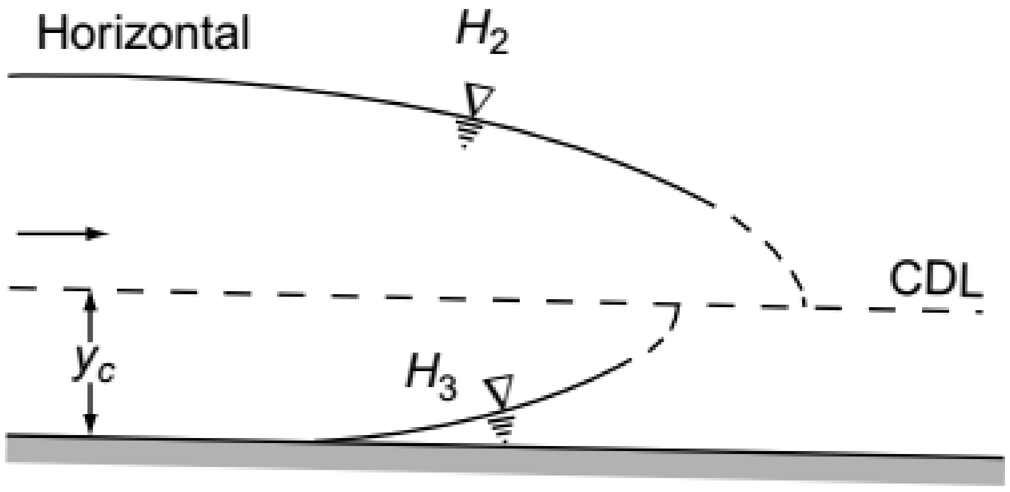
Critical Slope



(c) Critical slope

Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

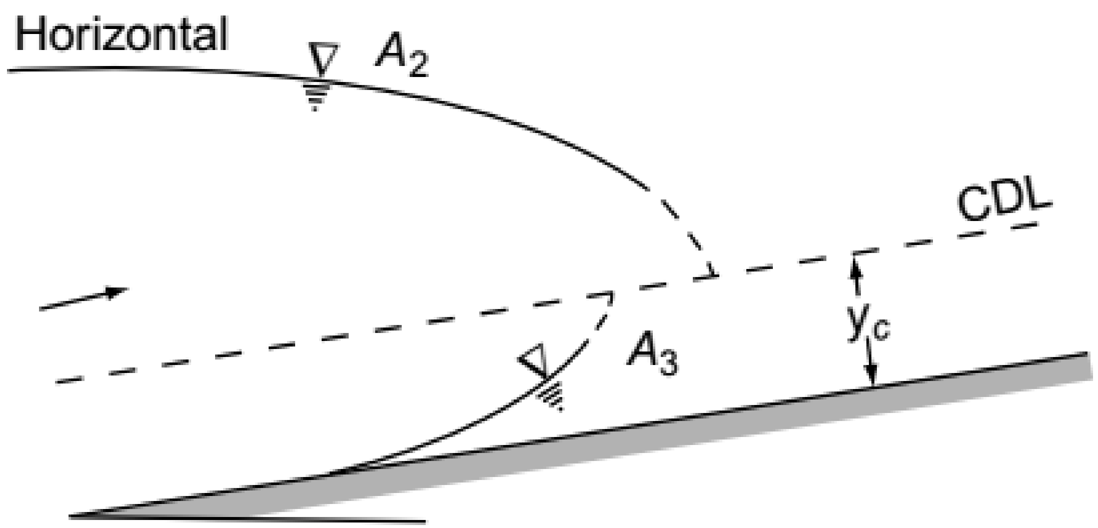
Horizontal Bed



(d) Horizontal bed

Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

Adverse Slope



(e) Adverse slope

Adapted from Subramanya, K. (1986). *Flow in Open Channels.* Tata McGraw- Hill Publishing Co. Ltd.

Problem- 1

- Find the rate of change of depth of water in a rectangular channel 10 m wide and 1.5 m deep, when the water is flowing with a velocity of 1 m/s. The flow of water through the channel of bed slope 1 in 4000, is regulated in such a way that energy line is having a slope of 0.00004.

Solution: $b = 10\,m$ $y = 1.5\,m$ $V = 1\,m/s$

$S_0 = 1/4000$ $S_f = 0.00004$

$$A = b \times y = 10 \times 1.5 = 15\ m^2 \qquad T = b = 10\ m \qquad Q = AV = 15\ m^3/s$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}} \qquad \frac{dy}{dx} = \frac{\frac{1}{4000} - 0.00004}{1 - \frac{15^2 \times 10}{9.81 \times 15^3}} \qquad \frac{dy}{dx} = 2.25 \times 10^{-4}$$

Problem- 2

- A rectangular channel with a bottom width of 4 m and a bottom slope of 0.0008 has a discharge of 1.5 m³/s. In a gradually varied flow in this channel, the depth at a certain location is found to be 0.30 m. Assuming Manning's *n* = 0.016, determine the type of GVF profile.

Solution: $b = 4\,m$ $y = 0.03\,m$ $Q = 1.5\ m^3/s$

$S_0 = 0.0008$ $n = 0.016$

Now, $\frac{Q}{b} = \frac{1.5}{4} = 0.0375\frac{m^3}{s}/m$

$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.375^2}{9.81}\right)^{1/3}$
$= 0.243m$

Now, $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$$1.5 = \frac{1}{0.016} 4 \times y_0 \left[\frac{4y_0}{4+2y_0}\right]^{2/3} (0.0008)^{1/3}$$

$$1.5 = \frac{4}{0.016} \times 4^{2/3} \times (0.0008)^{1/2} \frac{y_0 y_0^{2/3}}{(4+2y_0)^{2/3}}$$

$$\frac{y_0^{2/3}}{(4+2y_0)^{2/3}} = 0.0842$$

From trial and error

$$y_0 = 0.426\, m$$

$$y_0 > y_c$$ (Mild slope)

Also $y_0 > y > y_c$ → M_2

Class Question

A wide rectangular channel has a Manning's coefficient of 0.018. For a discharge intensity of $1.5\frac{m^3}{s}/m$, identify the possible types of gradually varied flow profiles produced in the following break in the grade of the channel.

$$S_{01} = 0.0004 \;\; and \;\; S_{02} = 0.016$$

Solution: **Discharge intensity q = 1.5** $\frac{m^3}{s}/m$

Critical depth $y_c = \left(q^2/g\right)^{1/3} = \left(1.5^2/9.81\right)^{1/3} = 0.612\, m$

Normal depth y_0 **: For a wide rectangular channel** $R = y_0$

$$q = \frac{1}{n} y_0 y_0^{2/3} s_0^{1/2}$$

$$y_0 = \left[\frac{nq}{\sqrt{S_0}}\right]^{3/5}$$

$$y_0 = \left[\frac{0.018 \times 1.5}{\sqrt{S_0}}\right]^{3/5}$$

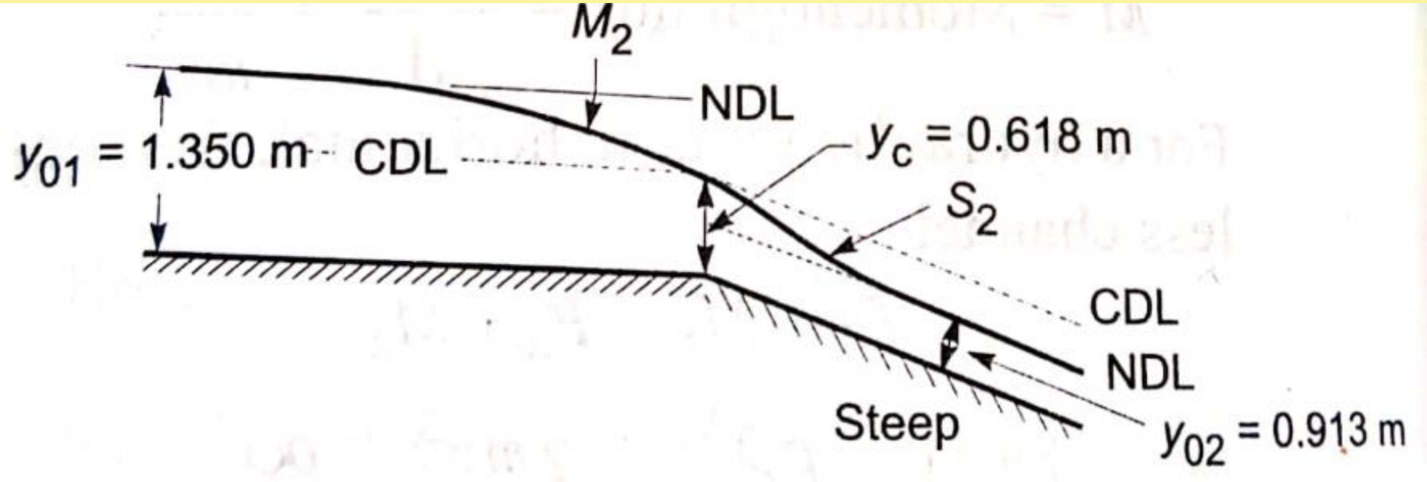
Slope	y_0
0.0004	1.197
0.016	0.396

$y_c(m)$	$y_{01}(\mathrm{m})$	$y_{02}(m)$
0.612	1.197	0.396

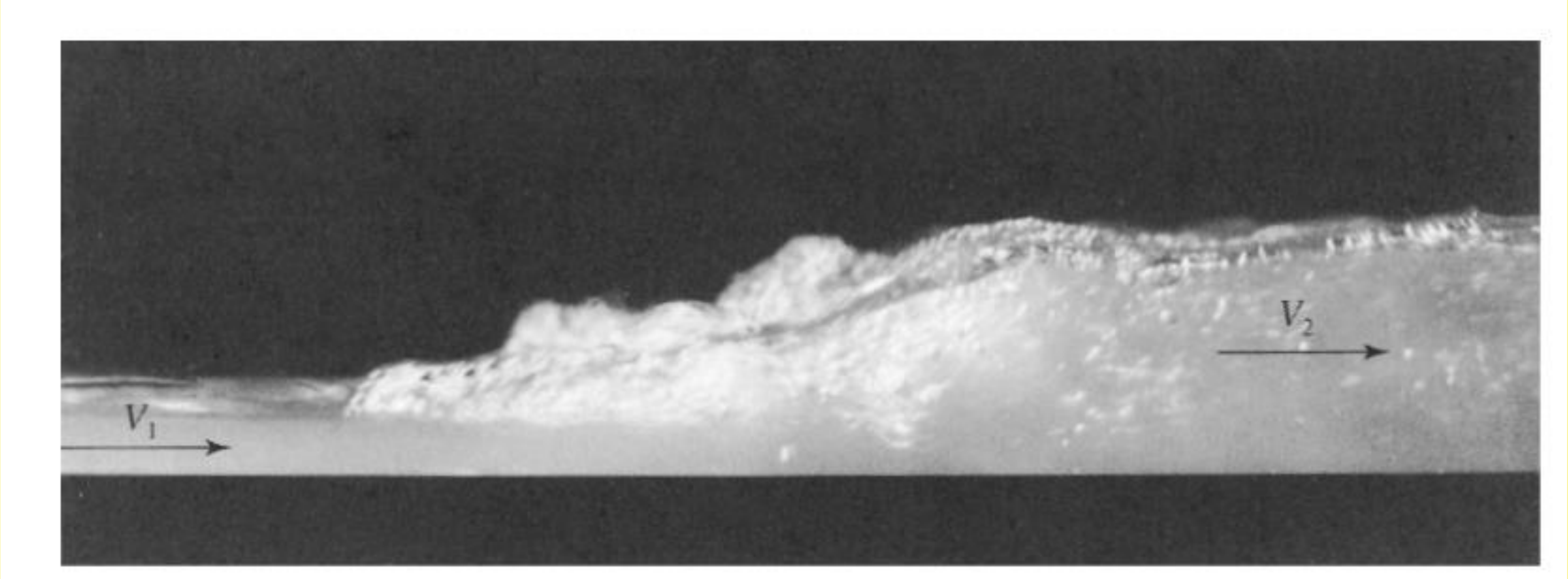
Type of grade change : Mild to Steep

The resulting water surface profiles are:

M_2 curve on Mild Slope and S_2 curve on steep slope

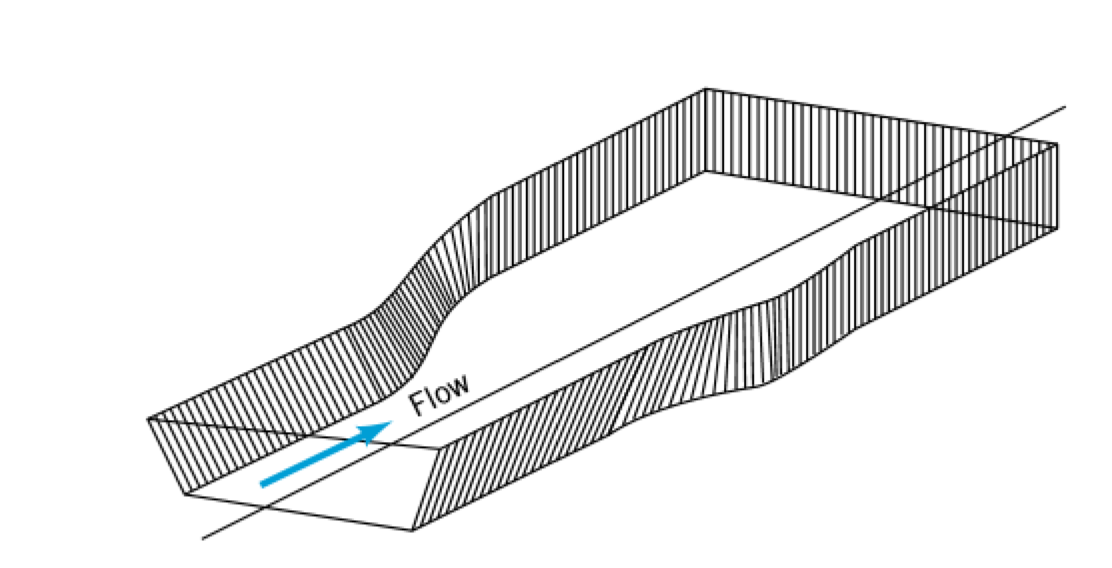


Rapidly varied flow



Hydraulic Jump due to change in bottom elevation

Rapidly varied flow



RVF due to transition

Hydraulic Jump

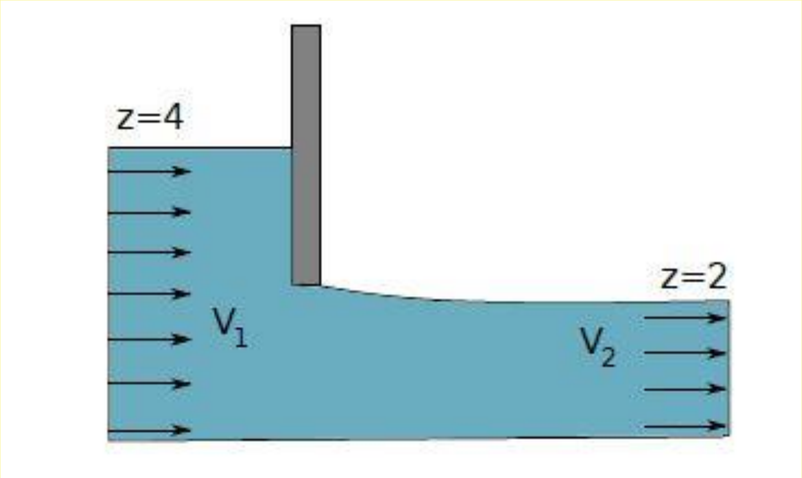
- For Rapidly varied flow (RVF) dy/dx ~1
  - Flow depth changes occur over a relatively short distance. One such example is *hydraulic jump*
  - These changes in depth can be regarded as discontinuity in free surface elevation (dy/dx → ∞)

- Hydraulic jump results when there is a conflict between upstream and downstream influences that control particular section of channel

Hydraulic Jump

- E.g. Sluice gate requires supercritical flow at upstream portion of channel whereas obstruction require the flow to be subcritical

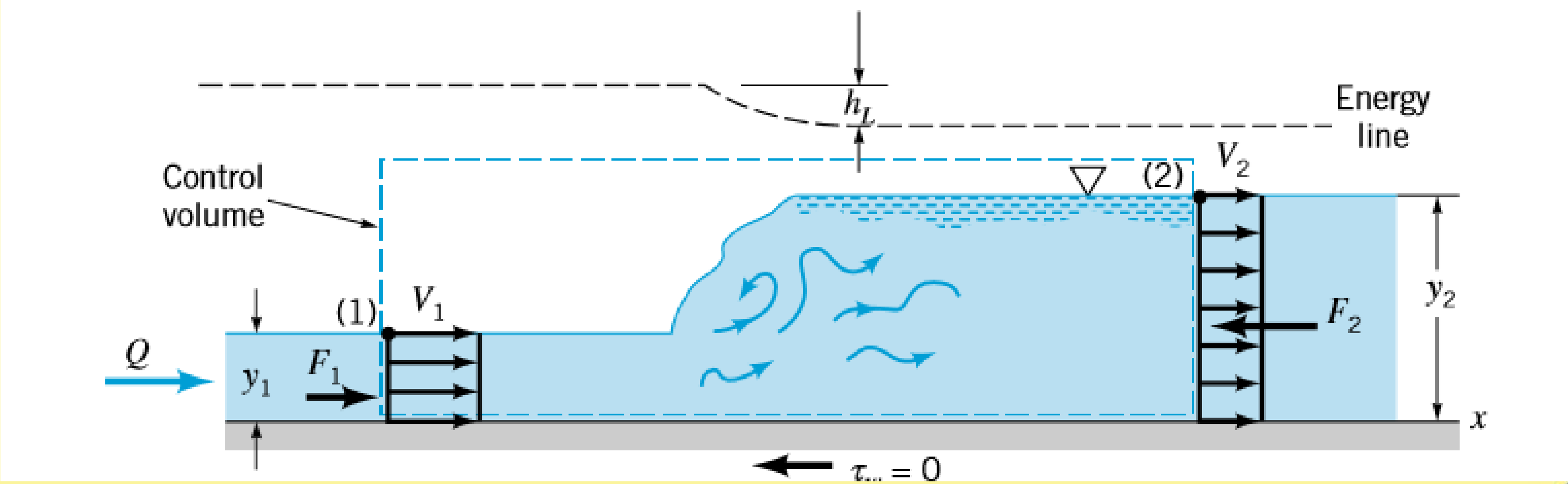
- Hydraulic jump provides the mechanism to make the transition between the two type of flows



Sluice Gate

Hydraulic Jump

- One of the most simple hydraulic jump occurs in a horizontal, rectangular channel as below



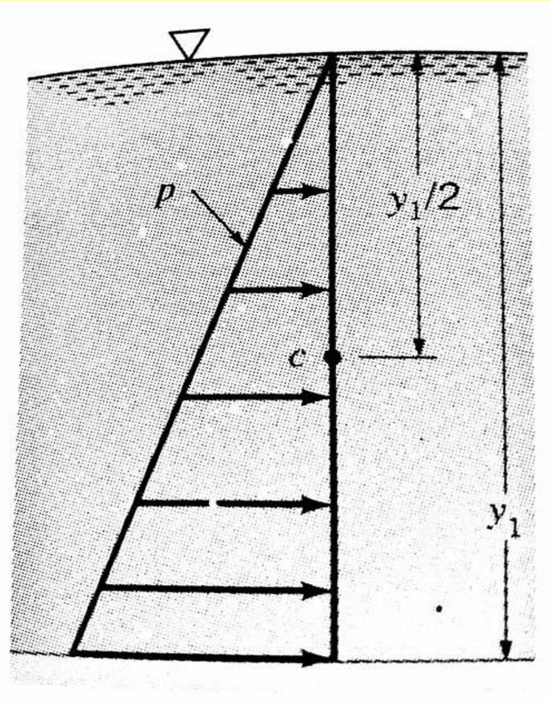
Hydraulic Jump Geometry

Hydraulic Jump Assumptions

- The flow within jump is complex but it is reasonable to assume that flow at sections 1 and 2 are nearly uniform, steady and 1D
- Neglect any wall shear stress τ_w, within relatively short segment between the sections
- Pressure force at either section is hydrostatic

Hydraulic Jump Derivation

- **x-component of momentum equation for control volume is written as**



$$F_1 - F_2 = \rho Q(V_2 - V_1) = \rho V_1 y_1 b(V_2 - V_1)$$

Where

$$F_1 = p_{c1}A_1 = \frac{\gamma y^2{}_1 b}{2} \qquad p_{c1} = \frac{\gamma y_1}{2}$$

$$F_2 = p_{c2}A_2 = \frac{\gamma y^2{}_2 b}{2} \qquad p_{c2} = \frac{\gamma y_2}{2}$$

b is the channel width

Hydraulic Jump Derivation

- **Momentum equation can be written as**

$$\frac{y^2{}_1}{2} - \frac{y^2{}_2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$$

Eq. 19

- **Conservation of mass (continuity) gives**

$$y_1 b V_1 = y_2 b V_2 = Q$$

Eq. 20

- **Energy conservation gives**

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

Eq. 21

h_L is the head loss

Hydraulic Jump Derivation

- Head loss is due to violent turbulent mixing and dissipation that occur during the jump.
- One obvious solution is y_1=y_2 and h_L=0 → NO JUMP
- Another solution : Combine Eq 19 and 20 to eliminate V_2

$$\frac{y^2{}_1}{2} - \frac{y^2{}_2}{2} = \frac{V_1 y_1}{g}\left(\frac{V_1 y_1}{y_2} - V_1\right) = \frac{V_1{}^2 y_1}{g y_2}(y_1 - y_2) \qquad \text{Eq. 21b}$$

Hydraulic Jump Derivation

$$\left(\frac{y_2}{y_1}\right)^2+\left(\frac{y_2}{y_1}\right)-2Fr_1^2=0$$

Eq. 21c

Where Fr_1 is upstream Froude number

Question : Obtain Eq. 21c from Eq. 21b

- **Using quadratic formula we get**

$$\frac{y_2}{y_1}=\frac{1}{2}\left(-1\pm\sqrt{(1+8Fr_1^2})\right)$$

Hydraulic Jump Derivation

- Solution with minus sign is neglected ??, Thus

$$\frac{y_2}{y_1} = \frac{1}{2}(-1 + \sqrt{(1 + 8Fr_1^2})$$

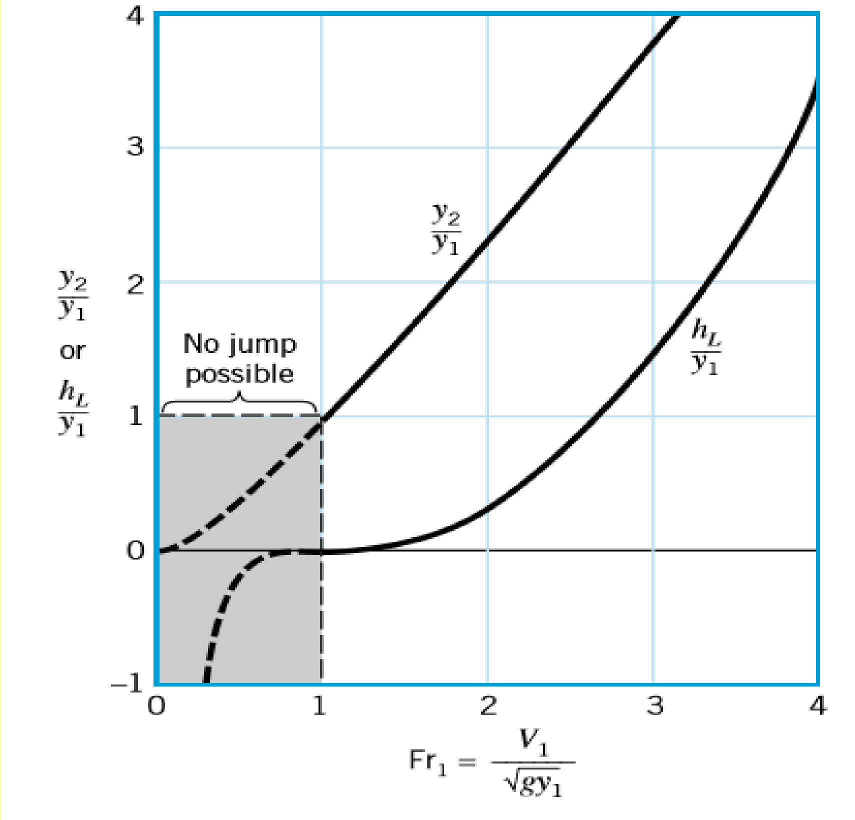
Eq. 22

- We can also obtain h_L / y_1 by using Eq. 21
  - The result is

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}[1 - (\frac{y_1}{y_2})^2]$$

Eq. 23

Hydraulic Jump Derivation



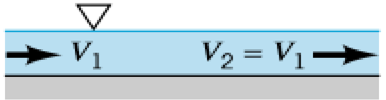
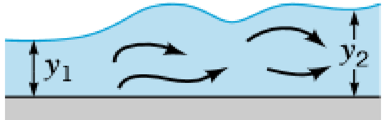
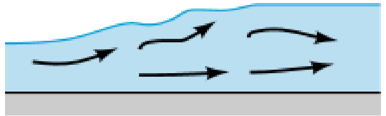
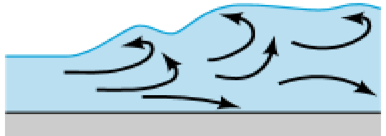
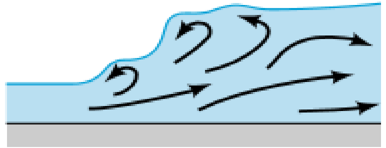
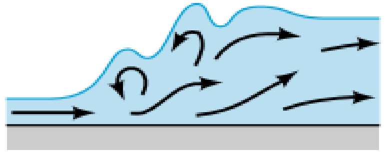
Question : Plot of Eq. 22 and corresponding Eq. 23

Hydraulic Jump Derivation

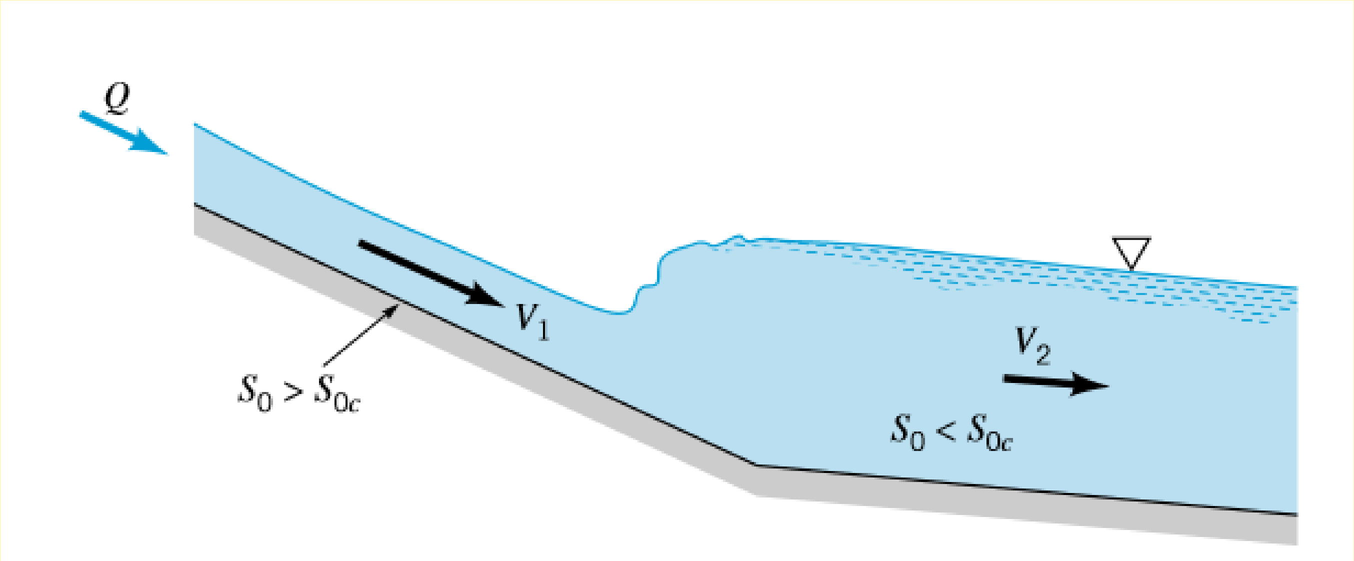
- **h_L cannot be negative since it violates the law of thermodynamics**
- **This means that y_2/y_1 cannot be less than 1 and Froude number upstream Fr_1 is always greater than 1 for hydraulic jump to take place.**

- ***A flow must be supercritical to produce discontinuity called a hydraulic jump.***

Classification of Hydraulic Jumps (Ref. 12)

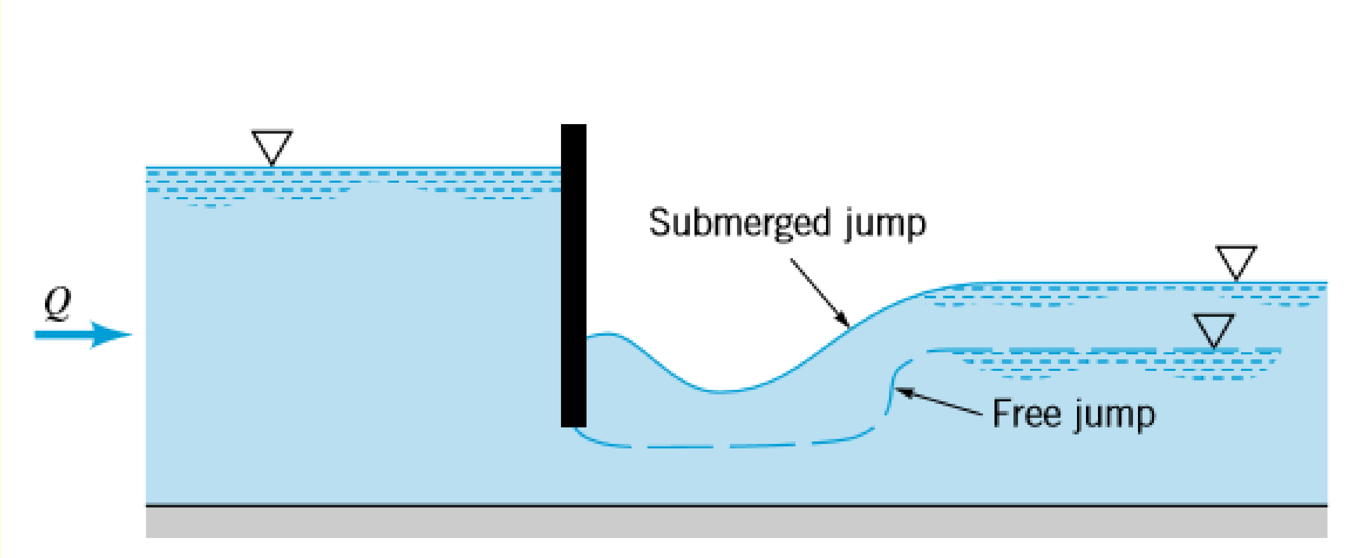
Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

Examples of hydraulic jump



Jump caused by a change in channel slope

Examples of hydraulic jump



Submerged hydraulic jumps that can occur just downstream of a sluice gate

Class Question

In a flow through rectangular channel for a certain discharge the Froude number corresponding to the two alternative depths are F_1 and F_2. Show that

$$\left(F_2/F_1\right)^{2/3} = \frac{2+F_2^{\,2}}{2+F_1^{\,2}}$$

Solution:

Let y_1 and y_2 be the alternative depths.

The specific energy $E_2 = E_1$

$$y_1 + \frac{V_1^{\,2}}{2g} = y_2 + \frac{V_2^{\,2}}{2g}$$

$$y_1\left(1+\frac{V_1^2}{2gy_1}\right)=y_2\left(1+\frac{V_2^2}{2gy_2}\right)$$

Since $\frac{V^2}{gy}=F^2=Froude\,number$

$$\frac{y_1}{y_2}=\frac{1+F_2^2/2}{1+F_1^2/2}=\frac{2+F_2^2}{2+F_1^2}$$

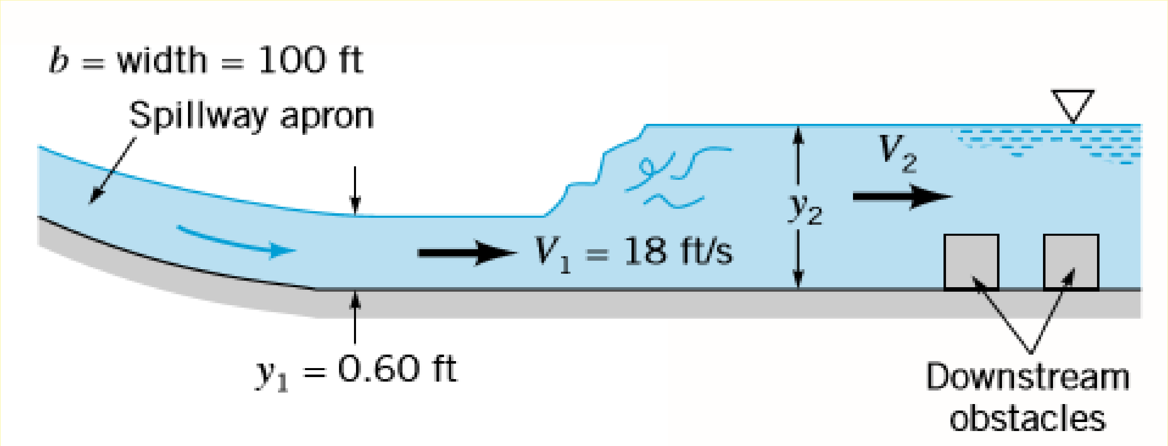
Also $F_1^2=\frac{Q^2}{B^2g\,y_1^3}$ *and* $F_2^2=\frac{Q^2}{B^2g\,y_2^3}$

Where Q = discharge in the channel and B = width of the channel, Hence

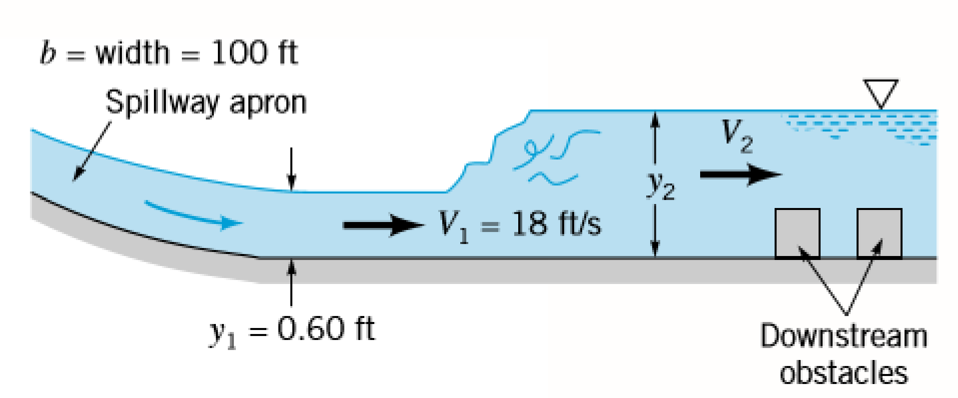
$$\frac{y_1^3}{y_2^3}=\frac{F_2^2}{F_1^2} \quad or \quad \left(\frac{y_1}{y_2}\right)=\left(\frac{F_2^2}{F_1^2}\right)^{2/3} \qquad \frac{y_1}{y_2}=\left(\frac{F_2^2}{F_1^2}\right)^{2/3}=\frac{2+F_2^2}{2+F_1^2}$$

Class Question

- Water on the horizontal apron of the 30 m wide spillway shown in Fig. has a depth of 0.20 m and a velocity of 5.5 m/s. Determine the depth, after the jump, the Froude numbers before and after the jump.



Class Question

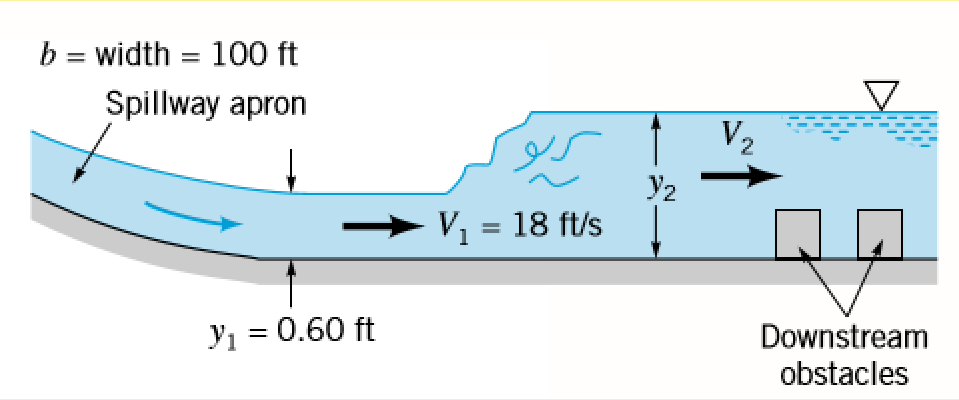


Conditions across the jump are determined by the upstream Froude number F_{r1}

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5.5}{\sqrt{9.8*0.2}} = 3.92$$

Upstream flow is super critical, and therefore it is possible to generate hydraulic jump

Class Question

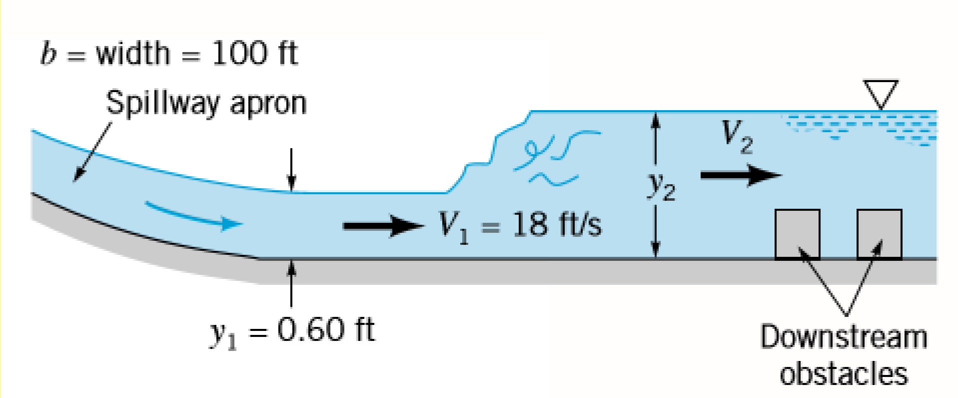


We obtain depth ratio across the jump as

$$\frac{y_2}{y_1} = \frac{1}{2}(-1+\sqrt{(1+8Fr_1^2})=\frac{1}{2}(-1+\sqrt{(1+8*3.92^2})=5.07$$

$$y_2 = 5.07*0.2 = 1.01 \text{ m}$$

Class Question

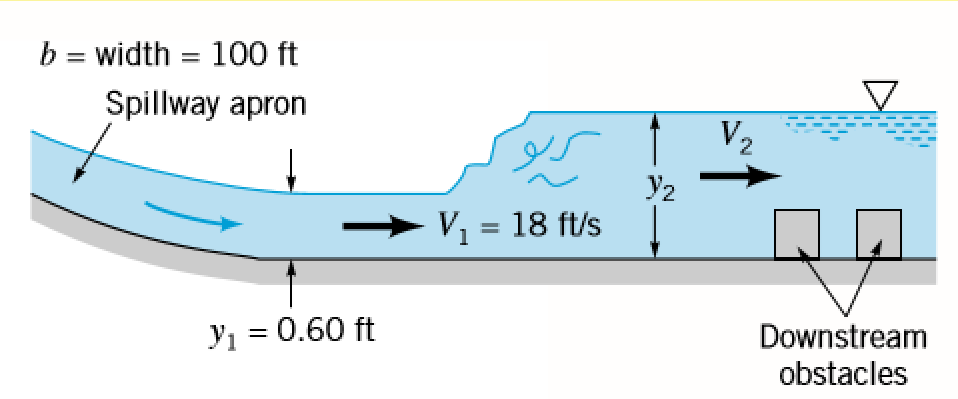


We obtain V_2 by equating the flow rate

$$V_2 = \frac{(y_1 V_1)}{y_2} = \frac{0.2*5.5}{1.01} = 1.08 \quad \text{m/s}$$

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{1.08}{\sqrt{9.8*1.01}} = 0.343 \qquad \text{Subcritical Flow}$$

Class Question



Head loss is obtained as

$$h_L = (y_1 + \frac{V_1^2}{2g}) - (y_2 + \frac{V_2^2}{2g})$$

$$h_L = 0.671 \text{ m}$$

Class Question

1) Prove that energy loss in a hydraulic jump occurring in a rectangular channel is

$$h_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2} \qquad \textbf{Eq. 24}$$

The loss of mechanical energy that takes place in a hydraulic jump is calculated by the application of energy equation (Bernoulli's equation). If loss of total head in the pump is h_L, then we can write by Bernoulli's equation neglecting the slope of the channel.

$$y_1 + \left(V_1^2/2g\right) = y_2 + \left(V_2^2/2g\right) + h_L$$

$$h_L = y_1 - y_2 + \left(V_1^2/2g\right) - \left(V_2^2/2g\right)$$

$$h_L = y_1 - y_2 + \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) \qquad q = V_1 y_1 = V_2 y_2$$

From Eq 21.c we are putting $V_1 = \frac{q}{y_1}$ $\left(F_{r1} = \frac{V_1}{\sqrt{g y_1}}\right)$

$$y_1 y_2^2 + y_1^2 y_2 - \frac{2q^2}{g} = 0$$

$$\frac{y_1 y_2^2 + y_1^2 y_2}{4} = \frac{q^2}{2g}$$

$$h_L = y_1 - y_2 + \left(\frac{y_1 y_2^2 + y_1^2 y_2}{4}\right)\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)$$

Which Finally gives

$$h_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

Class Question

If, in a hydraulic jump occurring in a rectangular channel, the Froude number before the jump is 10.0 and the energy loss is 3.20 m. Estimate (i) the sequent depths (ii) the discharge intensity and (iii) the Froude number after the jump.

Solution:

$$F_1 = 10.0 \quad and \quad E_L = 3.20\ m$$

The sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right] = \frac{1}{2}\left[-1+\sqrt{1+8\times(10.0)^2}\right] = 13.651$$

Energy loss

$$E_L = \frac{(y_2 - y_1)^3}{4\, y_1\, y_2}$$

$$\frac{E_L}{y_1} = \frac{(y_2/y_1 - 1)^3}{4(y_2/y_1)} \qquad \frac{3.20}{y_1} = \frac{(13.651-1)^3}{4(13.651)} = 37.08$$

(i) y_1 **=depth before the jump =** $\frac{3.20}{37.08} = 0.0863\, m$

y_2 **=depth after the jump =13.651×0.0863= 1.178 m**

(ii) $F_1 = \frac{V_1}{\sqrt{gy_1}} \qquad 10.0 = \frac{V_1}{\sqrt{9.81 \times 0.0863}} \qquad V_1 = 9.201\ m/s$

Discharge intensity $q = V_1 y_1 = 9.201 \times 0.0863 = 0.7941\ m^3/s/m$

(iii) Froude number after the jump $F_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{q}{y_2\sqrt{gy_2}} = \frac{0.7941}{1.178\sqrt{9.81 \times 1.178}} = 0.1983$

Class Question

A rectangular channel has a width of 1.8 m and carries a discharge of 1.8 at a depth of 0.2 m. Calculate (a) the specific energy, (b) depth alternate to the existing depth and (c) Froude numbers at the alternate depths.

Solution:

Let $y_1 = 0.20 m = Existing\ depth$

Area $A_1 = By_1 = 1.8 \times 0.20 = 0.36\ m^2$

Velocity $V_1 = Q \big/ A_1 = \frac{1.80}{0.36} = 5.0\ m^2/s$

(a) Specific energy $E_1 = y_1 + \frac{V_1^2}{2g} = 0.20 + \frac{(5.0)^2}{2 \times 9.81} = 1.4742\ m$

(b) Let y_2 **=depth alternate to** y_1

Then $E_2 = E_1$ $\quad y_2 + \frac{V_2^2}{2g} = 1.4742$ $\quad y_2 + \frac{(1.8)^2}{(2\times 9.81)\times (1.8)^2 \times y_2^2} = 1.4742, \quad as\, V_1 A_1 = V_2 A_2$

By trial and error, $y_2 = 1.45$

(c) Froude number for a rectangular channel is $F = V/\sqrt{gy}$

For $y_1 = 0.2m, \quad F_1 = \frac{5.0}{\sqrt{9.81 \times 0.2}} = 3.57$

For $y_2 = 1.45m, \quad V_2 = \frac{Q}{By_2} = \frac{1.80}{1.80 \times 1.45} = 0.69\ m/s$

$$F_2 = \frac{0.69}{\sqrt{9.81 \times 1.45}} = 0.1829$$

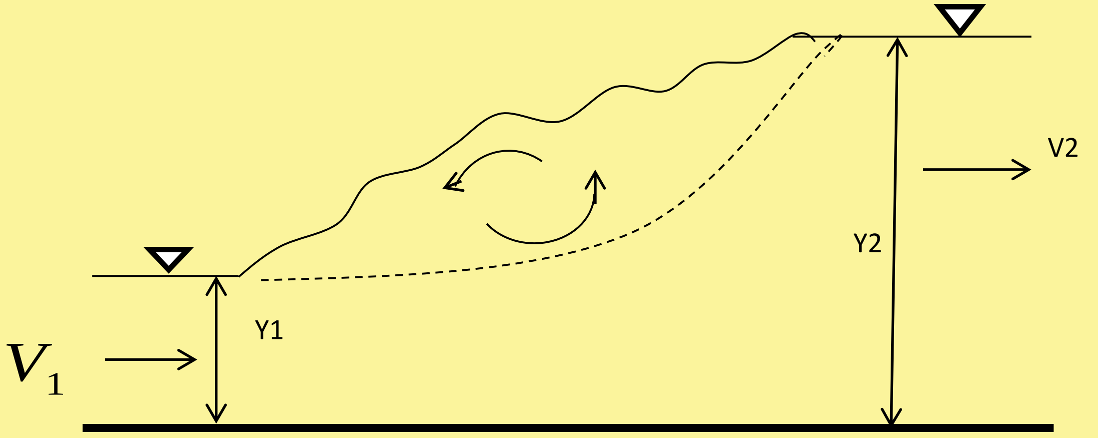
Class Question

In hydraulic jump occurring in a rectangular horizontal channel, the discharge per unit width is 2.5 m^3/s/m and the depth before the jump is 0.25 m. Estimate (i) the sequent depth and (ii) the energy loss

Solution:

$$q = 2.5\, m^3/s/m \quad and \quad y_1 = 0.25\, m$$

$$V_1 = \frac{q}{y_1} = \frac{2.5}{0.25} = 10.0\, m/s$$



Initial Froude number $$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{10.0}{\sqrt{9.81 \times 0.25}} = 6.386$$

(i) The sequent depth ratio y_2/y_1 is given by $\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right]$

$$\frac{y_2}{0.25} = \frac{1}{2}\left[-1+\sqrt{1+8\times(6.386)^2}\right]$$

$$y_2 = 2.136\ m = Sequent\ depth$$

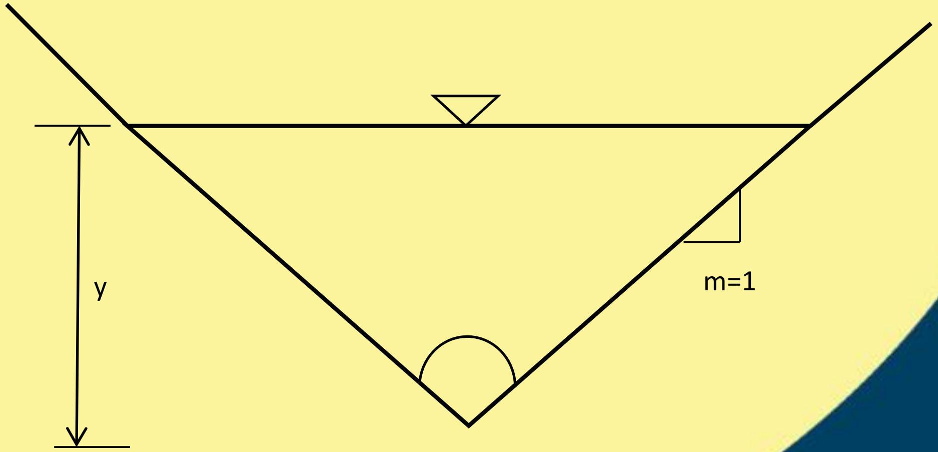
(ii) The energy loss E_L is given by

$$E_L = \frac{(y_2 - y_1)^3}{4\ y_1\ y_2} = \frac{(2.136-0.250)^3}{4\times 2.136\times 0.250} = 3.141\,m$$

Class Question

A hydraulic jump occur in a horizontal triangular channel. If the sequent depths in this channel are 0.60 m and 1.20 m respectively, estimate (i) the flow rate, (ii) Froude number at the beginning and end of the jump and (iii) energy loss in the jump.

Solution:



(i) Consider a triangular channel of side slope m horizontal: 1 vertical in fig (in the present case m=1)

P=pressure force= $\gamma A\bar{y} = \gamma\left(my^2\right)\frac{y}{3} = \gamma\, m\, y^3/3$

M=Momentum flux = $\frac{\rho Q^2}{A} = \frac{\rho Q^2}{my^2}$

For a hydraulic jump in horizontal, frictionless channel $P_1 + M_1 = P_2 + M_2$

$$\frac{\gamma m y_1^3}{3} + \frac{\rho Q^2}{m y_1^2} = \frac{\gamma m y_2^3}{3} + \frac{\rho Q^2}{m y_2^2}$$

$$\frac{Q^2}{m}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] = \frac{gm}{3}\left(y_2^3 - y_1^3\right)$$

On simplifying $\frac{Q^2}{g} = \frac{m^2}{3}\left[\frac{y_1^3\left(\eta^3 - 1\right)\eta^2 y_1^4}{\left(\eta^2 - 1\right)y_1^2}\right]$ *where* $\eta = \frac{y_2}{y_1}$

In the present problem m=1, $\eta = \frac{y_2}{y_1} = 1.2/0.6 = 2.0$

$$\frac{Q^2}{g} = \frac{1}{3}\left[\frac{(0.6)^5\left(2^3 - 1\right)2^2}{\left(2^2 - 1\right)}\right] = 0.24192$$

$$Q = 1.541\, m^3/s$$

II. **For triangular channel** $F = \frac{Q}{A\sqrt{g\,A/T}}$ **as such**

$$F^2 = \frac{Q^2 T}{gA^3} = \frac{Q^2(2my)}{gm^2 y^6} = \frac{2Q^2}{gm^2 y^5}$$

$$F_1^{\,2} = \frac{2(1.541)^2}{9.81 \times 1 \times (0.6)^5} = 6.222$$

$$F_1 = 2.494$$

Froude number at the end of the jump:

Since

$$F^2 = \frac{2Q^2}{gm^2 y^5}, \qquad \frac{F_1}{F_2} = \left(\frac{y_2}{y_1}\right)^{5/2} = \left(\frac{1.20}{0.60}\right)^{5/2} = 5.657$$

$$F_2 = 2.494/5.657 = 0.441$$

III. Energy loss

$$E_L = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$$

$$A_1 = 1 \times (0.6)^2 = 0.36\,m^2$$

$$V_1 = 1.541/0.36 = 4.281\,m/s$$

$$A_2 = 1 \times (1.2)^2 = 1.44\,m^2$$

$$V_2 = 1.54/1.44 = 1.070\,m/s$$

$$E_L = \left(0.6 + \frac{(4.281)^2}{2 \times 9.81} \right) - \left(1.2 + \frac{(1.070)^2}{2 \times 9.81} \right) = 1.534 - 1.258 = 0.276\,m$$

Class Question

Water flows in a wide channel at q =10 m^3/(s.m) and $y_1 = 1.25$ m. If the flow undergoes a hydraulic jump, compute (*a*) y_2, (*b*) V_2, (*c*) Fr_2, (*d*) h_f, (*e*) the percentage dissipation, (*f*) the power dissipated per unit width, and (*g*) the temperature rise due to dissipation if C_p 4200 J/(kg. K).

Solution:

$$V_1 = \frac{q}{y_1} = \frac{10m^3/(\text{s.m})}{1.25\ m} = 8.0\ m/s$$

The upstream Froude number is therefore

$$Fr_1 = \frac{V_1}{(gy_1)^{1/2}} = \frac{8.0}{[9.81(1.25)]^{1/2}} = 2.285$$

This is a weak jump. The depth y_2 is obtained from $\frac{2y_2}{y_1} = -1 + (1 + 8Fr_1^2)^{1/2}$

$$\frac{2y_2}{y_1} = -1 + (1 + 8(2.285)^2)^{\frac{1}{2}} = 5.54$$

$$y_2 = \frac{1}{2} y_1 (5.54) = 3.46\ m$$

The downstream velocity is $V_2 = \frac{V_1 y_1}{y_2} = \frac{8.0(1.25)}{3.46} = 2.89\ m/s$

The downstream Froude number is $Fr_2 = \frac{V_2}{(g y_2)^{1/2}} = \frac{2.89}{[9.81(3.46)]^{1/2}} = 0.496$

As expected, Fr_2 is subcritical, the dissipation loss is $h_f = \frac{(3.46 - 1.25)^3}{4(3.46)(1.25)} = 0.625\ m$

The percentage dissipation relates h_f to upstream energy:

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.25 + \frac{8^2}{2(9.81)} = 4.51$$

Hence percentage loss $= (100)\frac{h_f}{E_1} = \frac{100(0.625)}{4.51} = 14\ percent$

The power dissipated per unit width is

$$\text{Power} = \rho g q h_f = \left(9800 \frac{N}{m^3}\right)\left[10 \frac{m^3}{s.m}\right](0.625\ m) = 61.3\ kW/m$$

Finally, the mass flow rate is $\dot{m} = \rho q$ (1000 kg/m^3)[10 m^3/(s.m)] 10,000 kg/(s m), and the temperature rise from the steady flow energy equation is

$$\text{Power dissipated} = \dot{m} c_p \Delta T$$

$$61300 \frac{W}{m} = 10{,}000 \text{ kg/(s m)} \left[4200 \frac{J}{kg} . K\right] \Delta T$$

from which $\Delta T = 0.0015K$

The dissipation is large, but the temperature rise is negligible