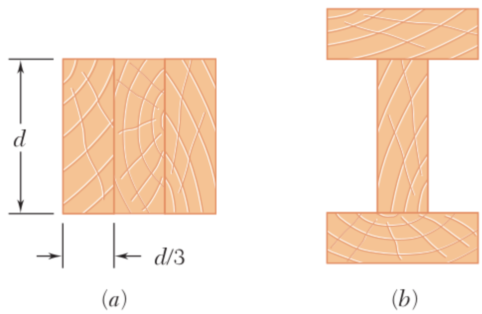
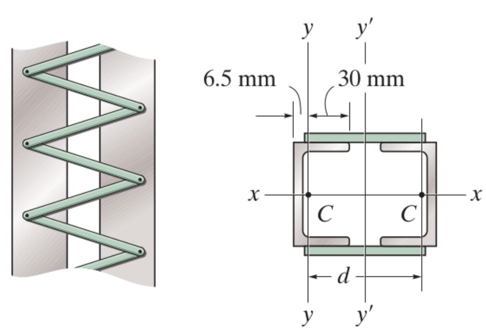
Tutorial Sheet 9: Buckling of Columns

1. A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b. [1.421]

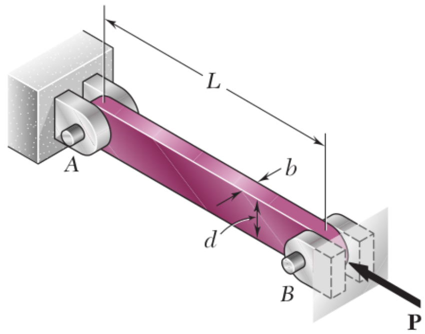


2. The two steel channels are to be laced together to form a 9 m long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 1950$ mm^2 and moments of inertia $I_x = 21.60 \times 10^6$ mm^4, $I_y = 0.15 \times 10^6$ mm^4. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that bucling occurs about the x-x and y'-y' axes due to the same load. What is the value of the critical load? Neglect the effect of the lacing. $E_{\text{st}} = 200$ GPa, $\sigma_y = 350$ MPa. [209.76 mm, 1052.8 kN]

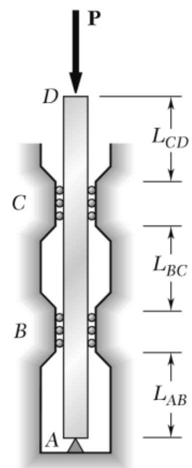


3. The uniform brass bar AB has a rectangular cross section and is supported by pins and brackets as shown. Each end of the bar can rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. Determine the ratio b/d for which the factor of safety is the same about the horizontal

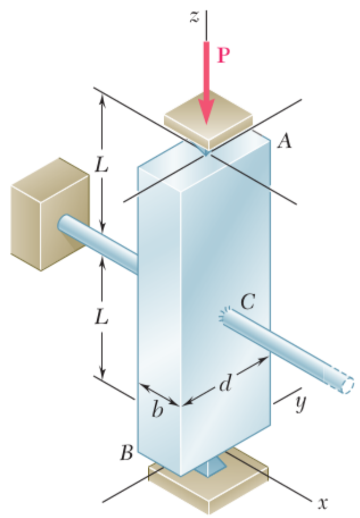
and vertical axes. Determine the factor of safetey if $P = 8$ kN. Here, $L = 2$ m, $d = 38$ mm, and $E = 105$ GPa. [1/2, 2.81]



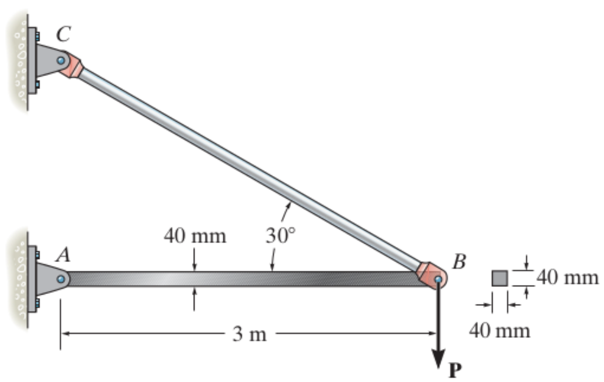
4. A 32 mm square aluminium strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 1.4$ m, determine the largest values of L_{BC} and L_{CD} that can be used if the allowable load P is to be as large as possible. Also, determine the magnitude of the corresponding allowable load if the factor of safety is 2.8. Consider only buckling in the plane of the figure and use $E = 72$ GPa. [1.96 m, 0.49 m, 23.1 kN]



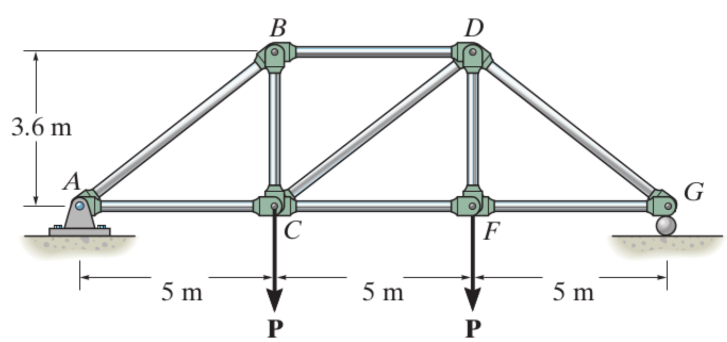
5. Column AB has a uniform cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the xz plane at its midpoint C and a carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L. Use $E = 200$ GPa.



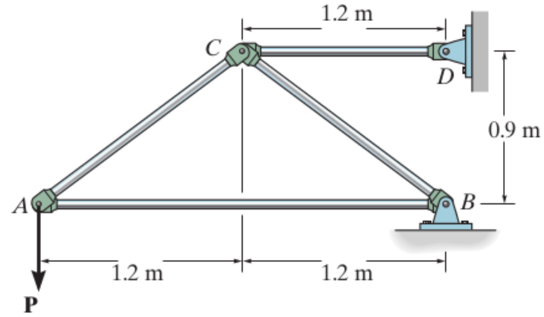
6. The steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P than can be applied to the frame. Use a factor of safety with respect to buckling of 2. [13.51 kN]



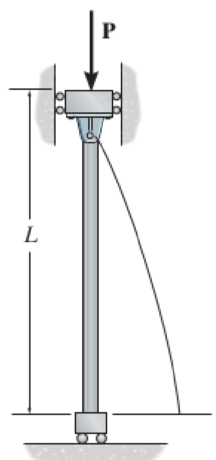
7. The members of the truss are assumed to be pin connected. If the member BD is a steel (E = 200 GPa) rod of radius 50 mm, determine the maximum load P that can be supported by the turss without causing the member to buckle. [315.4 kN]



8. The truss is made from steel ($E = 200$ GPa) bars, each of which has a circular cross section with a diameter of 40 mm. Determine the maximum force P that can be applied without causing any of the members to buckle. The members are pin connected at their ends. [25 kN]



9. The column with constant flexural rigidity (EI) has the end constraints shown in the figure. Determine the critical load for the column. $[\frac{\pi^2 EI}{4L^2}]$



10. For a column with clamped-clamped conditions, the critical load may be determined by solving the fourth-order equation:

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} + k^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0,$$

where $k^2 = \dfrac{P}{EI}$. Using four appropriate boundary conditions show that for non-trivial solutions to exist, two conditions are obtained:

$$\tan\frac{kL}{2} = \frac{kL}{2}, \quad \text{or,} \quad \sin\frac{kL}{2} = 0.$$

Determine the actual critical load from the second condition. Justify why the first condition is not necessary. $[\frac{\pi^2 EI}{(0.5L)^2}]$