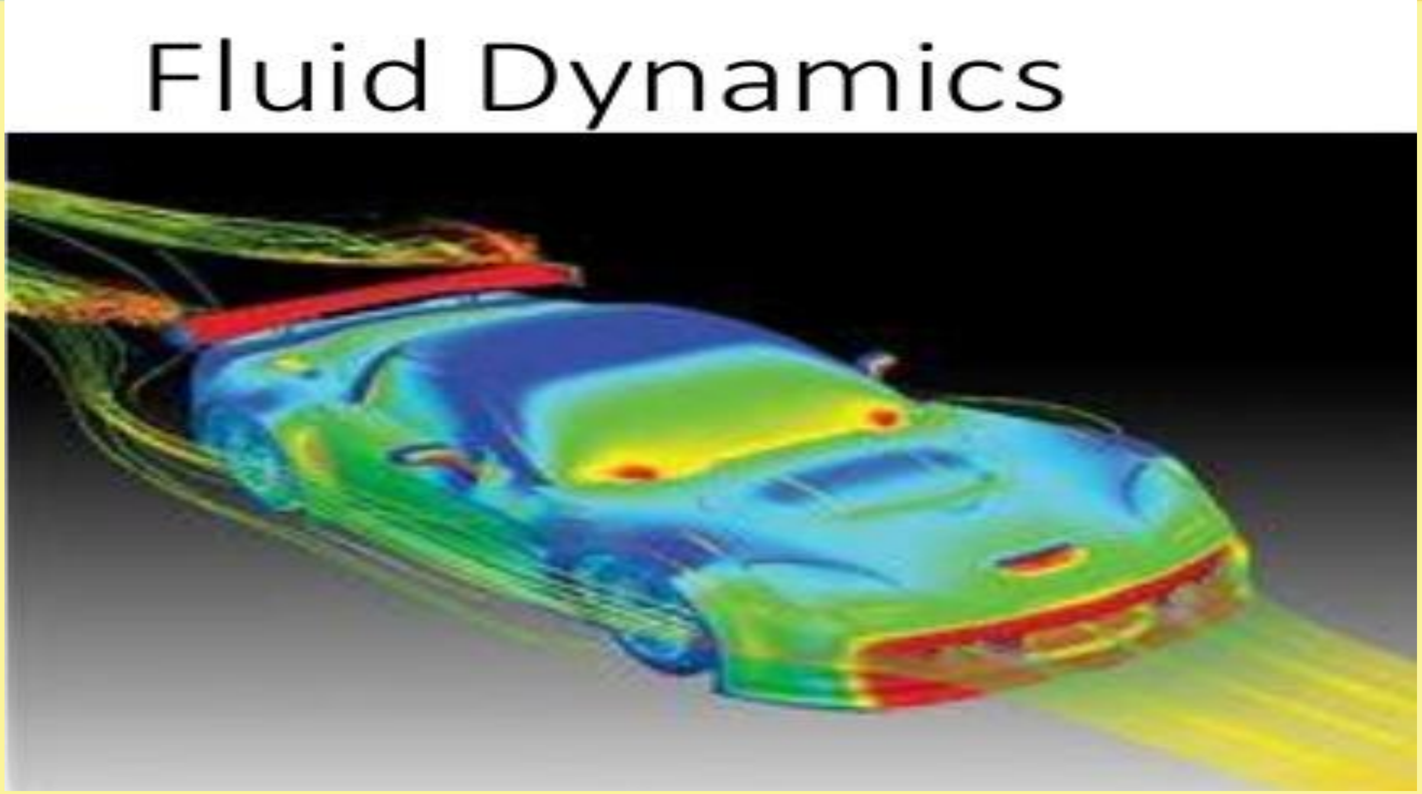


Hydraulics

Prof. Mohammad Saud Afzal

Department of Civil Engineering

Reynolds Transport Theorem (RTT)

- All physical laws are stated in terms of various physical parameters. Let B represent any of these (Velocity, acceleration, mass, temperature, and momentum etc.) fluid parameters and b represent the amount of that parameter per unit mass. That is

$$\boxed{\mathrm{B} = mb}$$

Where m is the mass of the portion of fluid of interest.

b = 1, if B = m

- The parameter B is termed as extensive property and the parameter b is termed as intensive property.

- The value of B is directly proportional to the amount of the mass being considered, whereas the value of b is independent of the amount of mass.

- The amount of an extensive property that a system possesses at a given instant, B_{sys} can be determined by adding up the amount associated with each fluid particle in the system.
- For infinitesimal fluid particles of size $\delta\forall$ and mass $\rho\delta\forall$ this summation (in the limit of $\delta\forall \to 0$) takes the form of an integration over all the particles in the system and can be written as

$$B_{sys} = \lim_{\delta\forall\to 0} \sum_i b_i(\rho_i \delta\forall_i) = \int_{sys} \rho b \, d\forall$$

- The limits of integration cover the entire system—a (usually) moving volume.
- We have used the fact that the amount of B in a fluid particle of mass $\rho\delta\forall$ is given in terms of b by

$$\delta B = b\rho\delta\forall$$

- Most of the laws governing fluid motion involve the time rate of change of an extensive property of a fluid system—the rate at which the momentum of a system changes with time, the rate at which the mass of a system changes with time, and so on. Thus, we often encounter terms such as

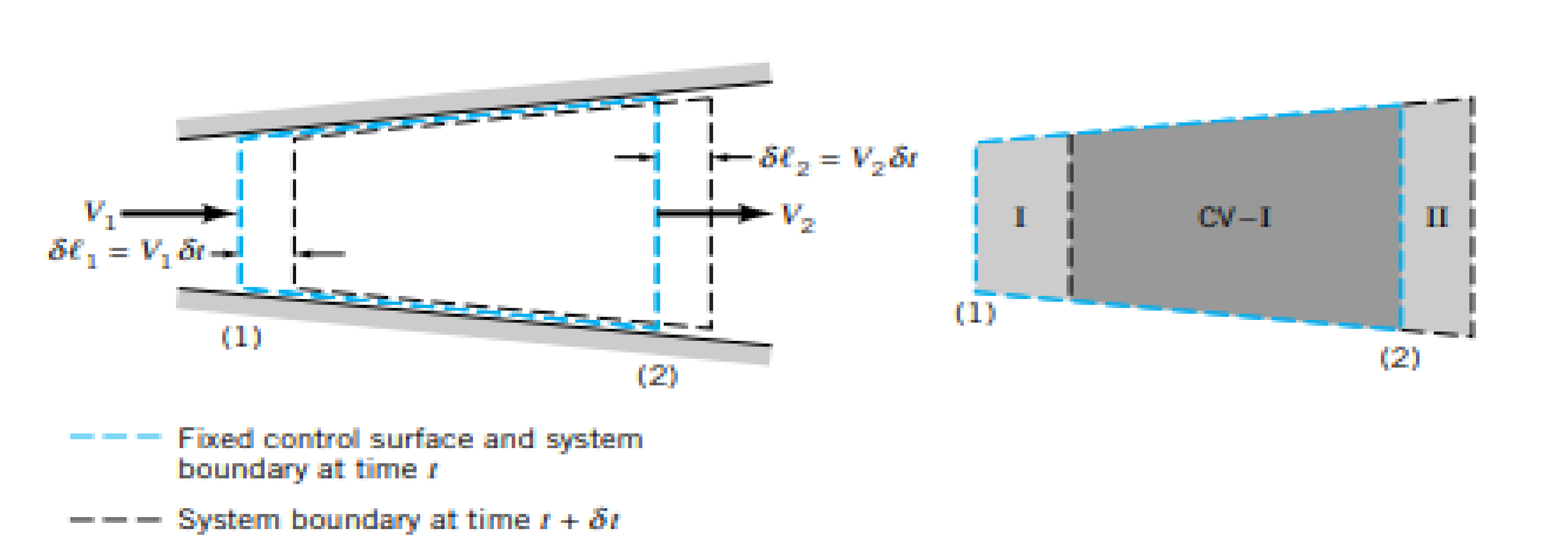
$$\frac{dB_{sys}}{dt} = \frac{d\left(\int_{sys} \rho b \, d\forall\right)}{dt}$$

- To formulate the laws into a control volume approach, we must obtain an expression for the time rate of change of an extensive property within a control volume, $B_{cv,}$ not within a system. This can be written as

$$\frac{dB_{cv}}{dt} = \frac{d\left(\int_{cv} \rho b \, d\forall\right)}{dt}$$

Derivation of the Reynolds Transport Theorem

- We consider the control volume to be that stationary volume within the pipe or duct between sections (1) and (2). The system that we consider is that fluid occupying the control volume at some initial time t



- A short time later, at time $t + \delta t$ the system has moved slightly to the right.
- The fluid particles that coincided with section (2) of the control surface at time t have moved a distance $\delta l_2 = V_2 \delta t$ to the right, where V_2 is the velocity of the fluid as it passes section (2). Similarly, the fluid initially at section (1) has moved a distance $\delta l_1 = V_1 \delta t$ where V_1 is the fluid velocity at section (1).
- If B is an extensive parameter of the system, then the value of it for the system at time t is

$$B_{sys}(t) = B_{cv}(t)$$

- since the system and the fluid within the control volume coincide at this time.

  Its value at time $t + \delta t$ is

Derivation Continue…

$$B_{sys}(t + \delta t) = B_{cv}(t + \delta t) - B_I(t + \delta t) + B_{II}(t + \delta t)$$

- Thus, the change in the amount of B in the system in the time interval δt divided by this time interval is given by
- If B is an extensive parameter of the system, then the value of it for the system at time t is

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{sys}(t + \delta t) - B_{sys}(t)}{\delta t}$$

$$= \frac{B_{cv}(t + \delta t) - B_I(t + \delta t) + B_{II}(t + \delta t) - B_{sys}(t)}{\delta t}$$

- By using the fact that at the initial time t we have $B_{sys}(t) = B_{cv}(t)$ this ungainly expression may be rearranged as follows.

Derivation Continue…

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{cv}(t+\delta t)-B_{cv}(t)}{\delta t} - \frac{B_I(t+\delta t)}{\delta t} + \frac{B_{II}(t+\delta t)}{\delta t}$$

- In the limit $\delta t \to 0$ the first term on the right-hand side of Eq. is seen to be the time rate of change of the amount of B within the control volume

$$\lim_{\delta t \to 0} \frac{B_{cv}(t+\delta t)-B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t} = \frac{\partial\left(\int_{cv} \rho b \, d\forall\right)}{\partial t}$$

- The third term on the right-hand side of Eq. represents the rate at which the extensive parameter B flows from the control volume, across the control surface. This can be seen from the fact that the amount of B within region II, the outflow region, is its amount per unit volume, ρb times the volume $\delta\forall_{II} = A_2 \delta l_2 = A_2(V_2 \delta t)$.Hence

Derivation Continue…

$$B_{II}(t+\delta t) = (\rho_2 b_2)(\delta \forall_{II} = \rho_2 b_2 A_2 V_2 \delta t)$$

- Where b_2 and ρ_2 are the constant values of b and ρ across section (2). Thus, the rate at which this property flows from the control volume, $\dot{B}_{out}$ is given by

$$\dot{B}_{out} = \lim_{\delta t \to 0} \frac{B_{II}(t+\delta t)}{\delta t} = \rho_2 A_2 V_2\, b_2$$

- Similarly, the inflow of B into the control volume across section (1) during the time interval δt corresponds to that in region I and is given by the amount per unit volume times the volume, $\delta\forall_I = A_1 \delta l_1 = A_1(V_1 \delta t)$.Hence

$$B_I(t+\delta t) = (\rho_1 b_1)(\delta \forall_I = \rho_1 b_1 A_1 V_1 \delta t)$$

Derivation Continue…

- Where b_1 and ρ_1 are the constant values of b and ρ across section (1). Thus, the rate at which this property flows from the control volume, $\dot{B}_{in}$ is given by

$$\dot{B}_{in} = \lim_{\delta t \to 0} \frac{B_I(t+\delta t)}{\delta t} = \rho_1 A_1 V_1 \, b_1$$

- If we combine all the equations we see that the relationship between the time rate of change of B for the system and that for the control volume is given by

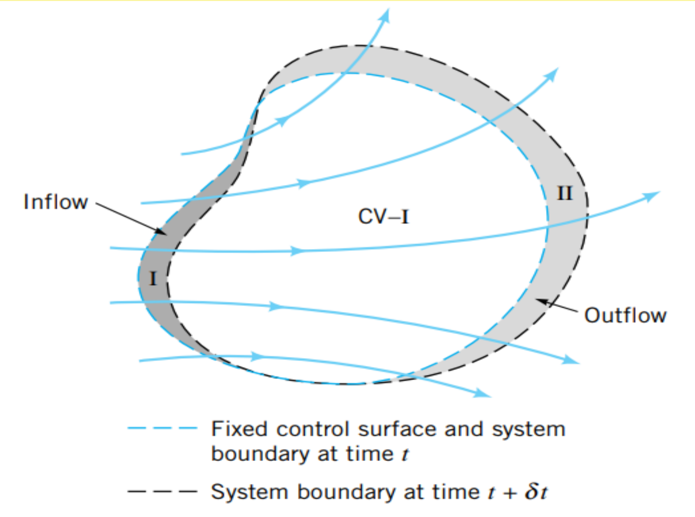
$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in} \qquad \frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$

- This is a simplified version of the Reynolds transport theorem.

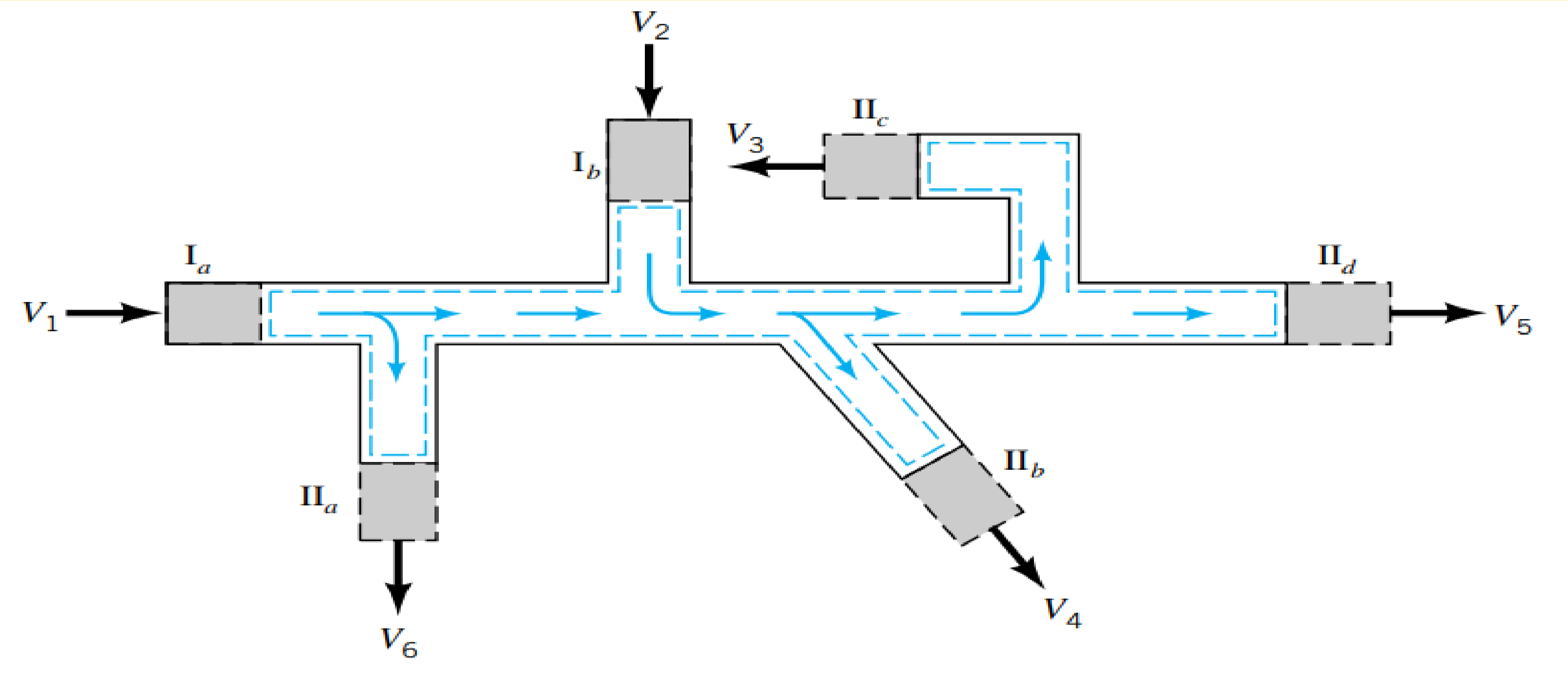
Control volume and system for flow through an arbitrary, fixed control volume.

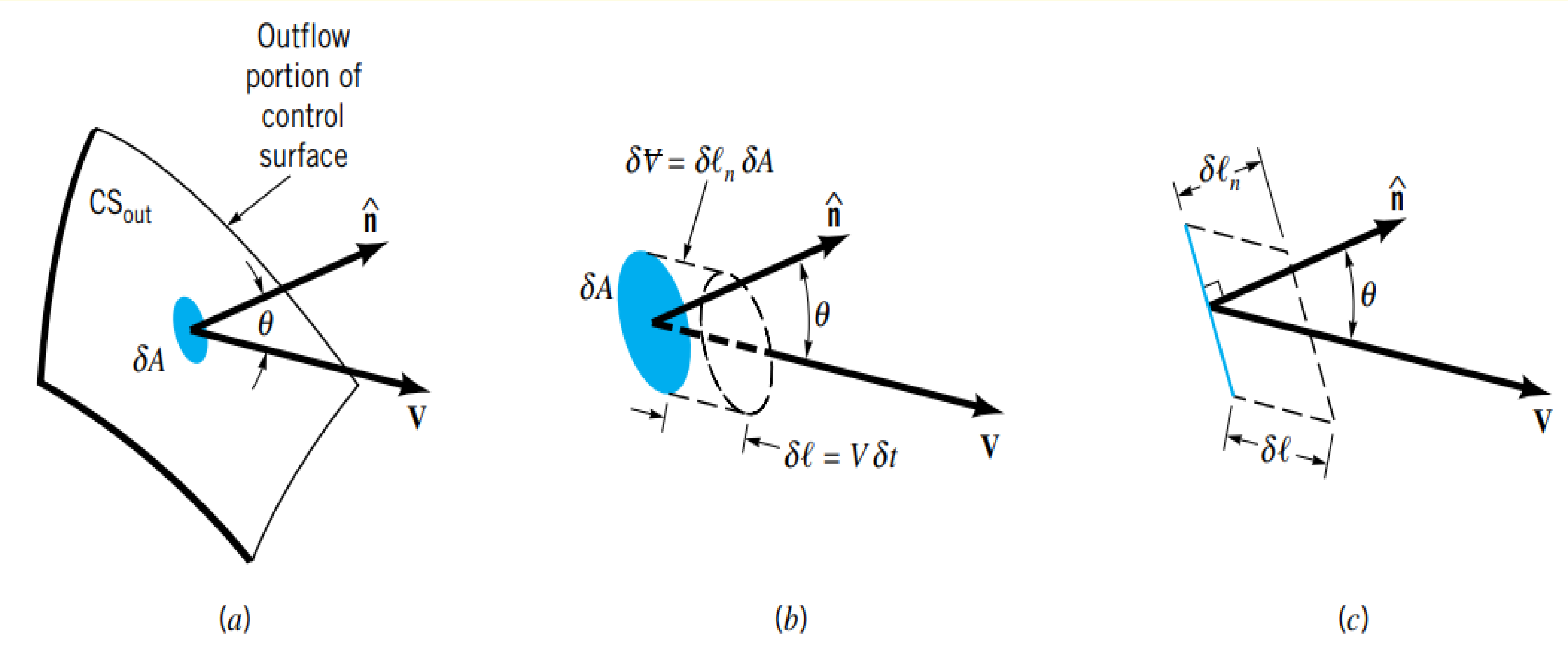
- We will now derive it for much more general conditions.

- We consider an extensive fluid property B and seek to determine how the rate of change of B associated with the system is related to the rate of change of B within the control volume at any instant.



- In general, the control volume may contain more (or less) than one inlet and one outlet. A typical pipe system may contain several inlets and outlets as are shown in Fig.




Outflow portion of control surface
CS_{out}
δA
θ
$\hat{n}$
V
(a)
$\delta \forall = \delta \ell_n \, \delta A$
δA
θ
$\hat{n}$
V
$\delta \ell = V \delta t$
(b)
$\delta \ell_n$
$\hat{n}$
θ
V
$\delta \ell$
(c)

- The term $\dot{B}_{out}$ represents the net flowrate of the property B from the control volume.
- Its value can be thought of as arising from the addition (integration) of the contributions through each infinitesimal area element of size on the portion δA of the control surface dividing region II and the control volume. This surface is denoted CS_{out} .

- In time δt the volume of fluid that passes across each area element is given by $\delta\forall = \delta l_n \delta A$, where $\delta l_n = \delta l cos\theta$ is the height (normal to the base, δA) of the small volume element, and θ is the rate at which B is carried out of the control volume across the small area element denoted is angle between the velocity vector and the outward pointing normal to the surface, $\hat{n}$. Thus, since $\delta l = V\delta t$ the amount of the property B carried across the area element δA in the time interval δt is given by

$$\delta B = b\rho\delta\forall = b\rho(V\ cos\theta\ \delta t)\ \delta A$$

- The rate at which B is carried out of the control volume across the small area element δA,denoted $\delta \dot{B}_{out}$, is

- In time δt the volume of fluid that passes across each area element is given by $\delta\forall = \delta l_n \delta A$, where $\delta l_n = \delta l cos\theta$ is the height (normal to the base, δA) of the small volume element, and θ is the rate at which B is carried out of the control volume across the small area element denoted is angle between the velocity vector and the outward pointing normal to the surface, $\hat{n}$. Thus, since $\delta l = V\delta t$ the amount of the property B carried across the area element δA in the time interval δt is given by

$$\delta \dot{B}_{out} = \lim_{\delta t \to 0} \frac{\rho b \delta\forall}{\delta t} = \lim_{\delta t \to 0} \frac{(\rho b V\ cos\theta\ \delta t)\delta A}{\delta t}$$

$$= \rho b V\ cos\theta\ \delta A$$

- The rate at which B is carried out of the control volume across the small area element δA,denoted $\delta \dot{B}_{out}$, is

- By integrating over the entire outflow portion of the control surface, CS_{out} we obtain

$$\dot{B}_{out} = \int_{CS_{out}} d\dot{B}_{out} = \int_{CS_{out}} \rho b V\, cos\theta\, \delta A$$

- The quantity $V\, cos\theta$ is the component of the velocity normal to the area element δA.

- From the definition of the dot product, this can be written as $V\, cos\theta = V.\hat{n}$. Hence, an alternate form of the outflow rate is

$$\dot{B}_{out} = \int_{CS_{out}} \rho b V.\hat{n}\, \delta A$$

- In a similar fashion, by considering the inflow portion of the control surface, CS_{in} can be written as

$$\dot{B}_{in} = -\int_{CS_{in}} \rho b V \cos\theta \, \delta A = -\int_{CS_{in}} \rho b V.\hat{n} \, \delta A$$

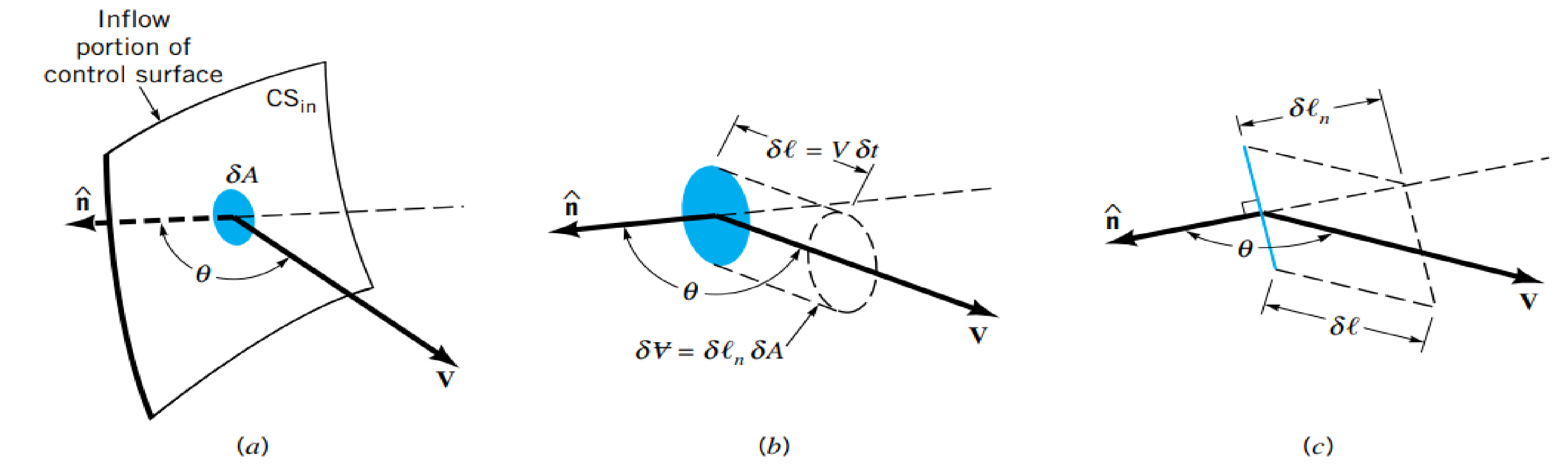
- For outflow regions (the normal component of V is positive)

$$V.\hat{n} > 0 \,, \; -90^0 < \theta < 90^0$$

- For inflow regions (the normal component of V is negative)

$$V.\hat{n} < 0 \,, \; 90^0 < \theta < 270^0$$

- The value of $cos\theta$ is, therefore, positive on the CV_{out} portions of the control surface and negative on the CV_{in} portions. Over the remainder of the control surface, there is no inflow or outflow, leading to $V.\hat{n} = V\ cos\theta = 0$ on those portions. On such portions either $V = 0$ (the fluid "sticks" to the surface) or $cos\theta = 0$ (the fluid "slides "along the surface without crossing it)



- Therefore, the net flux (flowrate) of parameter B across the entire control surface is

$$\dot{B}_{out} - \dot{B}_{in} = \int_{CS_{out}} \rho b V.\hat{n}\,\delta A - \left(-\int_{CS_{in}} \rho b V.\hat{n}\,\delta A\right)$$

$$= \int_{cs} \rho b V.\hat{n}\,\delta A$$

- where the integration is over the entire control surface. By combining above equations we obtain

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \int_{cs} \rho b V.\hat{n}\,\delta A$$

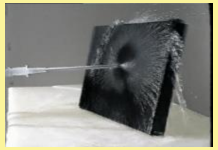
This can be written in a slightly different form by using so that

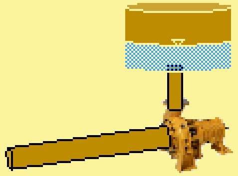
This can be written in a slightly different form by using $B_{cv} = \int_{cv} \rho b d\forall$ so that

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b d\forall + \int_{cs} \rho b V.\hat{n}\, \delta A$$

This is the general form of the Reynolds transport theorem for a fixed, nondeforming control volume.

Moving from a System to a Control Volume



- Mass
- Linear Momentum
- Moment of Momentum
- Energy
- Putting it all together!

Conservation of Mass

B = Total amount of mass in the system

b = mass per unit mass = 1

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b\, d\mathcal{V} + \int_{cs} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$$

cv equation

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$$

But $DM_{sys}/Dt = 0$!

$$\int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = -\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V}$$

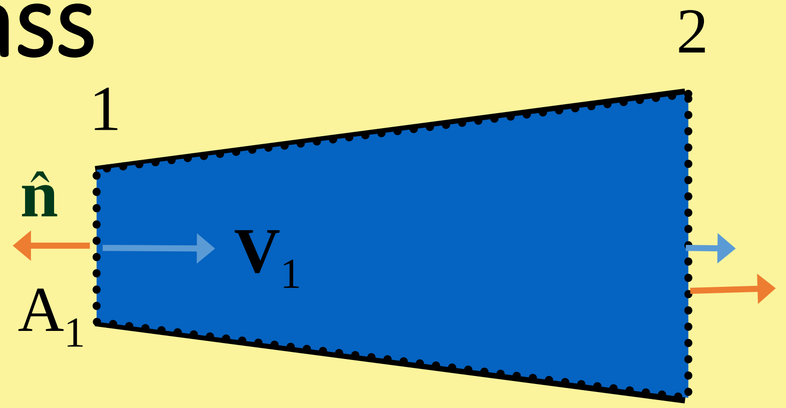
Continuity Equation

mass leaving - mass entering = - rate of increase of mass in cv

Conservation of Mass

$$\int_{cs} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\,dA = -\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall$$

If mass in cv is constant



$$\int_{cs}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA = \pm r\bar{V}A = \pm\dot{m} \quad [M/T]$$

Unit vector $\hat{\mathbf{n}}$ is normal to surface and pointed out of cv

We assumed uniform ρ on the control surface

$$\bar{V} = \left|\frac{\int_{cs}\mathbf{V}\cdot\hat{\mathbf{n}}\,dA}{A}\right|$$

$\bar{V}$ is the spatially averaged velocity normal to the cs

Continuity Equation for Constant Density and Uniform Velocity

$$\int_{cs_1} \rho_1 \mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 dA + \int_{cs_2} \rho_2 \mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 dA = 0$$

Density is constant across cs

$$-\rho_1 \overline{V}_1 A_1 + \rho_2 \overline{V}_2 A_2 = 0$$

Density is the same at cs_1 and cs_2

$$\overline{V}_1 A_1 = \overline{V}_2 A_2 = Q$$

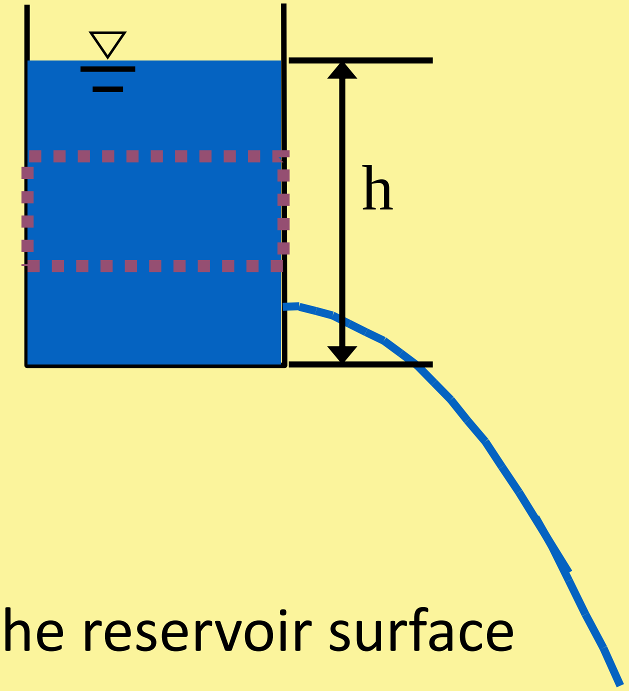
$[L^3/T]$

$$V_1 A_1 = V_2 A_2 = Q$$

Simple version of the continuity equation for conditions of constant density. It is understood that the velocities are either uniform or spatially averaged.

Example: Conservation of Mass?

The flow out of a reservoir is 2 L/s. The reservoir surface is 5 m x 5 m. How fast is the reservoir surface dropping?



$$\int_{cs} \cancel{\rho} \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial}{\partial t} \int_{cv} \cancel{\rho} d\forall$$

$$\int_{cs} \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\frac{\partial \forall}{\partial t}$$

$$Q_{out} - \cancel{Q_{in}} = -\frac{d\forall}{dt}$$

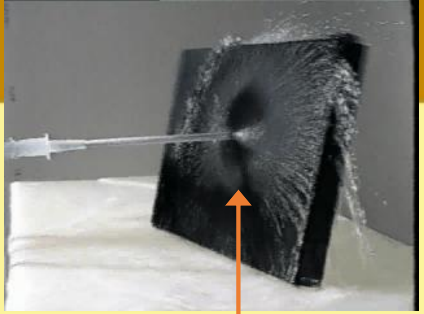
$$Q_{out} = -\frac{A_{res} dh}{dt}$$

Constant density

Velocity of the reservoir surface

$$\frac{dh}{dt} = -\frac{Q}{A_{res}}$$

Linear Momentum Equation



$\sum F \neq 0$

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b d\forall + \int_{cs} \rho b \mathbf{V}\cdot\hat{\mathbf{n}} dA$$ cv equation

$\mathbf{B} = m\mathbf{V}$ momentum $\mathbf{b} = \frac{m\mathbf{V}}{m}$ momentum/unit mass

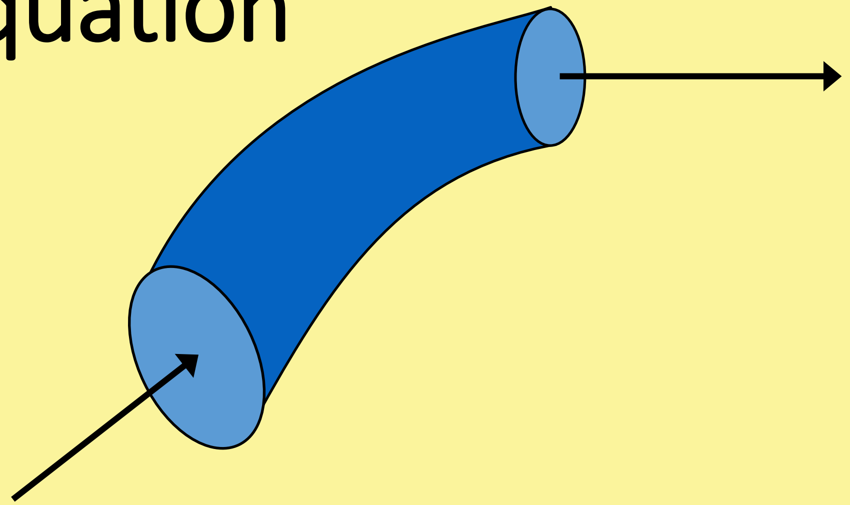
Vectors!

$$\frac{Dm\mathbf{V}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho \mathbf{V} d\forall + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}} dA$$

$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}} dA$$ Steady state

This is the "*ma*" side of the $F = ma$ equation!

Linear Momentum Equation



$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs} \mathbf{V}\rho\mathbf{V} \cdot \hat{\mathbf{n}} dA$$

$$\frac{Dm\mathbf{V}}{Dt} = \int_{cs_1} \mathbf{V}_1\rho_1\mathbf{V}_1 \cdot \hat{\mathbf{n}}_1 dA + \int_{cs_2} \mathbf{V}_2\rho_2\mathbf{V}_2 \cdot \hat{\mathbf{n}}_2 dA$$

$$\frac{Dm\underline{\mathbf{V}}}{Dt} = -(\rho_1 V_1 A_1)\underline{\mathbf{V}_1} + (\rho_2 V_2 A_2)\underline{\mathbf{V}_2}$$

$$\underline{\mathbf{M}}_1 = -(\rho_1 V_1 A_1)\underline{\mathbf{V}_1} = -(\rho Q)\underline{\mathbf{V}_1}$$

$$\underline{\mathbf{M}}_2 = (\rho_2 V_2 A_2)\underline{\mathbf{V}_2} = (\rho Q)\underline{\mathbf{V}_2}$$

Vectors!!!

Assumptions

Uniform density

Uniform velocity

$V \perp \mathbf{A}$

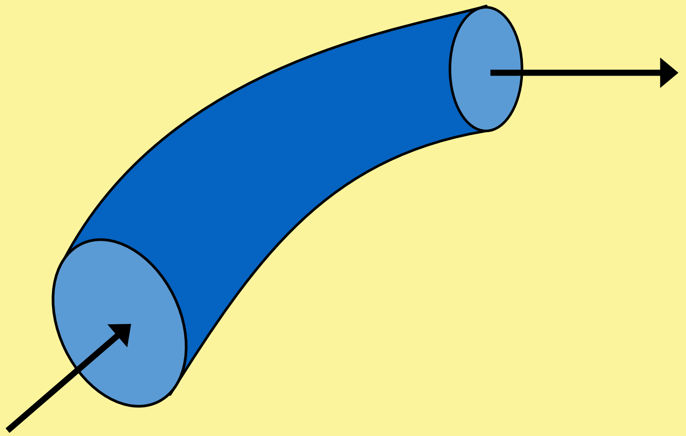
Steady

V fluid velocity relative to cv

Steady Control Volume Form of Newton's Second Law

$$\sum \mathbf{F} = \frac{D(m\mathbf{V})}{Dt} = \mathbf{M}_1 + \mathbf{M}_2$$

- What are the forces acting on the fluid in the control volume?
  - Gravity
  - Shear at the solid surfaces
  - Pressure at the solid surfaces
  - Pressure on the flow surfaces



$$\sum \mathbf{F} = \mathbf{M}_1 + \mathbf{M}_2$$

$$\sum \mathbf{F} = \mathrm{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{p_{wall}} + \mathbf{F}_{\tau_{wall}}$$

Why no shear on control surfaces? No velocity tangent to control surface

Linear Momentum Equation

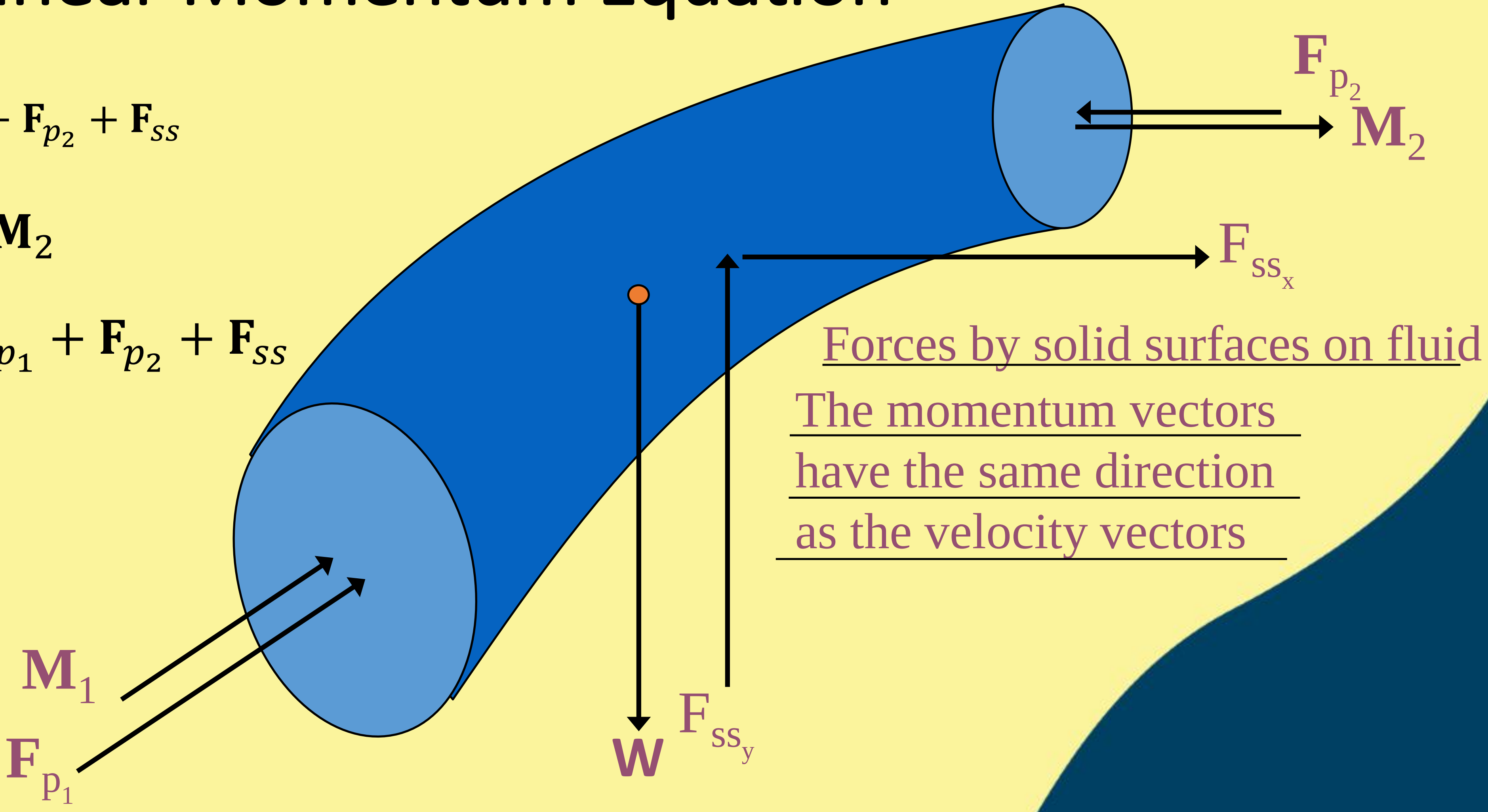
$$\sum \mathbf{F} = W + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

$$m\mathbf{a} = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_1 + \mathbf{M}_2 = W + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

$$\mathbf{M}_1 = -(\rho Q)\mathbf{V}_1$$

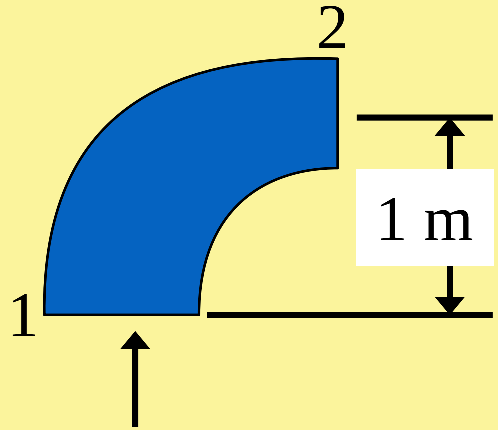
$$\mathbf{M}_2 = (\rho Q)\mathbf{V}_2$$



Example: Reducing Elbow

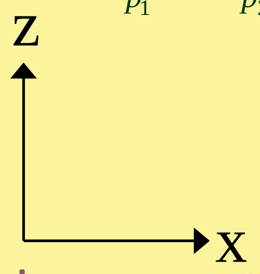
Reducing elbow in vertical plane with water flow of 300 L/s. The volume of water in the elbow is 200 L. Energy loss is negligible. Calculate the force of the elbow on the fluid.

W = -ρg*volume=-1961 N ↑



$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

	section 1	section 2
D	50 cm	30 cm
A	0.196 m^2	0.071 m^2
V	1.53 m/s ↑	4.23 m/s →
p	150 kPa	?
M	-459 N ↑	1270 N →
F_p	29,400 N ↑	?←



Direction of V vectors

Example: What is p_2?

$$\frac{p_1}{\gamma_1} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_2} + z_2 + \frac{V_2^2}{2g}$$

$$p_2 = p_1 + \gamma\left[z_1 - z_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right]$$

$$p_2 = (150 \text{ x } 10^3\text{Pa}) + (9810 \text{ N/m}^3)\left[0 - 1\text{ m} + \frac{(1.53\text{ m/s})^2 - (4.23\text{ m/s})^2}{2(9.8\text{ m/s}^2)}\right]$$

$P_2 = 132$ kPa

$F_{p2} = 9400$ N

Example: Reducing Elbow Horizontal Forces

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbb{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

$$\mathbf{F}_{ss} = \mathbf{M}_1 + \mathbf{M}_2 - \mathbb{W} - \mathbf{F}_{p_1} - \mathbf{F}_{p_2}$$

$$F_{ss_x} = \cancel{M_{1_x}} + M_{2_x} - \cancel{\mathbb{W}_x} - \cancel{F_{p_{1x}}} - F_{p_{2x}}$$

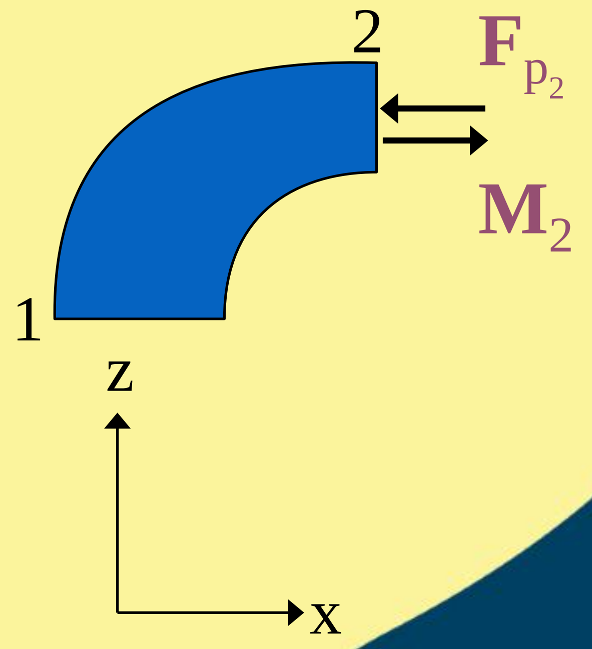
$$F_{ss_x} = M_{2_x} - F_{p_{2x}}$$

$$F_{ss_x} = (1270N) - (\text{-}9400N)$$

$$F_{ss_x} = 10.7\text{kN}$$

Force of pipe on fluid

Fluid is pushing the pipe to the left



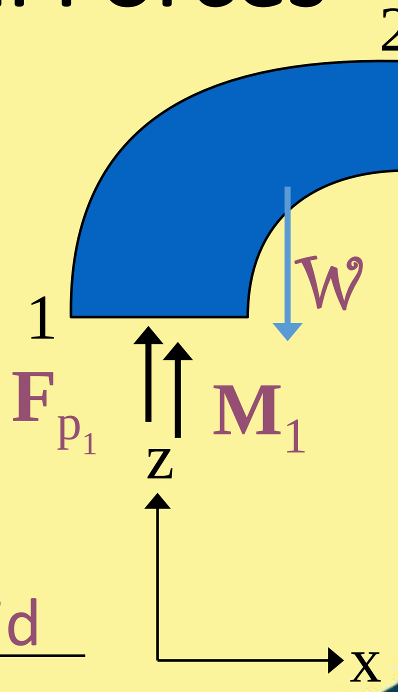
Example: Reducing Elbow Vertical Forces

$$F_{ss_z} = M_{1_z} + M_{2_z} - W_z - F_{p_{1z}} - F_{p_{2z}}$$

$$F_{ss_z} = M_{1_z} - W_z - F_{p_{1z}}$$

$$F_{ss_z} = -459\text{N} - (-1{,}961\text{N}) - (29{,}400\text{N})$$

$$F_{ss_z} = -27.9k\text{N}$$ 28 kN acting downward on fluid



Pipe wants to move up

Moment of Momentum Equation

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b d\forall + \int_{cs} \rho b \mathbf{V}\cdot\hat{\mathbf{n}} dA$$ cv equation

$$\mathbf{B} = m\mathbf{r}\times\mathbf{V}$$ Moment of momentum

$$\mathbf{b} = \frac{m\mathbf{r}\times\mathbf{V}}{m}$$ Moment of momentum/unit mass

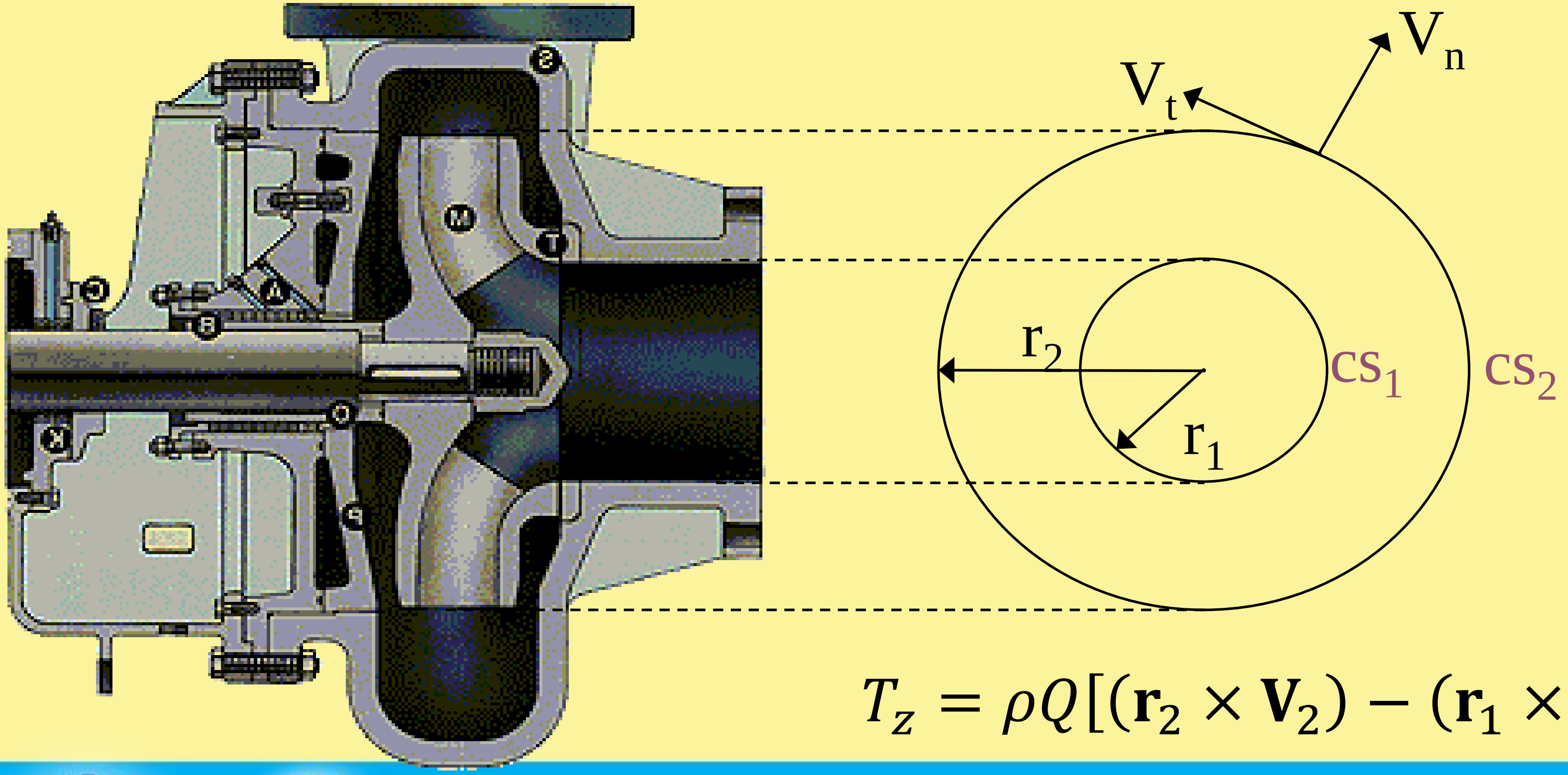
$$\frac{D(m\mathbf{r}\times\mathbf{V})}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho\mathbf{r}\times\mathbf{V} d\forall + \int_{cs} \rho(\mathbf{r}\times\mathbf{V})(\mathbf{V}\cdot\hat{\mathbf{n}}) dA$$

$$\mathbf{T} = \int_{cs} \rho(\mathbf{r}\times\mathbf{V})(\mathbf{V}\cdot\hat{\mathbf{n}}) dA$$ Steady state

Turbomachinery

$$\mathbf{T} = \int_{cs} \rho \overbrace{(\mathbf{r} \times \mathbf{V})}^{rV_t} \overbrace{(\mathbf{V} \cdot \hat{\mathbf{n}})}^{V_n} dA$$

$$\int_{cs} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = \rho Q$$



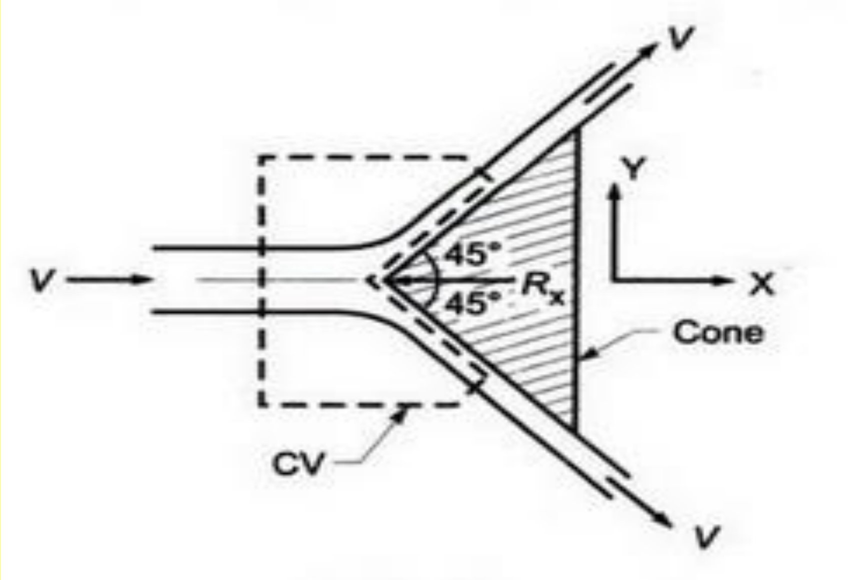
$$T_z = \rho Q[(\mathbf{r}_2 \times \mathbf{V}_2) - (\mathbf{r}_1 \times \mathbf{V}_1)]$$

Practice Problem

A jet of oil (RD = 0.80) issues from nozzle of 15 cm diameter with a velocity of 12 m/s. A smooth cone with vertex angle of 90° deflects the jet. The jet is horizontal and the vertex of the cone points towards the jet. Calculate the force required to hold the cone in position.

Solution:

Consider a control volume as shown in fig. Let R_x = reaction of the cone on the fluid in the control volume. The pressure is everywhere atmospheric. As the cone is smooth, by neglecting friction the velocity of the sheet of water over the cone is V everywhere. The inclination of the velocity V to axis is 90°/2 = 45°



$A = \frac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$ $\rho = 0.8 \text{ x } 998 = 798.4 \text{ kg/m}^3$

$Q = AV = 0.01767 \text{ X } 12 = 0.2121 \text{ m}^3/\text{s}$

By momentum Equation in X- direction:

0- R_x = ρQ (V cos 45°- V)

R_x = ρQV (1- cos 45°)

= 798.4 x 0.2121 x 12 x (1- cos 45°)

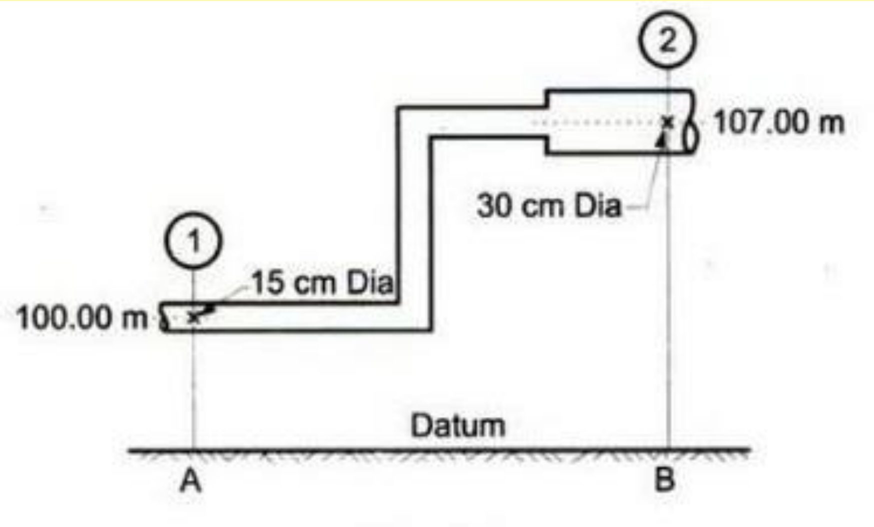
= 595 N

By symmetry, R_y = 0

Hence, the resultant reaction force on the fluid R= R_x . Thus the force required to hold the cone in position is F = R = R_x =595 N long (- X) direction.

Practice Problem

A pipeline is 15 cm in diameter and is at an elevation of 100.00 m at section A. At section B it is at an elevation of 107.00 m and has a diameter of 30 cm. When a discharge of 50 L/s of water is passed through this pipe the pressure at section A is observed to be 30 kPa. The energy loss in the pipe is 2 m. Calculate the pressure at B when the flow is (i) from A to B and (ii) from B to A.



Solution:

$$Q = 0.05\ m^3/s$$

Let suffixes 1 and 2 refer to sections A and B respectively.

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{\frac{\pi}{4} \times (0.15)^2} = 2.829 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{\frac{\pi}{4} \times (0.30)^2} = 0.7074 \text{ m/s}$$

γ = unit weight of water = 998 x 9.81/1000 = 9.79 kN/m^3

(i) When the flow is from A to B: Taking the atmospheric pressure as zero

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1\right) = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$\left(\frac{30}{9.79} + \frac{(2.829)^2}{2 \times 9.81} + 100.00\right) = \frac{p_2}{\gamma} + \frac{(0.7074)^2}{2 \times 9.81} + 107.0 + 2.0$$

$$3.064 + 0.4080 + 100.00 = \frac{p_2}{\gamma} + 0.0255 + 107.0 + 2.0$$

$$\frac{p_2}{\gamma} = — 5.554 \text{ m (gauge)}$$

$$p_2 = -5.554 \text{ x } 9.79 = — 54.37 \text{ kPa (gauge)}$$

(ii) When the flow is from B to A: Taking atmospheric pressure as zero,

$$\left(\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+Z_2\right)-H_L=\left(\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+Z_1\right)$$

$$\frac{p_2}{\gamma}+\frac{(0.7074)^2}{2\times 9.81}+107.0-2.0=\left(\frac{30}{9.79}+\frac{(2.829)^2}{2\times 9.81}+100.00\right)$$

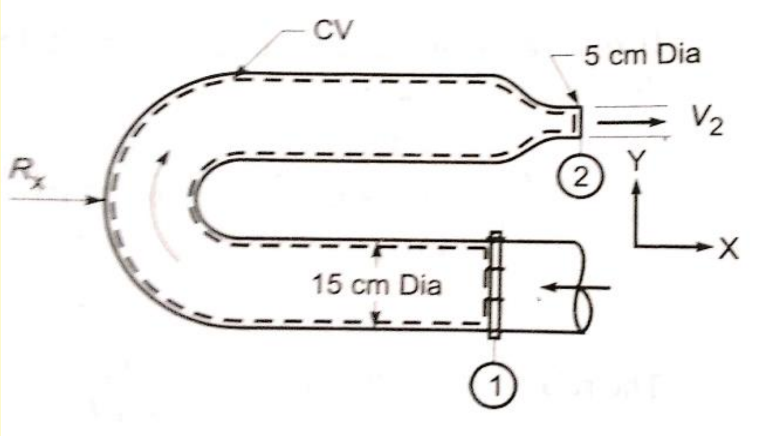
$$\frac{p_2}{\gamma}+0.0255+107.0-2.0=3.064+0.4080+100.00$$

$$\frac{p_2}{\gamma}=—1.554\text{ m (gauge)}$$

$$p_2=-1.554\text{ x }9.79\ =—\mathbf{15.21\ kPa\ (gauge)}$$

Practice Problem

A discharge of 0.06 m^3/s flows through a horizontal bend as shown in fig. Calculate the force on the bolts in section 1.



The control volume is shown in dotted lines. The Reaction on the control volume fluid is shown as R_x in positive x- direction.

Discharge $Q = \frac{\pi}{4} \text{ x } (D_2)^2 \text{ x } V_2 = 0.06 \text{ m}^3 \text{ /s}$

$$V_2 = \frac{0.06}{\frac{\pi}{4}\text{x}(0.05)^2} = 30.56 \text{ m/s} \qquad V_1 = V_2\left(\frac{D_2}{D_1}\right)^2 = 30.56 \text{ x } \left(\frac{5}{15}\right)^2 = 3.395 \text{ m/s}$$

By applying Bernoulli equation to sections 2 and 1, by assuming the bend to be in horizontal plane,

$$0 + \frac{(30.56)^2}{2\,x\,9.81} = \frac{P_1}{\gamma} + \frac{(3.395)^2}{2\,x\,9.81} \text{ .....}(P_2 = \text{atmospheric})$$

$$\frac{P_1}{\gamma} = 47.59 - 0.59 = 47.00 \text{ m}$$

$$P_1 = 47.00 \text{ X } 9.79 = 460.2 \text{ kPa}$$

By momentum equation in the x- direction,

$$-P_1 A_1 + R_x - 0 = \rho Q\,(V_2 - (-V_1))$$

$$-(460.2 \text{ x } 10^3) \text{ x } \frac{\pi}{4} \text{ x } (0.15)^2 + R_x = 998 \text{ x } 0.06 \text{ x}(30.56 + 3.395)$$

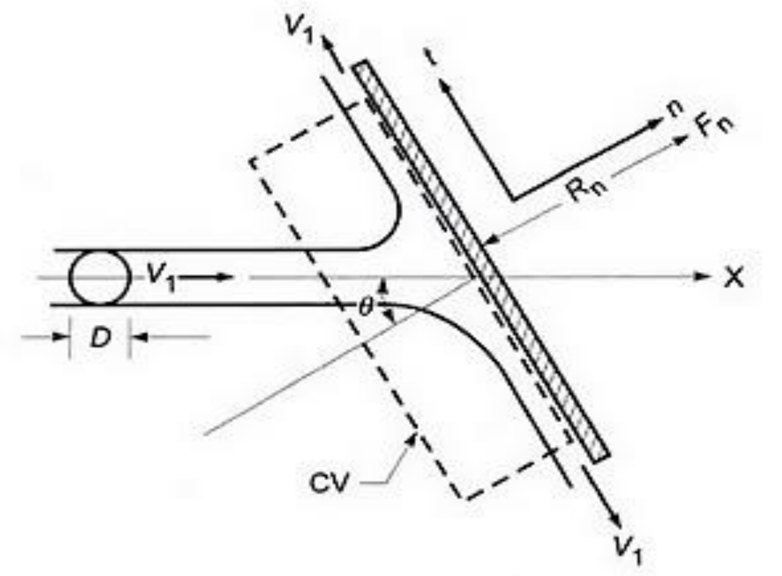
$$R_x = 8132 + 2033 = 10165 \text{ N}$$

The force *F* exerted by the fluid on the pipe and hence on the bolts in section 1, is equal and opposite to R_x .Thus **F = 10165 N and acts to left , i.e., in the negative x-direction** , as a pull (tension) on the joint.

Practice Problem

A 7.5 cm diameter water jet having a velocity of 12 m/s impinges on a plane, smooth plate at an angle of 60° to the normal to the plate. What will be the impact force when (i) the plate is stationary and (ii) when moving in the direction of the jet at 6 m/s. Estimate the work done per unit time on the plate in each case.

Solution: Consider the normal and tangential directions and a control volume as shown in Fig . Let R_n be the normal reaction on the fluid in the control volume. Consider the normal direction *n.* The pressure in the jet is atmospheric.



(i) When the plate is stationary:

$$0 - R_n = \rho Q\,(0 - V_l \cos\theta)$$

$$R_n = \rho Q V_l \cos\theta$$

$$= 998 \times \left[\frac{\pi}{4} \times (0.075)^2 \times 12\right] \times 12 \cos 60° = 317.45\ \text{N}$$

The normal force of the jet on plate is $\boldsymbol{F_n}$ **= 317.45 N in the positive n direction** (opposite to R_n).

(ii) **When the plate moves in the x-direction with u = 6 m/s.**

Considering normal direction and relative velocities:

$$-R_n = \rho Q_r\,(0 - V_{1r} \cos\theta)$$

$$R_n = \rho A\, V_{1r}^{\,2} \cos\theta$$

$$V_{1r} = 12.0 - 6.0 = 6.0\ \text{m/s}$$

$$R_n = 998 \times \left[\frac{\pi}{4} \times (0.075)^2 \times 6^2\right] \times \cos 60°$$

$$= 79.36\ \text{N}$$

The normal force of the water jet on the plate F_n will be equal and opposite to R_n.

Hence **F_n = 79.36 N and acts in positive n direction**.

Work done $W = F_n \times u_n = F_n \cos 60° \times u$

In case (i) u = 0 , W=0

(ii) u = 6 m/s,

W= 79.36 x 0.5 x 6.0

= 238.08 N.m/s