Journal of Hydrology

Distribution of nonuniform sediment in suspension for a steady two-dimensional transport through an open channel

--Manuscript Draft--

Manuscript Number:	
Article Type:	Research paper
Keywords:	Nonuniform Sediment; Grain size distribution; Open channel flow; Hindered settling; Numerical Methods; Sediment Transport
Corresponding Author:	SHIV MOHAN Indian Institute of Technology Kharagpur INDIA
First Author:	Koeli Ghoshal
Order of Authors:	Koeli Ghoshal
	SHIV MOHAN
	Jitendra Kumar
Abstract:	A theoretical model for determining the concentration of various grain sizes in suspension over erodible beds of different configuration, has been developed based on advection-diffusion equation for sediment concentration in a steady two-dimensional transport through an open channel. The effect of hindered settling over grain-size distribution due to increased concentration in suspension is also considered into the model. In the evaluation of reference concentration, the influence of non-ceasing probability, pick-up probability and incipient motion probability of the sediment particles has been acknowledged. The non-linear partial differential equation thus obtained, has been solved numerically. The present study also shows the concentration of different grain sizes along the stream-wise direction at different heights and finds that the finer particles reach equilibrium at a larger distance along main flow direction in comparison to that of the coarser particles. It is found that hindered settling effect is prominent in case of medium size particles and the grain-size distribution has bimodal nature with a strong peak and another weaker secondary peak at higher levels in suspension for higher velocities. Obtained numerical solution of the proposed model has been interpreted physically and compared with the experimental data.
Suggested Reviewers:	Vijay P Singh Texas A and M University: Texas A&M University vsingh@tamu.edu He is a leading researcher in the field of water engineering.
	Manotosh Kumbhakar Texas A&M University College Station: Texas A&M University manotosh.kumbhakar@gmail.com He is a very good researcher in the sediment-laden flows.
	Robert van Gorder Robert.VanGorder@maths.ox.ac.uk He is an expert in applied mathematics. He has worked extensively in the area of analytical methods applied to different kinds of science and engineering problems.
	Snehasis Kundu snehasis18386@gmail.com He is an expert in the field of sediment transport.
	Durga Charan Dalal durga@iitg.ac.in He is a leading researcher in the area of grain size distribution.

Distribution of nonuniform sediment in suspension for a steady two-dimensional transport through an open channel

Koeli Ghoshal*, Shiv Mohan† and Jitendra Kumar‡

October 1, 2021

Abstract

A theoretical model for determining the concentration of various grain sizes in suspension over erodible beds of different configuration, has been developed based on advection-diffusion equation for sediment concentration in a steady two-dimensional transport through an open channel. The effect of hindered settling over grain-size distribution due to increased concentration in suspension is also considered into the model. In the evaluation of reference concentration, the influence of non-ceasing probability, pick-up probability and incipient motion probability of the sediment particles has been acknowledged. The non-linear partial differential equation thus obtained, has been solved numerically. The present study also shows the concentration of different grain sizes along the stream-wise direction at different heights and finds that the finer particles reach equilibrium at a larger distance along main flow direction in comparison to that of the coarser particles. It is found that hindered settling effect is prominent in case of medium size particles and the grain-size distribution has

*Department of Mathematics, IIT Kharagpur, Kharagpur-721302, India, Email:koeli@maths.iitkgp.ac.in
†Department of Mathematics, IIT Kharagpur, Kharagpur-721302, India, Email:shivmohan58@gmail.com
‡Department of Mathematics, IIT Kharagpur, Kharagpur-721302, India, Email: jkumar@maths.iitkgp.ac.in

bimodal nature with a strong peak and another weaker secondary peak at higher levels in suspension for higher velocities. Obtained numerical solution of the proposed model has been interpreted physically and compared with the experimental data.

Key words: Nonuniform sediment; Open channel flow; Grain-size distribution; Hindered settling; Flow velocity

1 Introduction

Problems related to transportation of sediments in an open channel have received much attention of the researchers in the area of fluvial hydrodynamics due to its practical concern. In the study of sediment transportation, movement of non-cohesive sediment particles like gravel, silt or sand under hydraulic conditions is an essential field of research related to the process of grain-sorting. Bed load and suspended load are the two broad categories of transportation of sediments over erodible beds which influenced the particle size distribution heavily. The bed load is a thin layer of sediments close to the bed where particles movement take place through the process of sliding, rolling and saltation. On the other hand, suspended load refers to the relatively low amount of sediment particles in suspension surrounded by fluid for a considerable time interval. Study of the concentration of grain sizes in the suspension region under certain hydrodynamic state is a subject of great interest for the solution of real life problems in the field of coastal sediment transport, two phase flow and pollutant transport.

Numerous experimental and theoretical studies have been carried out by many investigators under different sedimentary conditions for better understanding of grain-size distribution of suspended load in an open channel ([1], [2], [3],[4], [5], [6]). Controlled experimental investigations had shown that the distribution of grain-size in suspended load under unidirectional flow condition is related to flow velocity, nature of the bed material and height of suspension ([1], [2], [7]). Unimodal size distribution in suspension was the key finding of these experimental studies. For evaluating the concentration of grain sizes in suspension, Ghosh et. al [2] proposed a mathematical model by considering the sorting process in two ways, first from bed to bed layer and then from the bed layer to suspension. Later Ghosh et. al [8] investigated the deposition of sand from both the theoretical and experimental view points. The study of Purkait [9] for the grain-size distribution of point-bar in the Usri River, India, reported that the log normal distribution is the suitable model for a certain size of bedforms. Mazumder [5] proposed a theoretical model from the continuity equations of water and sediment for the evaluation of grain-size distribution into suspension after including the effect of several factors. Mazumder et. al [4] did flume experiments with five beds of heterogeneous sediment mixtures having different values of bed roughness. They developed mathematical models for computing actual amount of material in suspension and also the grain-size distribution of suspended materials over different sediment beds. Pal and Ghoshal [10] investigated the influence of flow velocity, suspension height and bed roughness on the distribution of grain-size in suspension over five sediment beds of Mazumder et. al [4]. Out of the five sediment beds of Mazumder et. al [4], two beds were of sand and gravel mixture and Ghoshal and Pal [11] made further study on those beds. They determined the inverse of Schmidt number for different individual grain size and through regression analysis, connected the inverse of Schmidt number to the normalized settling velocity and suspension height. A numerical model to examine the dynamics of sand/gravel deposits in the Sandy River for grain-size distribution was developed by Cui [12] and the study is capable of simulating the dynamics of bed materials. From statistical viewpoints, Ghoshal et. al [13] investigated the grain-size distribution of suspended load and bed load experimentally and showed that the distribution of grain-size is leading from log-normal to log-skew-Laplace distribution with the increasing flow velocity and there is no effect of bed roughness in changing the pattern of size distribution from log-normality. Pal and Ghoshal [14] studied the suspended grain size distribution for the sediment beds considered by Mazumder et. al [4] through mixing length approach. Recently, Sun et. al [15] proposed a new model for vertical concentration profile of nonuniform sediment which is suitable for any fraction of nonuniform sediment.

Due to increased sediment concentration in suspension, the settling velocity of the particles get reduced. This phenomenon of hindered settling was investigated by several researchers in their models ([16], [17], [3], [6], [18], [19]). A theoretical model has been developed by Mazumder [3] based on diffusion equation for the evaluation of the suspended concentration of grain sizes by incorporating the hindered settling effect because of higher concentration in suspension. Later Pal and Ghoshal [6], studied a mathematical model on the concentration of grain sizes in suspension over sand-gravel bed after incorporating the effect of hindered settling and stratification. All these aforementioned studies mainly focused on the steady vertical distribution of suspended grain-size. The change in the grain-size distribution along main flow direction in a steady two-dimensional transport or with time in an unsteady one-dimensional transport, still remains unclear. To apply in real life problems, it is more realistic to find concentration of different grain-sizes for these kind of transports as any natural flow contains nonuniform sediment sizes. Several models have been proposed for the distribution of suspended sediment concentration in vertical as well as in stream-wise direction or unsteady vertical distribution of suspended sediments in the case of uniform sediment([19], [20], [21], [22]). However, the transportation of sediments in case of nonuniform sediments is different from that of uniform due to the collision of different size particles between each other. So it is worthy to develop a new model on suspended grain-size distribution from bed materials along vertical as well as in main flow direction together with important physical effects of turbulence in an open channel.

Keeping all these in mind, the primary aim of this study is to provide a theoretical model for the evaluation of concentration distribution of grain sizes from bed materials along vertical as well as in stream-wise direction incorporating the hindered settling effect for high concentrated flows. Also that, the transportation of nonuniform sediment is different from that of uniform sediment due to the collision among the particles of different sizes ([23], [24]). So influence of particle-particle interactivity has been considered in the determination of reference level. The effect of pick-up probability, non-ceasing probability and incipient motion probability of the sediment particles in the evaluation of reference concentration are also considered. The obtained governing equation based on advection-diffusion equation is a nonlinear partial differential equation with variable coefficients. It is always a challenging task to solve a nonlinear partial differential equation. To get the solution of the governing equation, a well defined numerical approach has been adopted and the obtained result is compared with the experimental results performed in the Fluvial Mechanics Laboratory (FML) of Indian Statistical Institute (ISI), Kolkata for the grain-size distribution at far field condition.

2 Mathematical model

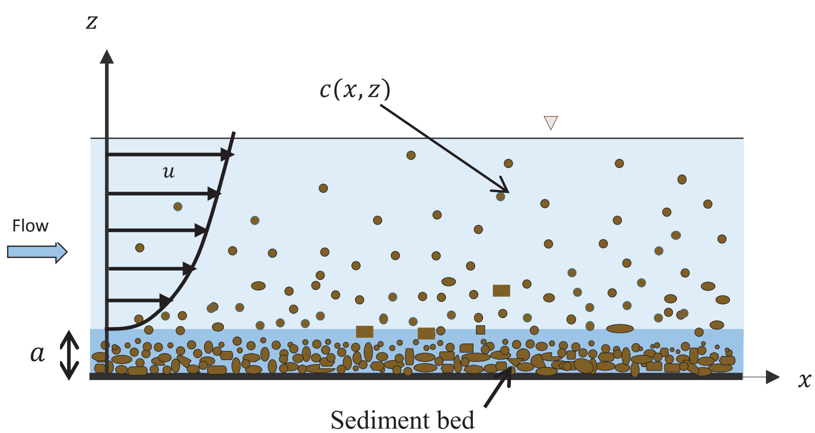


Figure 1: Schematic diagram for the grain-size distribution over a nonuniform sediment bed. Above the reference level a movement of sediment is termed as suspended load and below it as bed load.

Generally, the motion of suspended sediment particles is governed by an advection-diffusion equation. The governing equation for a steady uniform flow in a wide open channel for non-equilibrium suspended sediment concentration can be written as [21]:

$$u(z)\frac{\partial c}{\partial x} - \frac{\partial}{\partial z}\big(\omega_s c\big) = \frac{\partial}{\partial z}\left[\epsilon_s(z)\frac{\partial c}{\partial z}\right] \tag{2.1}$$

where x and z represent stream-wise and vertical directions of the flow respectively; $u(z)$ is the flow velocity in stream-wise direction which is a function of vertical coordinate only; c is the volumetric sediment concentration; ω_s is the settling velocity of sediment particle; and $\epsilon_s(z)$ is the sediment diffusion coefficient in vertical direction. Therefore, for a random jth fragment of nonuniform sediment, the governing equation of non-equilibrium suspended sediment concentration in a two-dimensional steady uniform flow can be described as:

$$u(z)\frac{\partial c_j}{\partial x} - \frac{\partial}{\partial z}\left(\omega_{s_j} c_j\right) = \frac{\partial}{\partial z}\left[\epsilon_s(z)\frac{\partial c_j}{\partial z}\right] \tag{2.2}$$

where c_j is the volumetric suspended sediment concentration and ω_{s_j} is the settling velocity of sediment particles for jth fraction of nonuniform sediment. To solve Eq. (2.2), expressions for flow velocity, settling velocity and sediment diffusion coefficients are needed. In the present work, more realistic log-law velocity profile is considered which is

$$u(z) = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{2.3}$$

where u_* is the shear velocity, κ is the von-Karman constant and z_0 is the start elevation of log-law. Due to the presence of different size particles in nonuniform sediment, the settling behavior of sediment particles will be different from that of uniform sediment particles. According to Richardson [25], the expression for reduced settling velocity of particles in sediment-fluid mixture can be expressed as

$$\omega_{s_j} = \omega_{0_j}(1 - c_j)^{n_j} \tag{2.4}$$

where ω_{0_j} is the settling velocity of particles in clear fluid and n_j is the reduction exponent for jth fraction of nonuniform sediment in sediment-fluid mixture. Its value mainly depends on particle Reynolds number and varies from 2 to 5 for different size of sediment particles. There are many expressions available for n_j. But, it can be observed from the literature ([3], [5]) that the value of reduction exponent n_j does not affect much and hence an average value $n_j = 3$ can be taken to avoid the computational difficulty. The present study also consider the same. The sediment diffusion coefficient $\epsilon_s(z)$ in sediment-fluid mixture is generally different from the turbulent diffusivity of the fluid, $\epsilon(z)$. The relationship between them can be described as

$$\epsilon_s(z) = \beta\epsilon(z) \tag{2.5}$$

where β is a proportionality parameter, known as inverse Schmidt number. Using Boussinesq hypothesis, turbulent shear stress τ in a steady turbulent flow can be expressed as

$$\tau = \rho\epsilon(z)\frac{du}{dz} = \rho {u_*}^2\Big(1 - \frac{z}{h}\Big) \tag{2.6}$$

where ρ is the mass density of the fluid and h is the flow depth. Using Eq. (2.3) in Eq. (2.6), eddy viscosity $\epsilon(z)$ can be expressed as

$$\epsilon(z) = \kappa u_* z\Big(1 - \frac{z}{h}\Big) \tag{2.7}$$

Using Eqs. (2.3, 2.4, 2.5 and 2.7) into Eq. 2.2, the governing equation of non-equilibrium suspended sediment concentration for jth fragment of nonuniform sediment can be written as

$$\frac{1}{\kappa}\ln\Big(\frac{z}{z_0}\Big)\frac{\partial c_j}{\partial x} - \frac{\omega_{0_j}}{u_*}\frac{\partial}{\partial z}\Big[c_j(1-c_j)^{n_j}\Big] = \beta\kappa\frac{\partial}{\partial z}\Big[z\Big(1-\frac{z}{h}\Big)\frac{\partial c_j}{\partial z}\Big] \tag{2.8}$$

Introducing the non-dimensional variables $X = \frac{x}{h}$, $Z = \frac{z}{h}$ and $Z_0 = \frac{z_0}{h}$, Eq. (2.8) can be rewritten as

$$\frac{1}{\kappa}\ln\Big(\frac{Z}{Z_0}\Big)\frac{\partial c_j}{\partial X} - \frac{\omega_{0_j}}{u_*}\frac{\partial}{\partial Z}\Big[c_j(1-c_j)^{n_j}\Big] = \beta\kappa\frac{\partial}{\partial Z}\Big[Z(1-Z)\frac{\partial c_j}{\partial Z}\Big] \tag{2.9}$$

It is clear from Eq. (2.9) that the governing equation is a parabolic type partial differential equation (PDE). To solve it, a set of initial and boundary conditions are required. To that purpose, the following initial and boundary conditions are used:

$$c_j(X, Z) = 0 \;\; \text{at} \;\; X = 0, A_j \leq Z \leq 1 \tag{2.10}$$

$$c_j(X,Z) = c_{a_j} \quad \text{at} \quad Z = A_j,\ X > 0 \tag{2.11}$$

and

$$c_j(X,Z) = 0 \quad \text{at} \quad Z = 1,\ X > 0 \tag{2.12}$$

where A_j represents the non-dimensional reference level and c_{a_j} is the reference concentration at reference level A_j for jth fraction of nonuniform sediment.

3 Result and Discussion

3.1 Input functions and parameters

In order to get the solution from Eq. (2.9), it is necessary to compute the non-dimensional reference level A_j, reference concentration c_{a_j} at the reference level A_j and the settling velocity ω_{0_j}, for jth fraction of nonuniform sediment. To that purpose, formulas suggested by Sun et. al [26] to calculate reference level A_j and reference concentration c_{a_j}, for jth fraction of nonuniform sediment are used.

The mathematical expression of reference level A_j after including the influence of particle-particle interaction of the sediments is given by

$$A_j = \frac{{u_*}^2}{(s-1)gh} \frac{{D_{*j}}^3}{\left(\sqrt{25 + 1.2{D_{*j}}^2} - 5\right)^{1.5}} \left(\int_0^{c_m} \frac{E_{*j}}{c_j \mu_{r_j} \omega_{r_j}} dc_j \right) \tag{3.1}$$

where $s(= 2.65)$ is the relative mass density of sediment particle, g is the gravitational acceleration, $D_{*j} = D_j[(s-1)g/\nu^2]^{1/3}$ is the non-dimensional particle diameter for jth fraction of nonuniform sediment in which ν is the kinematic viscosity of fluid and D_j is the particle diameter for jth fraction of nonuniform sediment; c_m is the maximum bed concentration and E_{*j}, μ_{r_j} and ω_{r_j} are the dimensionless diffusion coefficients, relative viscosity and relative settling velocity for jth fraction of nonuniform sediment, respectively. According to Cheng [27], the expressions of E_{*j}, μ_{r_j} and ω_{r_j} are given by

$$E_{*j} = \alpha_1 ({c_j}^{-1/3} - 1)^{-2} \tag{3.2}$$

$$\mu_{r_j} = e^{\frac{2.5}{\beta_1}[(1-c_j)^{-\beta_1}-1]} \tag{3.3}$$

$$\omega_{r_j} = \frac{\mu_{r_j}}{1+\triangle c_j}\left(\frac{\sqrt{25+1.2{D_{*j}}^2(1-c_j)^{\frac{2}{3}}(1+\triangle c_j)^{\frac{2}{3}}{\mu_{r_j}}^{\frac{-4}{3}}}-5}{\sqrt{25+1.2{D_{*j}}^2}-5}\right)^{1.5} \tag{3.4}$$

where α_1 and β_1 are constants and $\triangle = s - 1$. According to Cheng [27], the values of constants α_1, β_1 and c_m are taken as 0.02, 2.5 and 0.6, respectively.

Sun et. al [26] developed a theoretical model to calculate the reference concentration of any fraction for a nonuniform sediment bed. In his study, he considered the effect of non-ceasing probability, pick-up probability and incipient motion probability of a sediment particle together with the grain-size distribution of the bed materials in suspension distribution. Following Sun et. al [26], the expression of reference concentration c_{a_j} for jth fragment of nonuniform sediment can be written as

$$c_{a_j} = c_m P_j \frac{B_j F_j}{1 + B_j F_j} \tag{3.5}$$

where P_j represents the percentage of the bed material for jth fraction of nonuniform sediment,

$$B_j = 10^{-5} {\theta_{n_j}}^2 {D_{*j}}^{1.84} \tag{3.6}$$

$$\theta_{n_j} = \frac{{u_*}^2 {\sigma_g}^{0.25}}{\triangle g (D_m D_j)^{0.5}} \tag{3.7}$$

where $D_m = (D_{84} D_{16})^{0.5}$ represents the geometric diameter of bed material and $\sigma_g = (D_{84}/D_{16})^{0.5}$ represents geometric standard deviation of the bed material. D_{16} and D_{84} are the sieve sizes of which 16% and 84% of the mixture by weight is finer. The expression of F_j for jth fragment of nonuniform sediment can be written as

$$F_j = \frac{\alpha_j \lambda_j}{(1-\gamma_j)(1-\lambda_j)(1+\gamma_j \lambda_j)} \tag{3.8}$$

where

$$\alpha_j = 1 - \frac{1}{\sqrt{2\pi}} \int_{-2.7(\sqrt{0.0822/\theta_{n_j}}+1)}^{2.7(\sqrt{0.0822/\theta_{n_j}}-1)} e^{-0.5\chi^2} d\chi \tag{3.9}$$

$$\lambda_j = \frac{2}{\sqrt{2\pi}} \int_{\omega_{0_j}/u_*}^{\infty} e^{-0.5\chi^2} d\chi \tag{3.10}$$

$$\gamma_j = 1 - \frac{1}{\sqrt{2\pi}} \int_{-2.7(\sqrt{0.0571/\theta_{n_j}}+1)}^{2.7(\sqrt{0.0822/\theta_{n_j}}-1)} e^{-0.5\chi^2} d\chi \tag{3.11}$$

The above expressions were also used by Pal and Ghoshal [6] in their steady one-dimensional (i.e. vertical) concentration distribution of nonuniform sediment in an open channel flow. To evaluate the settling velocity of particles for jth fraction of nonuniform sediment in clear fluid, several expressions are available in the literature. In the present study, the expression given by Cheng [28] is used.

$$\omega_{0_j} = \frac{\nu}{D_j}\left(\sqrt{25 + 1.2{D_{*j}}^2} - 5\right)^{1.5} \tag{3.12}$$

To solve Eq. (2.9) which is a parabolic type nonlinear partial differential equation together with initial and boundary conditions Eqs. (2.10-2.12), numerical scheme based on fourth order Runge-Kutta method has been applied. To that purpose, first, PDE Eq. (2.9) is discretized using second order central finite difference approximation for the derivatives with respect to vertical variable Z and then obtained system of coupled first order ODEs is solved using fourth order Runge-Kutta method due to its prediction accuracy. A maple code has been prepared according to the above defined procedure to get the numerical solution of the theoretical model developed in this study.

3.2 Variation of concentration of different grain sizes along stream-wise direction

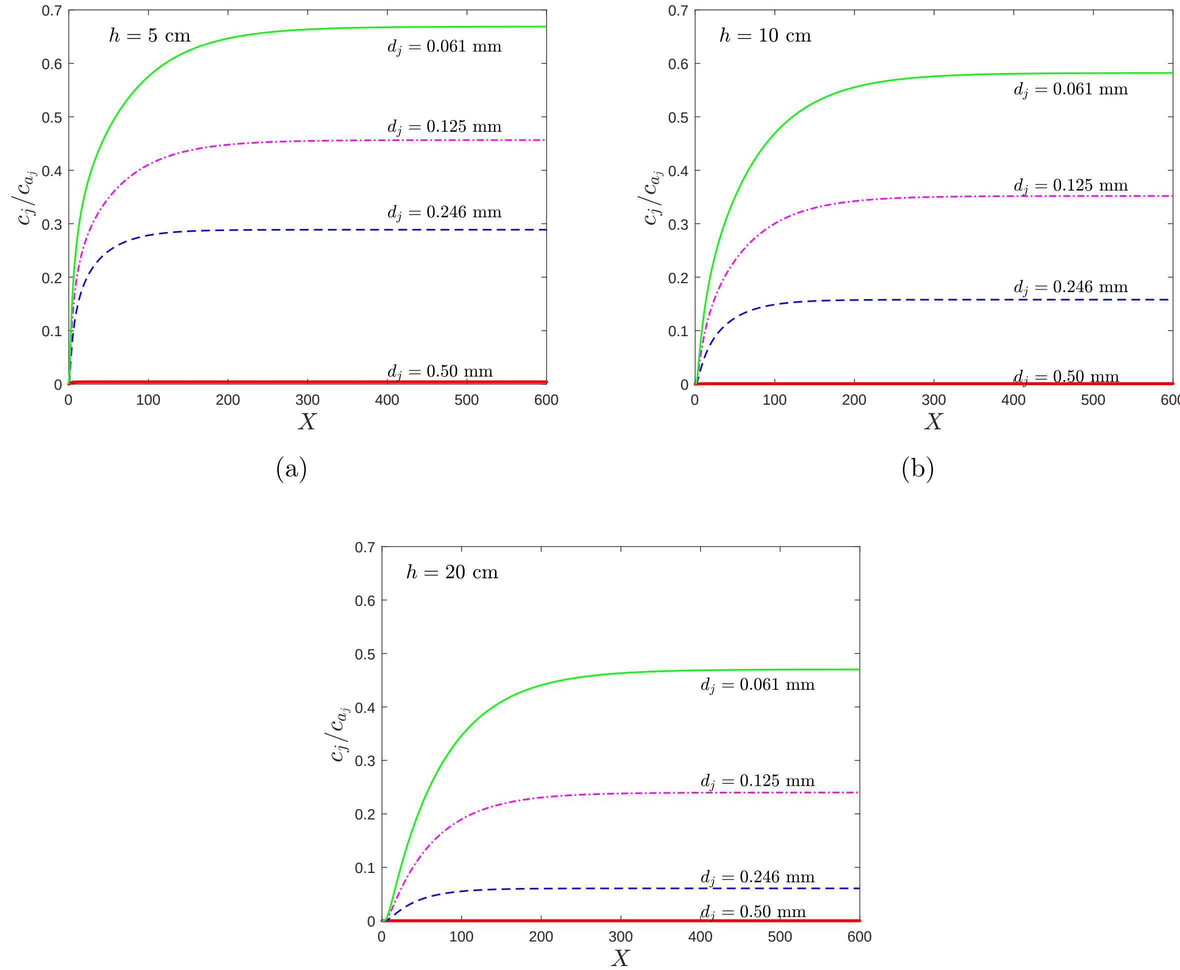
In Figure 2, grain-size distribution in suspension along stream-wise direction is plotted at different heights for a particular velocity. The required parameters are taken as $\beta = 1$, $u_* = 7.35$ cm/s and $A_j = 0.01$. The values of other parameters which are used, are given in Table 1. Figure 2 shows that more amount of finer particles go in suspension comparison to that of coarser particles at all heights and as the height increases, concentration of all size particles in suspension decreases. It can be observed that the finer particles of diameter $D_j = 0.061$ mm, are reaching in equilibrium position at a larger distance along stream-wise direction in comparison to the coarser particles. It is also clear from Figure 2, that approximately, after $X = 300$, all grain sizes reach equilibrium along stream-wise direction at any flow depth i.e. there is no change in the concentration of grain sizes with respect to X.

ϕ	D_j (mm)	c_{a_j}	ω_{0_j} (mm/s)	n_j
0	1.0	0.0191	111.4413	3
1	0.50	0.0932	60.6991	3
2	0.25	0.3438	26.6814	3
3	0.125	0.0642	9.0112	3
4	0.063	0.0156	2.5175	3
5	0.031	0.0057	0.6498	3

Table 1: Parameter Values

3.3 Variation of concentration of different grain sizes along vertical direction

In this subsection, grain-size distribution along vertical direction at fixed $X = 300$ is plotted. The required parameters are taken as $n_j = 3$, $\beta = 1$, $u_* = 7.35$ cm/s and $A_j = 0.01$. Figure 3 shows that for heavy particles (like particles of diameter 1 mm or more), concentration in suspension is very low at all heights; because to uplift the heavy particles into suspension, flow with higher velocity



(a) (b) (c)

Figure 2: Distribution of grain-size along stream-wise direction at different heights

is required. But for particles of smaller diameter, concentration along a vertical is comparatively much more. It is happening because even the flow with less shear velocity is competent enough to uplift the finer particles in suspension.

3.4 Effect of hindered settling over different grain sizes

Effect of hindered settling over the distribution of grain-size particles through the exponent parameter n_j is shown in Figure 4 at fixed $X = 300$. The required parameters are given in Table 1. Figure 4(a) and Figure 4(b) show that the effect of hindered settling is almost negligible in case

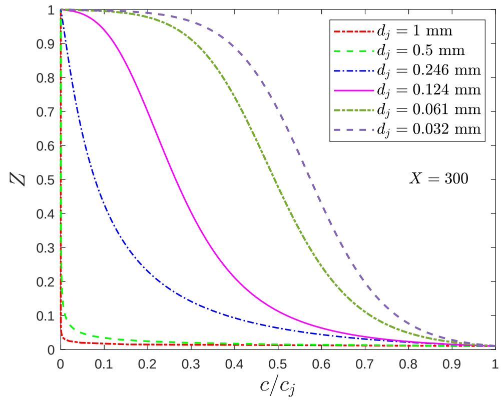


Figure 3: Distribution of grain-size along vertical direction at $X = 300$.

of coarser pericles. It is happening because for these particles, concentration in suspension is very low and for that reason, the settling behavior of particles is almost same as that of in clear fluid. Also, in the case of finer particles, (Figure 4(e) and Figure 4(f)), no hindered effect is observed. This is because the size of these particles is so small that no hindered effect take place during their settling. On the other hand, it can be observed from Figure 4(c) and Figure 4(d) that for medium size sediment particles, effect of hindered settling mechanism is clearly visible.

3.5 Comparison with Experimental data

In this section, the solution of present theoretical model has been validated with the experimental data of Mazumder et. al. [4] at far field condition as grain-size data for two-dimensional transport is not available in literature. A series of experiments were conducted in the Fluvial Mechanics Laboratory (FML) of Indian Statistical Institute (ISI), Kolkata to observe the influence of bed roughness on the distribution of grain-size in suspension through an experimental channel. Five different sediment beds (10C1-10C5) of different bed roughness and the same modal grain-size of at 2.0ϕ (where $\phi = -\log_2 D_j$; D_j is the particle diameter in mm for jth fraction of nonuniform sediment) were taken to perform the experiments. The beds 10C1, 10C2 and 10C3 were consisted of 100% sand having the range from 0.032 to 2.0 mm. On the other hand, the proportion of sand-gravel mixture (range 0.032 to 8.0 mm) for two beds 10C4 and 10C5 were 14% gravel, 86%

sand and 25% gravel, 75% sand, respectively. Cumulative percentage plots, weight (kg) plots and percentage plots of grain-size distribution for all these five beds are given in Figure 5. The plots can also be found in [4] and [10]. For each of the experiments, flow depth h was considered at a constant height of 35 cm. A specific mixture of grain-size was distributed uniformly on the flume base to maintain an uniform bed for each experiments. At different heights and for different velocities (see Table 2), samples for grain-size distribution of suspended sediments were collected with the help of siphon tubes. The more detailed explanations of these beds and experimental setup can be found in Mazumder et. al. [4] and Ghoshal [29].

Run No.	u_{max} (cm/s)	u_* (cm/s)
Run 1	68	4.94
Run 2	101	6.45
Run 3	116	7.35

Table 2: Parameters for Velocity Profile

To get the solution of Eq. (2.9) together with boundary conditions (2.10-2.12), the reference level A_j, reference concentration c_{a_j} and the settling velocity ω_{0_j} for jth class of grain-size are calculated according to formula given by Eq. (3.1), Eq. (3.5) and Eq. (3.12), respectively. In the present study, the reference level is calculated for a representative grain-size, which is taken as D_{65} instead of calculating the reference level for each jth grain-size. For all five beds, the evaluated reference concentration c_{a_j} for jth grain-size of nonuniform sediment according to Eq. (3.5) is plotted in Figure 6. Apart from these input parameters, β is another parameter present in the governing equation Eq. (2.9) that has to be evaluated. Many researchers have taken $\beta = 1$ in their study by assuming that the sediment diffusion coefficient $\epsilon_s(z)$ is identical to the turbulent diffusivity $\epsilon(z)$ of the fluid to avoid the computational difficulty. However, many researchers concluded based on experimental results that the value of β depends on the size of sediment particles ([30], [31], [32]). Mazumder et. al. [4] provided four empirical relations between β and the normalized settling velocity $\frac{\omega_{0_j}}{u_*}$ of jth class of grain-size for all five beds. These empirical relations are given as follows

$$\ln\beta = -0.181 + 0.08\left(\ln\frac{\omega_{0_j}}{u_*}\right) - 0.175\left(\ln\frac{\omega_{0_j}}{u_*}\right)^2 \tag{3.13}$$

for bed 10C1-10C3 and $u_{max} = 101$ cm/s,

$$\ln\beta = -0.32 + 0.35\left(\ln\frac{\omega_{0_j}}{u_*}\right) - 0.07\left(\ln\frac{\omega_{0_j}}{u_*}\right)^2 \tag{3.14}$$

for bed 10C4-10C5 and $u_{max} = 101$ cm/s,

$$\ln\beta = 0.234 + 0.39\left(\ln\frac{\omega_{0_j}}{u_*}\right) - 0.09\left(\ln\frac{\omega_{0_j}}{u_*}\right)^2 \tag{3.15}$$

for bed 10C1-10C3 and $u_{max} = 116$ cm/s and

$$\ln\beta = 0.096 + 0.655\left(\ln\frac{\omega_{0_j}}{u_*}\right) - 0.012\left(\ln\frac{\omega_{0_j}}{u_*}\right)^2 \tag{3.16}$$

for bed 10C4-10C5 and $u_{max} = 116$ cm/s. In the present study, above defined empirical relations are used to compute β for each grain-size to validate the experimental data.

After obtaining the numerical solution of PDE (2.9), vertical suspension concentration of each grain-size at a larger value of $X = 300$ is calculated and then the relative suspension concentration c_{r_j} of jth grain-size, which can be expressed as

$$c_{r_j} = \frac{c_j}{\sum_j c_j} \tag{3.17}$$

is evaluated for each jth fraction of nonuniform sediment. In Figure 7, the computed relative suspension concentration c_{r_j} is validated with the observed values of experimental data [4] at two maximum velocities ($u_{max} = 101$ cm/s and $u_{max} = 116$ cm/s) for two different heights. In this figure, at higher velocities, a bimodal nature of grain-size distribution is clearly visible with the occurrence of a strong peak and a weaker peak. These experiments reveals that the bed is better represented at lower levels for higher velocities and the bi-modality in suspension becomes prominent at higher levels and at higher velocities. The strong peak of this bimodal distribution is observed around 2.5ϕ and another weak peak is found around 4.0ϕ. It can be observed from Figure 7 that despite irregular behavior of individual grain-size in suspension, present model gives good agreement with the experimental data.

4 Conclusions

In the present study, a theoretical model for the determination of concentration of different grain sizes present in suspension from bed materials is proposed for steady two-dimensional transport through an open channel. The log-law velocity profile is used for stream-wise velocity component and an important turbulent feature, hindered settling effect due to high sediment concentration in suspension, has been taken into account. The influence of particle-particle interaction in the evaluation of reference level and the effect of pick-up probability, non-ceasing probability and incipient motion probability of the sediment particles in the computation of reference concentration are also considered. Finally, the obtained non-linear partial differential equation has been solved by fourth order Runge-Kutta method using numerical techniques. The variation of concentration of different grain-sizes along the main flow direction at different heights has been shown in this study and found that the finer particles reach equilibrium at a larger distance than the coarser particles. The effect of hindered settling on different grain-sizes through the reduction exponent has been discussed and interpreted from physical point of view. It is found that the effect is negligible for finer and coarser particles and is prominent in case of medium size particles. Due to unavailability of similar experimental data in literature, the obtained numerical solution is validated with the experimental data at far field condition and it shows a good agreement with the experimental data irrespective of irregular behavior of grain-size.

References

[1] S. Sengupta, Size-sorting during suspension transportationlognormality and other characteristics, Sedimentology 22 (2) (1975) 257–273.

[2] J. Ghosh, B. Mazumder, S. Sengupta, Methods of computation of suspended load from bed materials and flow parameters, Sedimentology 28 (6) (1981) 781–791.

[3] B. Mazumder, Grain size distribution in suspension from bed materials, Sedimentology 41 (2) (1994) 271–277.

[4] B. Mazumder, K. Ghoshal, D. Dalal, Influence of bed roughness on sediment suspension: experimental and theoretical studies, Journal of Hydraulic Research 43 (3) (2005) 245–257.

[5] B. Mazumder, R. Ray, D. Dalal, Size distributions of suspended particles in open channel flow over bed materials, Environmetrics: The official journal of the International Environmetrics Society 16 (2) (2005) 149–165.

[6] D. Pal, K. Ghoshal, Mathematical model on grain-size distribution in suspension over sand-gravel bed, Journal of Hydrology 511 (2014) 640–647.

[7] S. Sengupta, J. Ghosh, B. Mazumder, Experimental-theoretical approach to interpretation of grain size frequency distributions, Principles, methods and application of particle size analysis (1991) 264–280.

[8] J. K. Ghosh, B. Mazumder, M. Saha, S. Sengupta, Deposition of sand by suspension currents; experimental and theoretical studies, Journal of Sedimentary Research 56 (1) (1986) 57–66.

[9] B. Purkait, Patterns of grain-size distribution in some point bars of the usri river, india, Journal of Sedimentary Research 72 (3) (2002) 367–375.

[10] D. Pal, K. Ghoshal, Effect of bed roughness on grain-size distribution in an open channel flow, Journal of Hydro-environment Research 8 (4) (2014) 441–451.

[11] K. Ghoshal, D. Pal, Grain-size distribution in suspension over a sand-gravel bed in open channel flow, International Journal of Sediment Research 29 (2) (2014) 184–194.

[12] Y. Cui, Examining the dynamics of grain size distributions of gravel/sand deposits in the sandy river, oregon with a numerical model, River research and applications 23 (7) (2007) 732–751.

[13] K. Ghoshal, B. Purkait, B. Mazumder, Size distributions in suspension over sand–pebble mixture: An experimental approach, Sedimentary Geology 241 (1-4) (2011) 3–12.

[14] D. Pal, K. Ghoshal, Grain-size distribution in open channel flow by mixing length approach, Environmetrics 26 (2) (2015) 107–119.

[15] Z. Sun, H. Zheng, D. Xu, C. Hu, C. Zhang, Vertical concentration profile of nonuniform sediment, International Journal of Sediment Research 36 (1) (2021) 120–126.

[16] W. Thacker, J. Lavelle, Two-phase flow analysis of hindered settling, The physics of Fluids 20 (9) (1977) 1577–1579.

[17] H. Woo, P. Julien, E. Richardson, Suspension of large concentrations of sands, Journal of Hydraulic Engineering 114 (8) (1988) 888–898.

[18] S. Mohan, M. Kumbhakar, K. Ghoshal, J. Kumar, Semianalytical solution for simultaneous distribution of fluid velocity and sediment concentration in open-channel flow, Journal of Engineering Mechanics 145 (11) (2019) 04019090.

[19] S. Mohan, M. Kumbhakar, K. Ghoshal, J. Kumar, Semi-analytical solution for one-dimensional unsteady sediment transport model in open channel with concentration-dependent settling velocity, Physica Scripta 95 (5) (2020) 055204.

[20] A. t. Hjelmfelt, C. W. Lenau, Nonequilibrium transport of suspended sediment, in: Journal of the Hydraulics Division: Proceedings of the American Society of Civil Engineers, no. HY7, 1970.

[21] X. Liu, Analytical solutions for steady two-dimensional suspended sediment transport in channels with arbitrary advection velocity and eddy diffusivity distributions, Journal of Hydraulic Research 54 (4) (2016) 389–398.

[22] X. Liu, M. Nayamatullah, Semianalytical solutions for one-dimensional unsteady nonequilibrium suspended sediment transport in channels with arbitrary eddy viscosity distributions and realistic boundary conditions, Journal of Hydraulic Engineering 140 (5) (2014) 04014011.

[23] M. He, Q. Han, Mechanism and characteristics of nonuniform sediment transport, in: Third International Symposium on River Sedimentation, the University of Mississippi, 1986, pp. 844–853.

[24] F. Holly Jr, J.-L. Rahuel, New numerical/physical framework for mobile-bed modelling: Part 1: Numerical and physical principles, Journal of hydraulic research 28 (4) (1990) 401–416.

[25] J. t. RICHARDSON, Sedimentation and fluidisation: Part i, Transactions of the institution of chemical engineers 32 (1954) 35–53.

[26] Z.-l. Sun, Z.-f. Sun, J. Donahue, Equilibrium bed-concentration of nonuniform sediment, Journal of Zhejiang University-SCIENCE A 4 (2) (2003) 186–194.

[27] N.-S. Cheng, A diffusive model for evaluating thickness of bedload layer, Advances in water resources 26 (8) (2003) 875–882.

[28] N.-S. Cheng, Simplified settling velocity formula for sediment particle, Journal of hydraulic engineering 123 (2) (1997) 149–152.

[29] K. Ghoshal, On velocity and suspension concentration in a sediment-laden flow: experimental and theoretical studies, Ph.D. thesis, Indian Statistical Institute Kolkata (2004).

[30] L. C. v. Rijn, Sediment transport, part ii: suspended load transport, Journal of hydraulic engineering 110 (11) (1984) 1613–1641.

[31] H. E. Jobson, W. W. Sayre, Vertical transfer in open channel flow, Journal of the Hydraulics Division 96 (3) (1970) 703–724.

[32] N. L. Coleman, Flume studies of the sediment transfer coefficient, Water Resources Research 6 (3) (1970) 801–809.

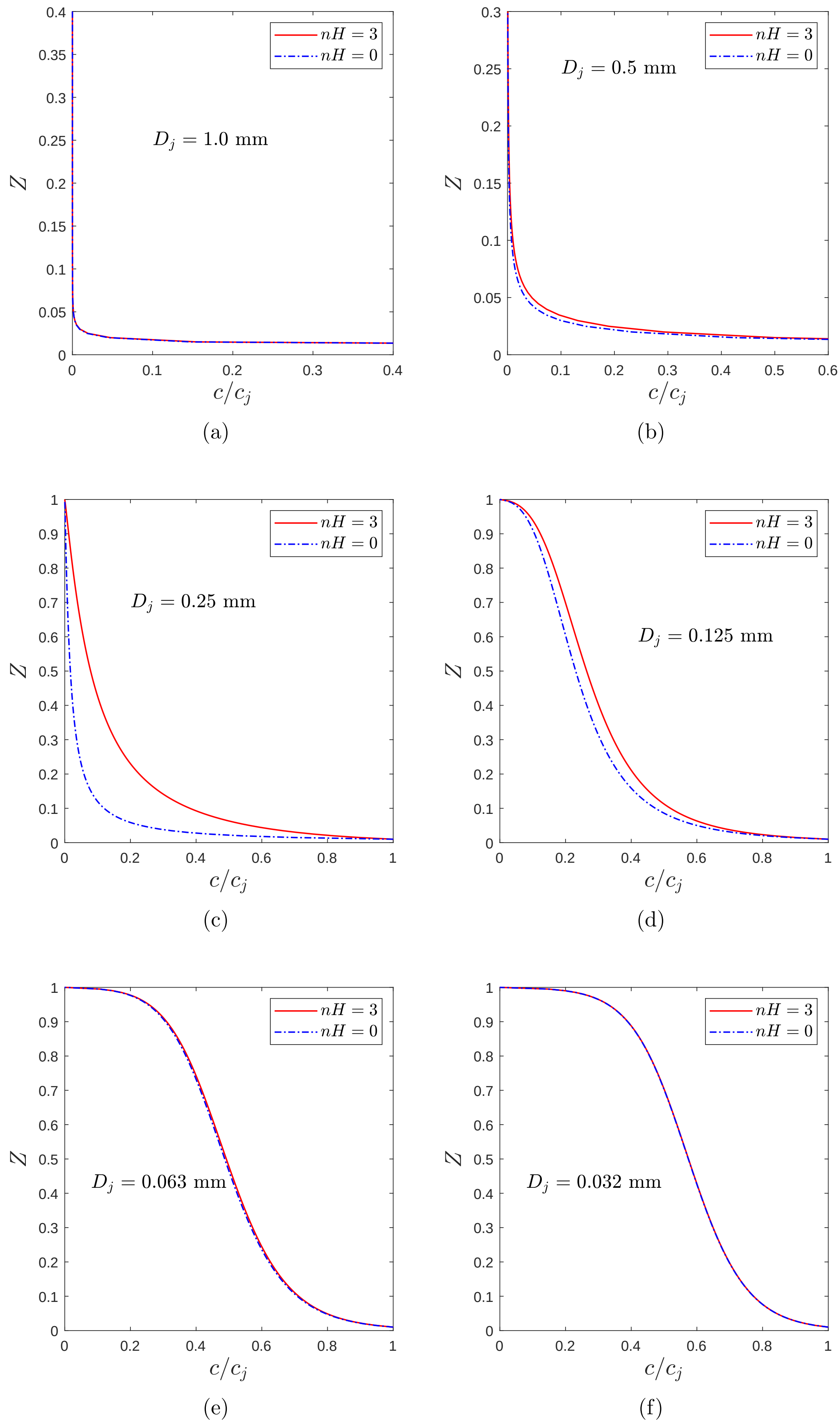


Figure 4: Hindered effect for different grain-sizes in vertical direction at $X = 300$

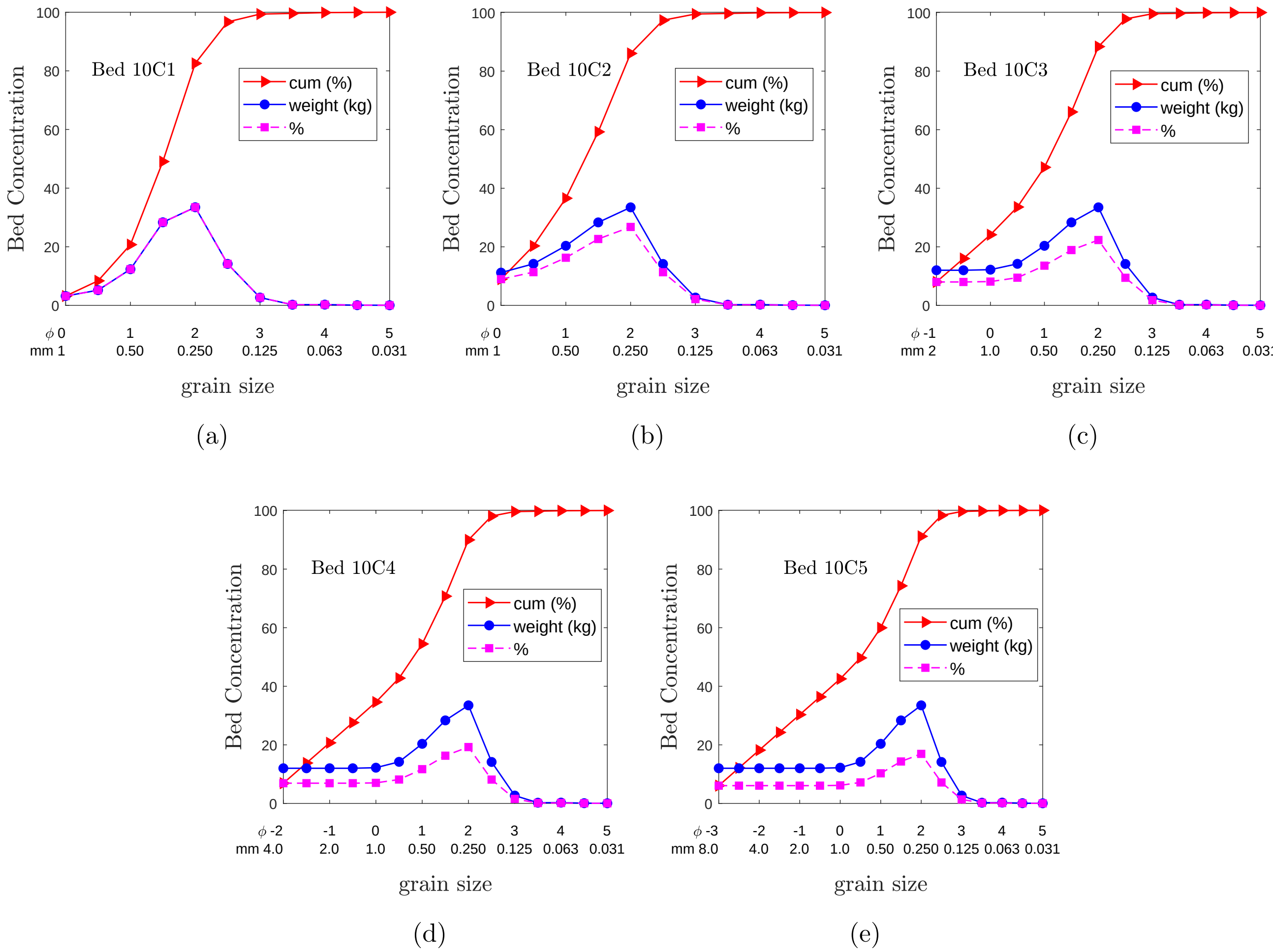


Figure 5: Grain-size distribution of different beds [4]

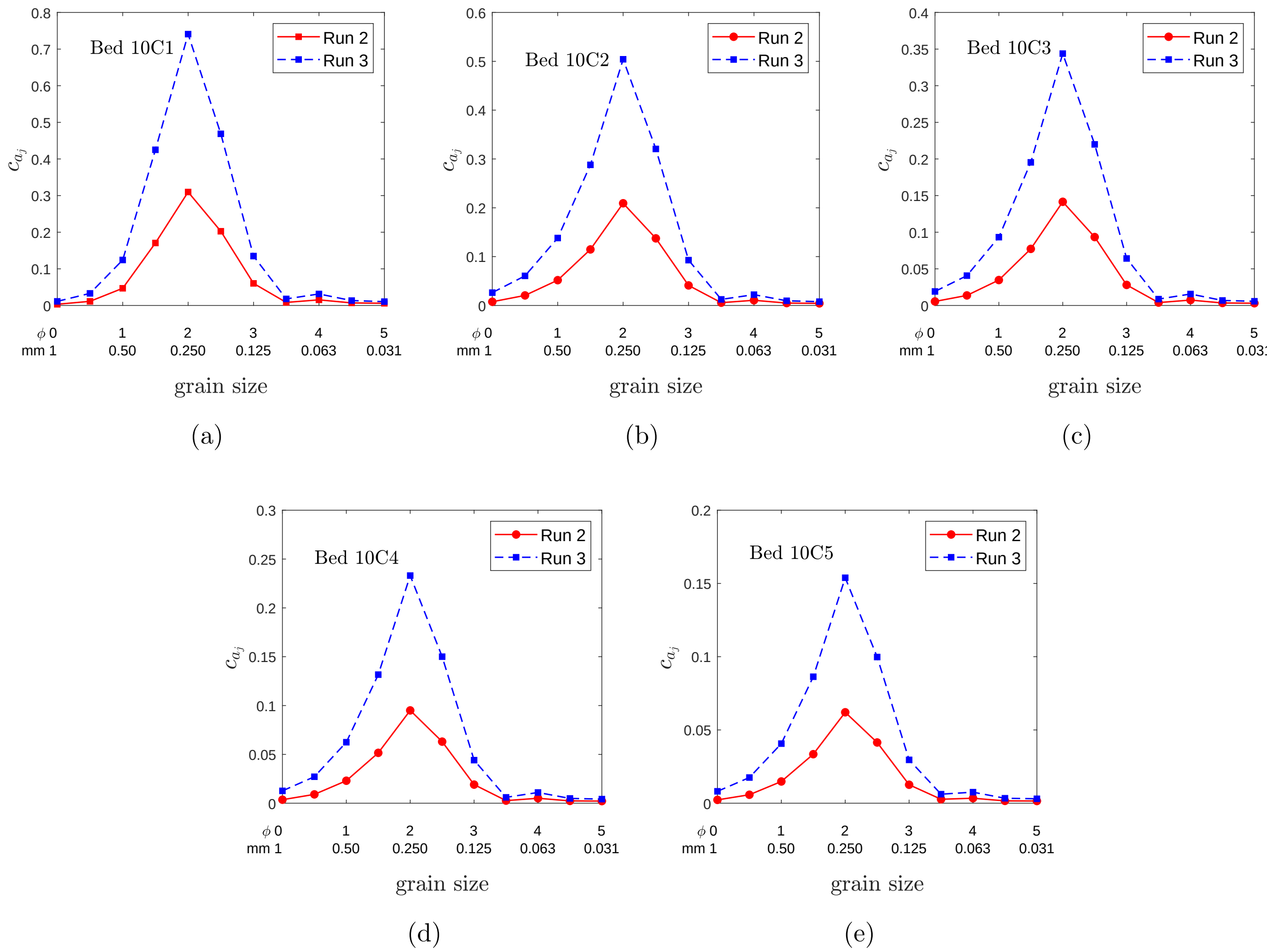


Figure 6: Reference concentration c_{a_j} at the reference level for grain-size according to Sun et. al. [26] formula given by Eq. (3.5) for different beds.

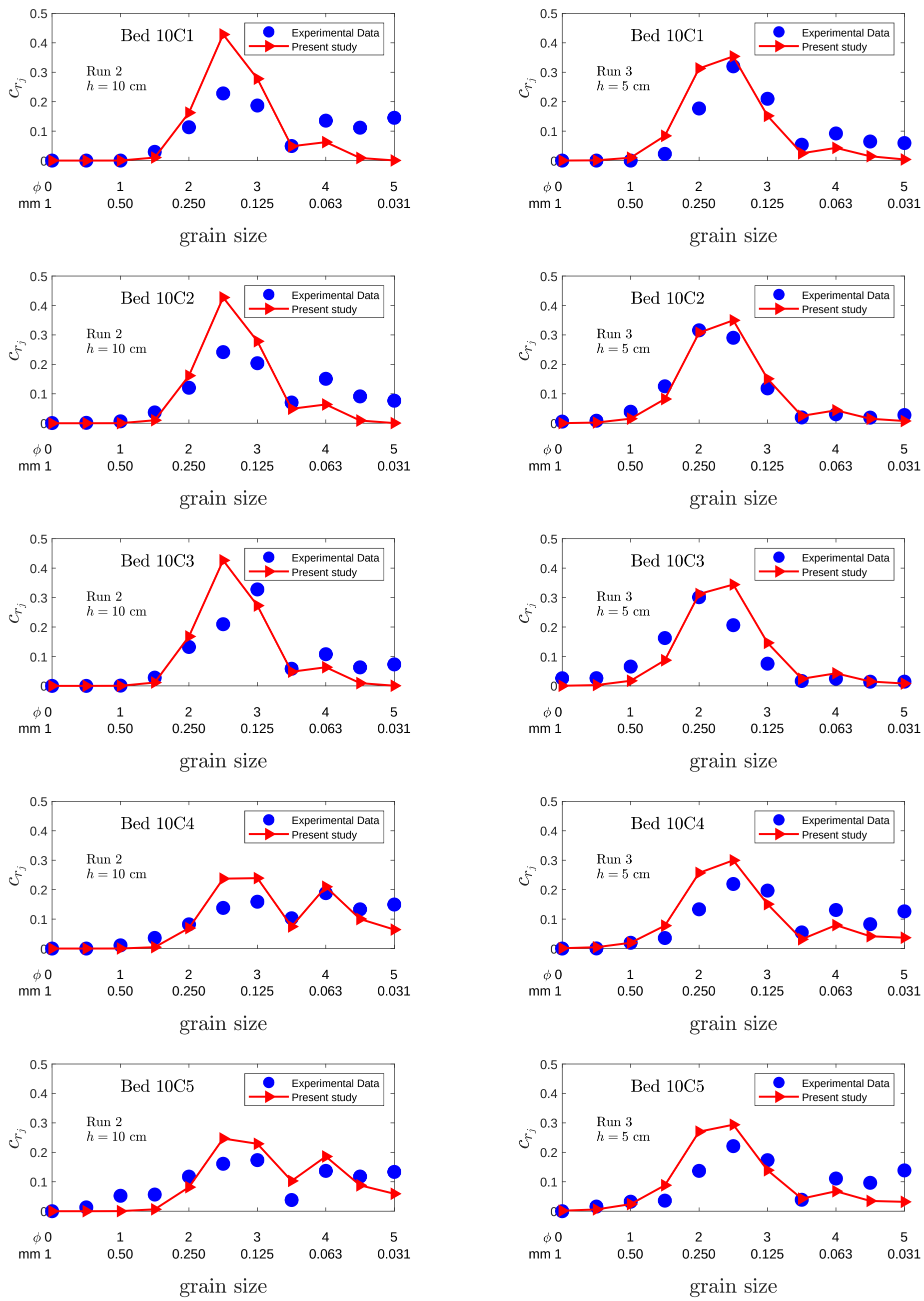


Figure 7: Comparison of computed and experimental relative suspension concentration of grain-size distribution for Run 2 (u_{max} = 101 cm/s) and Run 3 (u_{max} = 116 cm/s) at different heights.

Highlights (3 to 5 bullet points (maximum 85 characters including spaces per bullet point)

Highlights

- A theoretical model for grain size distribution over erodible beds of different configuration has been developed in a steady two-dimensional transport through an open channel.
- The effect of hindered settling on the grain-size distribution has been incorporated in the model.
- Obtained non-linear partial differential equation has been solved by a numerical scheme.
- Present study also shows the variation of concentration of different grain sizes along stream-wise direction at different heights.

Abstract

A theoretical model for determining the concentration of various grain sizes in suspension over erodible beds of different configuration, has been developed based on advection-diffusion equation for sediment concentration in a steady two-dimensional transport through an open channel. The effect of hindered settling over grain-size distribution due to increased concentration in suspension is also considered into the model. In the evaluation of reference concentration, the influence of non-ceasing probability, pick-up probability and incipient motion probability of the sediment particles has been acknowledged. The non-linear partial differential equation thus obtained, has been solved numerically. The present study also shows the concentration of different grain sizes along the stream-wise direction at different heights and finds that the finer particles reach equilibrium at a larger distance along main flow direction in comparison to that of the coarser particles. It is found that hindered settling effect is prominent in case of medium size particles and the grain-size distribution has bimodal nature with a strong peak and another weaker secondary peak at higher levels in suspension for higher velocities. Obtained numerical solution of the proposed model has been interpreted physically and compared with the experimental data.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: