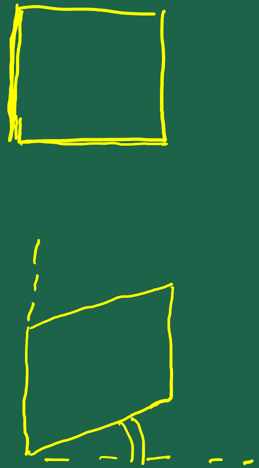
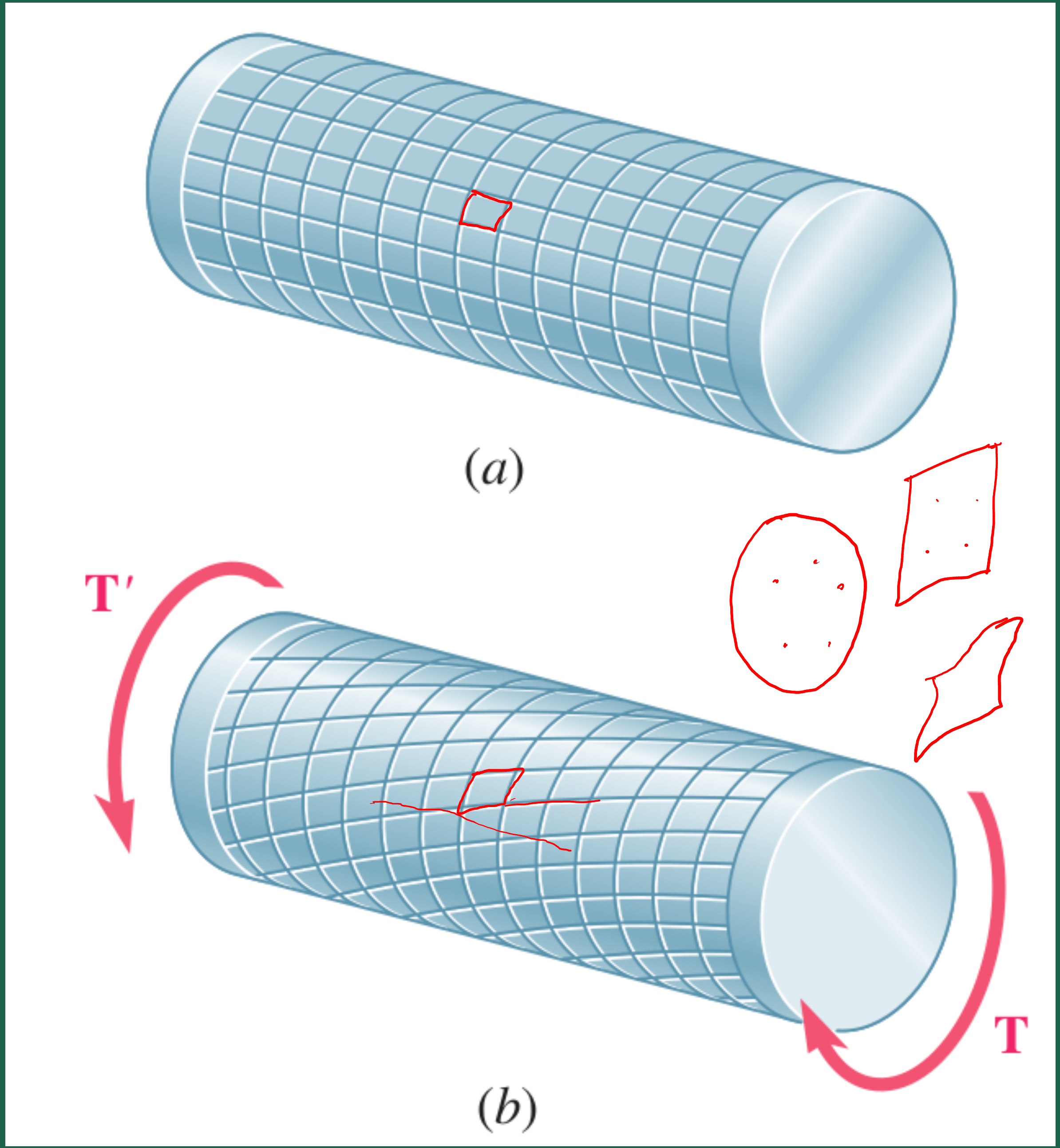
TORSION



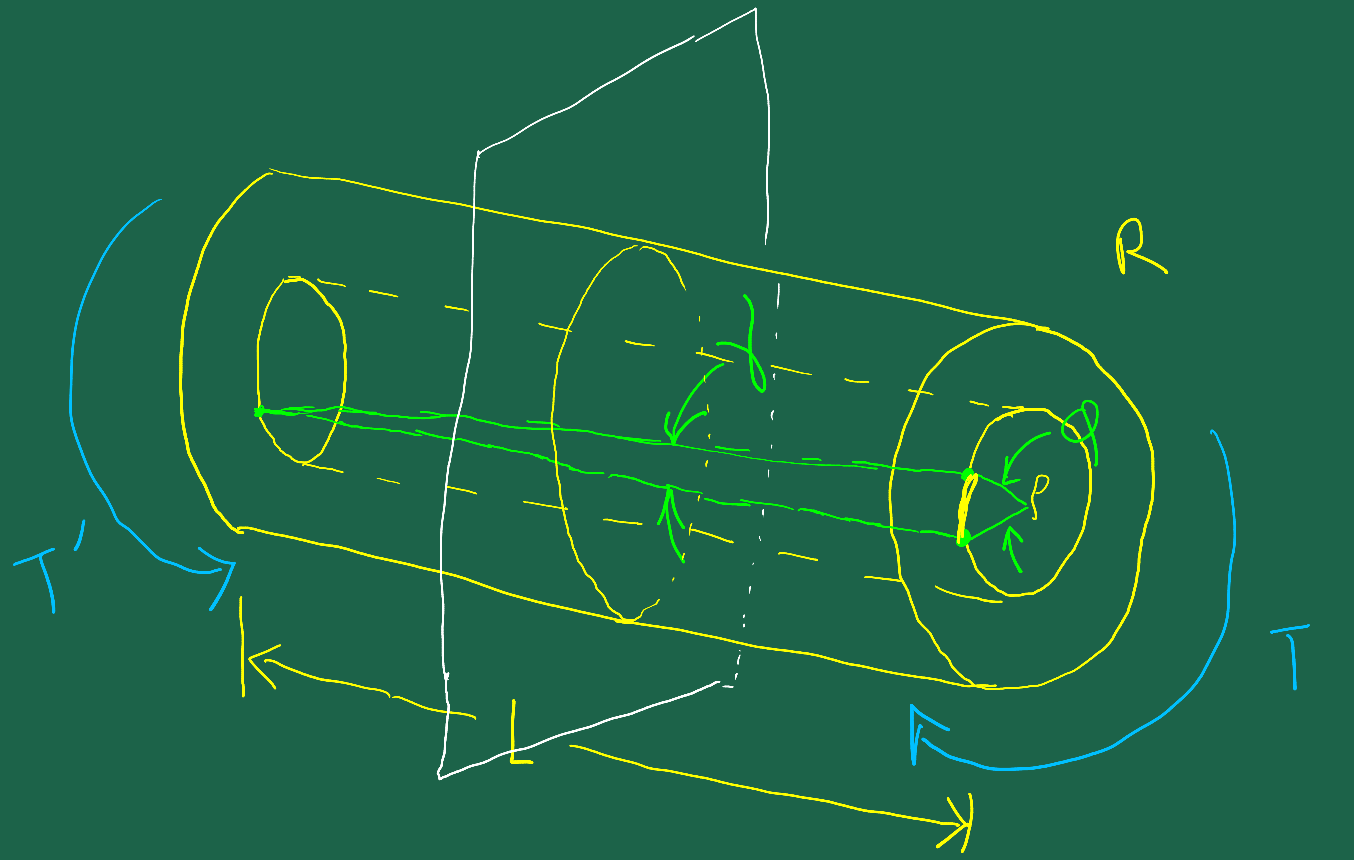
$$\tau = G\gamma$$



$$\tau_{xy} = G\gamma_{xy}$$

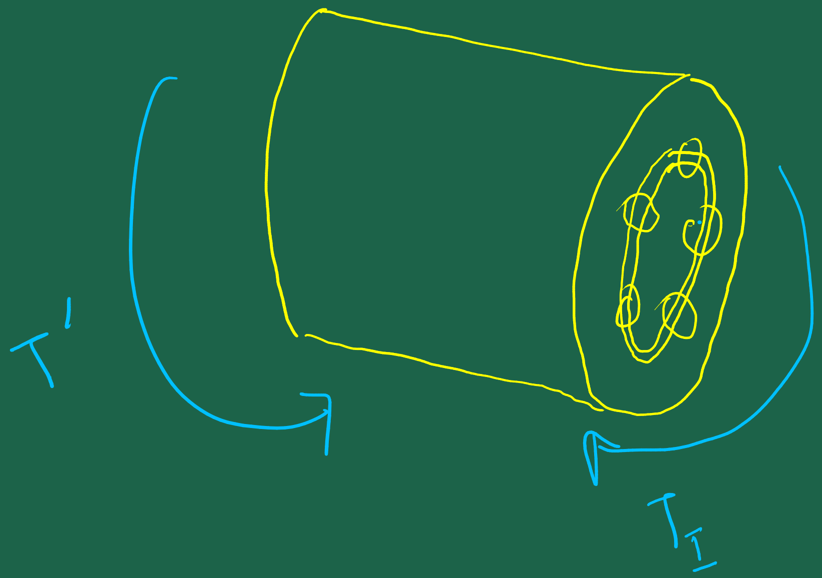
$$\tau_{yz} = G\gamma_{yz}$$

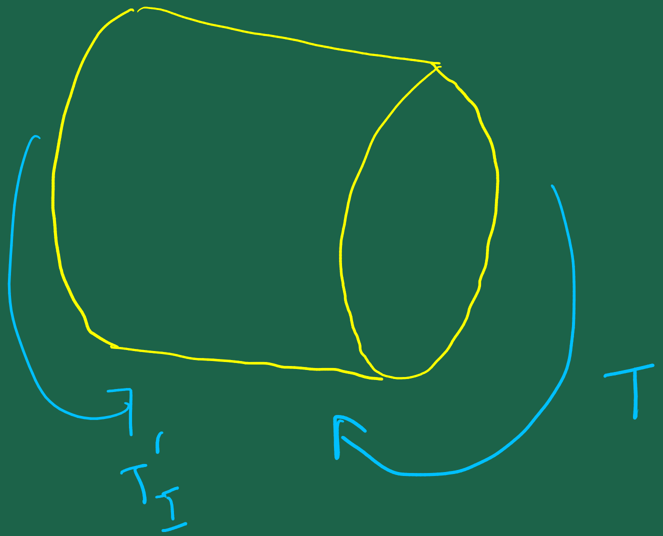
$$\tau_{zx} = G\gamma_{zx}$$

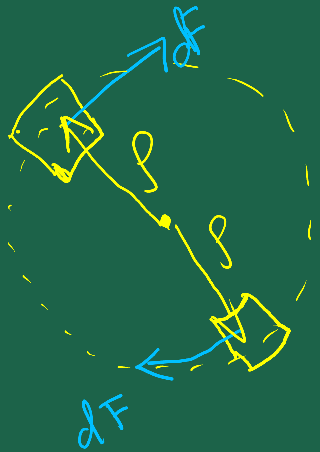


$$\gamma L = \rho\varphi$$

$$\Rightarrow \gamma = \frac{\rho\varphi}{L} \rightarrow \tau = G\gamma = \frac{G\rho\varphi}{L}$$







$$\int_0^R \int_0^{2\pi} \rho \, dF$$

$$dF = \tau \, dA$$

$$\underbrace{T'}_{=T} = T_I = \int_0^R \int_0^{2\pi} \rho \, \tau \, dA$$

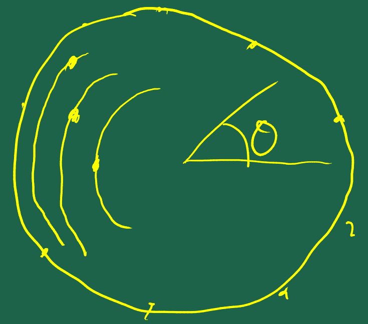
$$= \int_0^R \int_0^{2\pi} \rho \, \tau \, \rho \, d\theta \, d\rho$$

$$= \int_0^R \tau \rho^2 \left(\int_0^{2\pi} d\theta \right) d\rho$$

$$= \int_0^R 2\pi \rho^2 \, \frac{G \rho \varphi}{L} \, d\rho$$

$$= \frac{2\pi G \varphi}{L} \int_0^R \rho^3 \, d\rho = \frac{2\pi G \varphi}{L} \frac{R^4}{4} = \frac{\varphi G}{L} \underbrace{\left(\frac{\pi R^4}{2} \right)}_{J} = \frac{\varphi G J}{L}$$

J : polar moment of inertia



$$T = \frac{\varphi G J}{L} \Rightarrow \boxed{\varphi = \frac{TL}{GJ}}$$

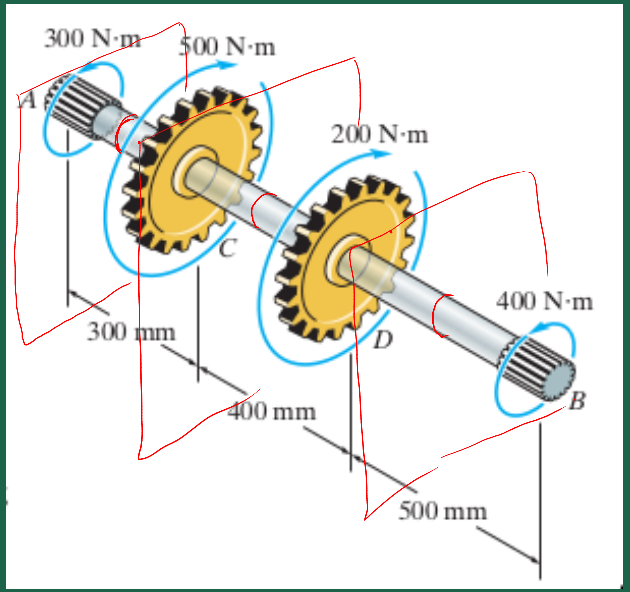
$$\tau = \frac{G \rho \varphi}{L} = \frac{\cancel{G} \rho}{\cancel{L}} \frac{T \cancel{L}}{\cancel{G} J} = \boxed{\frac{T \rho}{J} = \tau}$$

Power Transmission

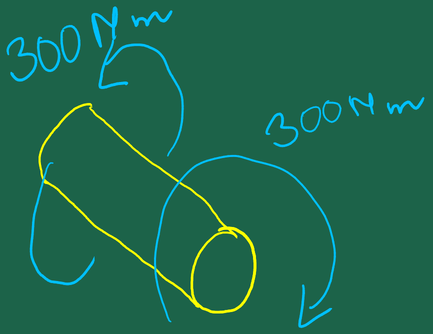
$$P = T\omega$$

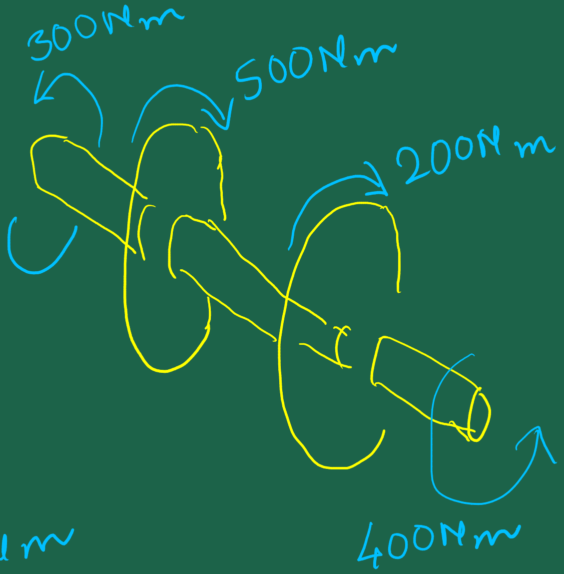
$$\varphi \qquad\qquad \tau$$

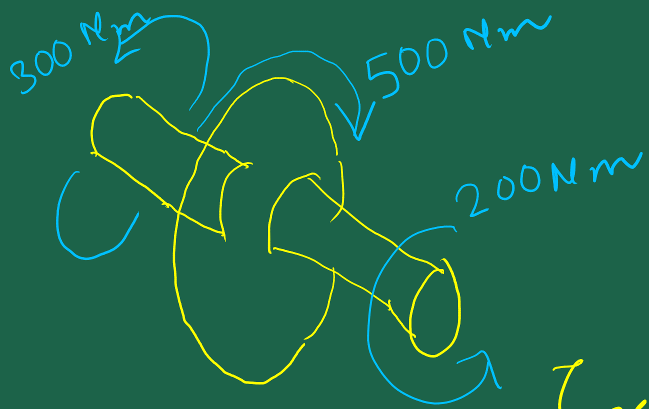
1. The solid 30 mm diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft. [75.5 MPa]



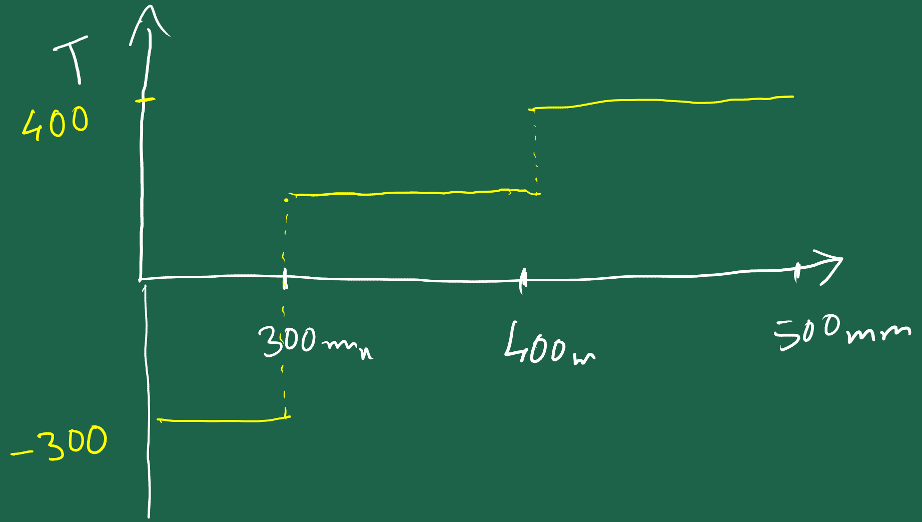
$$\tau_{max} = \frac{T\rho}{J}$$

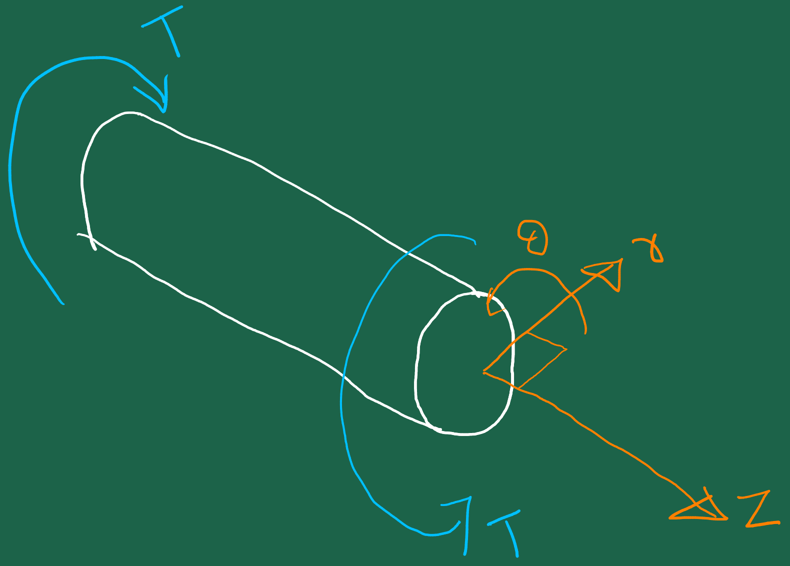




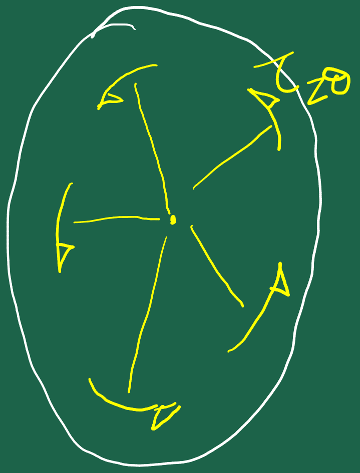


$$\tau_{max} = \frac{(400\ \mathrm{N\,m})(15\times10^{-3}\ \mathrm{m})}{\frac{\pi}{2}\times(15\times10^{-3}\ \mathrm{m})^4}$$

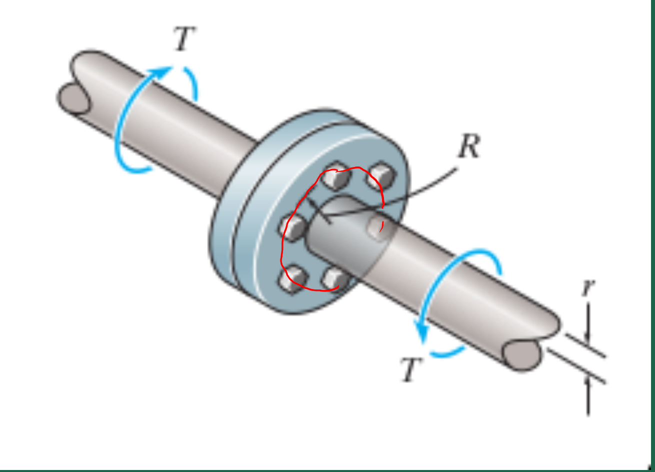




$\tau_{z\theta}$ $\tau_{z\theta}$



2. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is uniform, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shar stress in the bolts. Each bolt has a diameter d. $[(2r^3)/(Rd^2)]$



$$F = \frac{T}{R}$$

(N: no. of bolts)

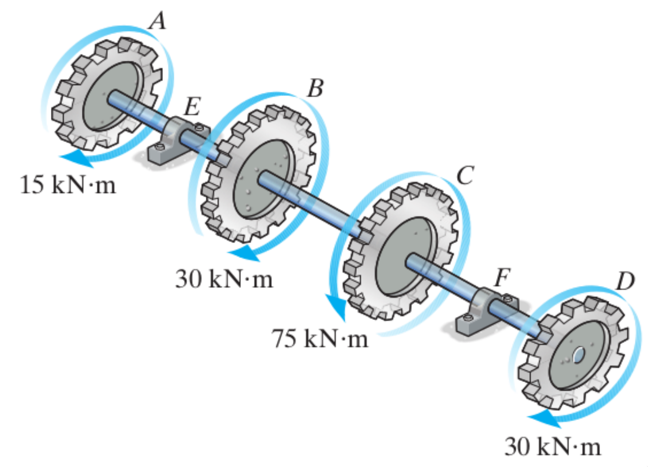
$$f = \frac{F}{N}$$

$$\tau_{bolt} = \frac{f}{\pi d^2/4}$$

$$\tau_{shaft,max} = \frac{Tr}{J} = \frac{Tr}{\frac{\pi}{2} r^4} = \frac{2T}{\pi r^3}$$

$$\tau_{shaft,max} = \tau_{bolt} \Rightarrow$$

3. If the tubular shaft is made from material having an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa, determine the required minimum wall thickness of the shaft to the nearest millimeter. The shaft has an outer diameter of 150 mm. [25 mm]



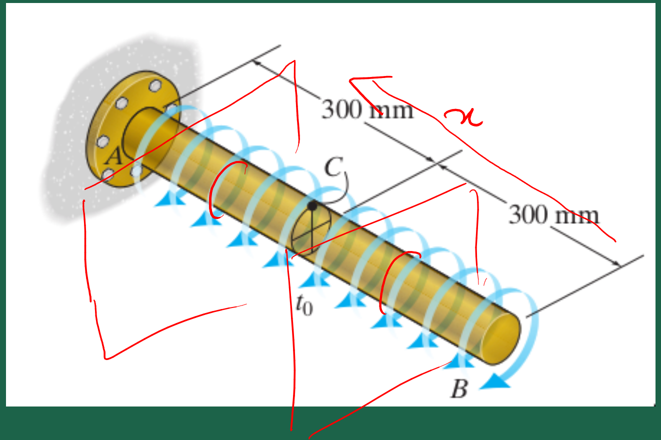
$$\tau = \frac{T r}{J} \qquad J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) \qquad t = r_o - r_i$$

AB : $+15$ kN

BC : $+45$ kN

CD : -30 kN

4. If the rod is subjected to a uniform distributed torque of $t_0 = 1.5$ kN·m/m (torque per unit length), determine the minimum required diameter d of the rod if the material has an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. [39.4 mm]



$$\tau = \frac{T r}{J}$$

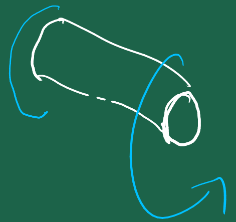
$T = 1.5 \frac{kN \cdot m}{m} \times x$ (from B to A) [x is in m]

$$T_{max} = \frac{1.5 \, kN \cdot m}{m} \times 0.6 \, m$$

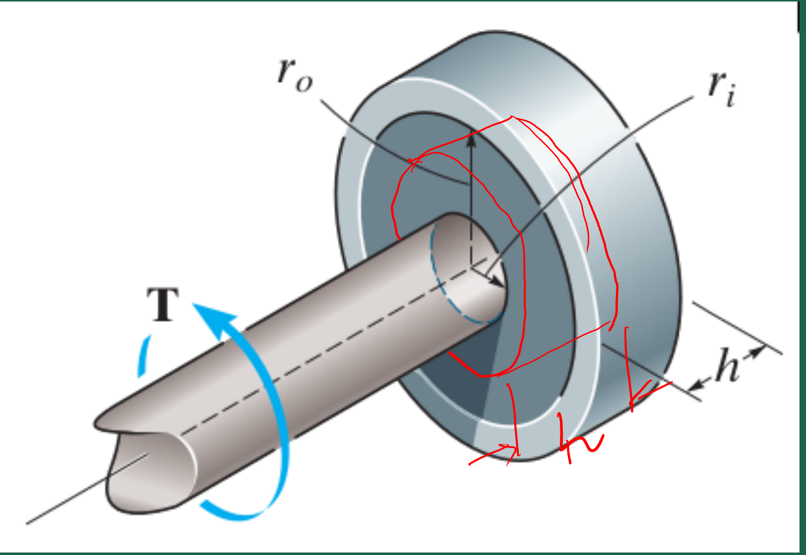
from B:

$$T_{B-A} = t_0 x$$

$$T_{A-B} = t_0 (0.6 - x) - t_0 (0.6)$$



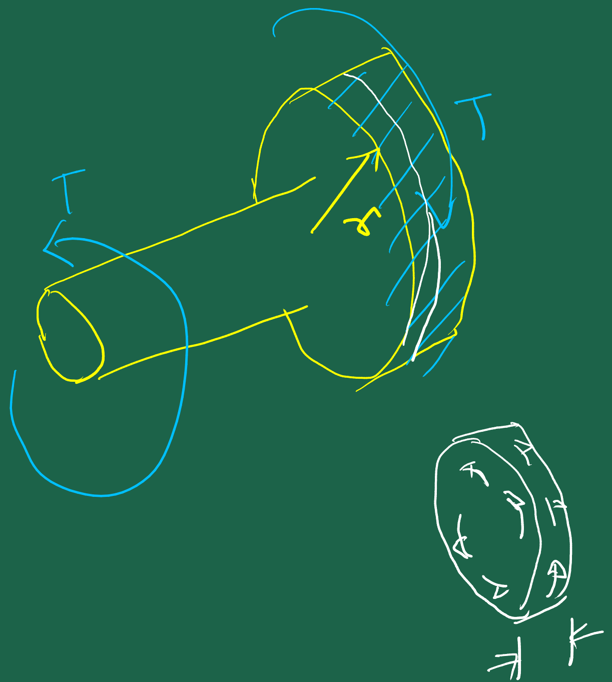
5. A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the shaft, determine the maximum shear stress in the rubber. $\left[\dfrac{T}{2\pi r_i^2 h}\right]$



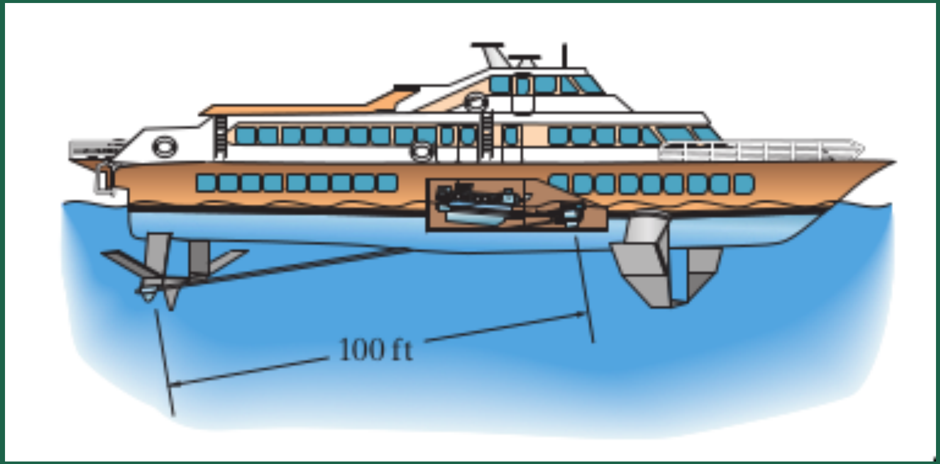
$$\tau = \frac{F}{A} = \frac{T/r}{2\pi r h}$$

$$= \frac{T}{2\pi r^2 h}$$

$$\tau_{max} = \frac{T}{2\pi r_i^2 h}$$



6. The hydrofoil boat has an A992 steel propeller shaft that is 100 ft long. It is connected to an in-line diesel engine that delivers a maximum power of 2500 hp and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 8 in. and the wall thickness is $\frac{3}{8}$ in., determine the maximum shear stress developed in the shaft. Also, what is the angle of twist in the shaft at full power? [2.83 ksi; 4.43°]



$$\tau = \frac{Tr}{J}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)$$

$$\phi = \frac{TL}{GJ}$$

$$P = T\omega \Rightarrow T$$

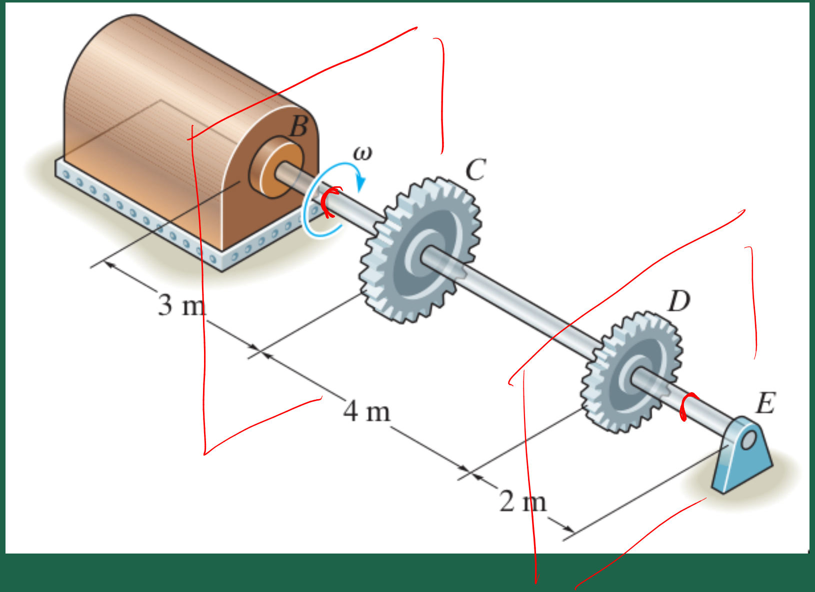
$$G = 11 \times 10^6 \text{ psi}$$

$$1700 \text{ rpm} \rightarrow \frac{2\pi \times 1700}{60} \text{ rad/s}$$

$$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ ft} = 12 \text{ in}$$

7. The turbine develops 150 kW of power, which is transmitted to the gears such that the gear C receives 70% and gear D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis. [9.12 MPa; 0.585°]



$$P_C = 70\% \text{ of } 150 \text{ kW} \rightarrow T_C$$

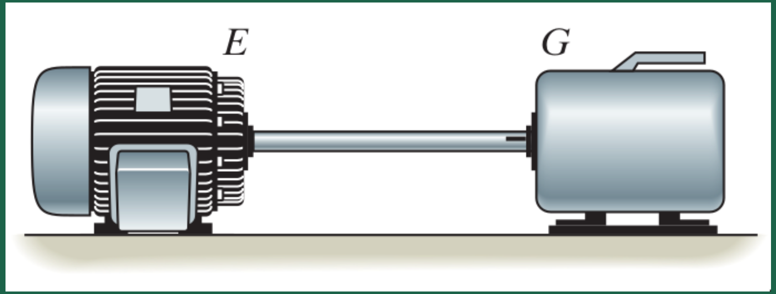
$$P_D = 30\% \text{ of } 150 \text{ kW} \rightarrow T_D$$

$$T_{BC} = \frac{P}{\omega} \rightarrow \tau$$

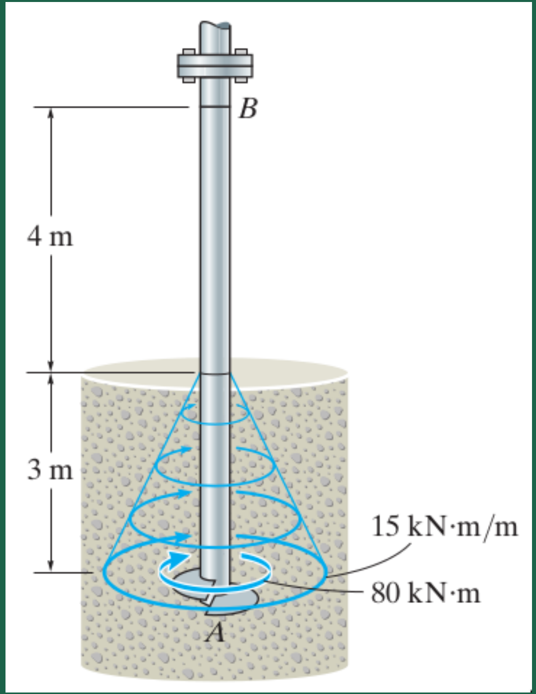
$$\phi_{E/B} = \phi_{D/B} = \phi_{D/C} + \phi_{C/B}$$

$$= \sum \frac{TL}{GJ} = \frac{T_{DC} L_{DC}}{GJ} + \frac{T_{CB} L_{CB}}{GJ}$$

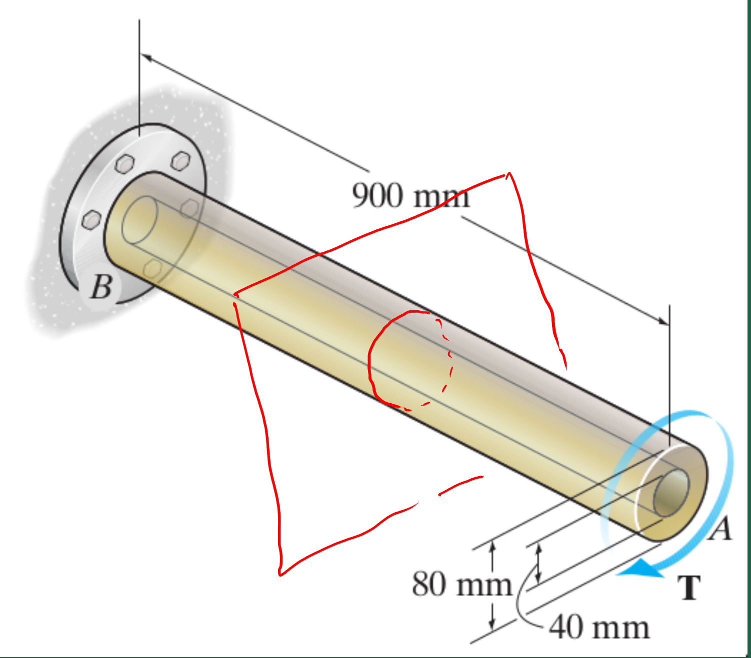
8. The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{\text{allow}} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad. [7.53 mm]

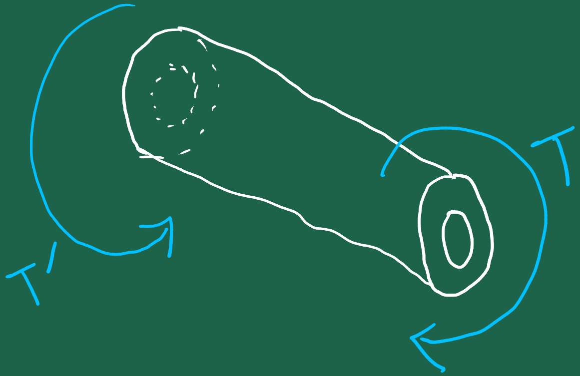


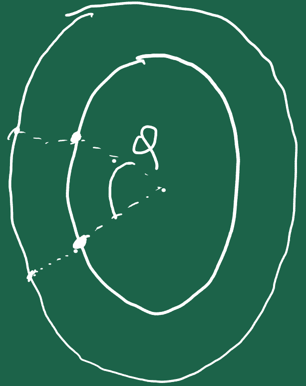
9. The A992 steel posts are "drilled" at constant angular speed into the soil using the rotary installer. If the post has an inner diameter of 200 mm and an outer diameter of 225 mm, determine teh relative angle of twist of end A of the post with respect to end B when the post reaches the depth indicated. Due to soil friction, assume the torque along the post varies linearly as shown, and a concentrated torque of 80 kN·m acts at the bit. [5.62°]



10. The magnesium tube is bonded to the A-36 steel rod. If the allowable shear stresses for the magnesum and steel are $(\tau_{\text{allow}})_{\text{mg}} = 45$ MPa and $(\tau_{\text{allow}})_{\text{st}} = 75$ MPa, respectively, determine the maximum allowable torque that can be applied at A. Also, find the corresponding angle of twist of end A. [4.34 kN·m; 2.58°]







$$\phi_{st} = \phi_{Mg}$$

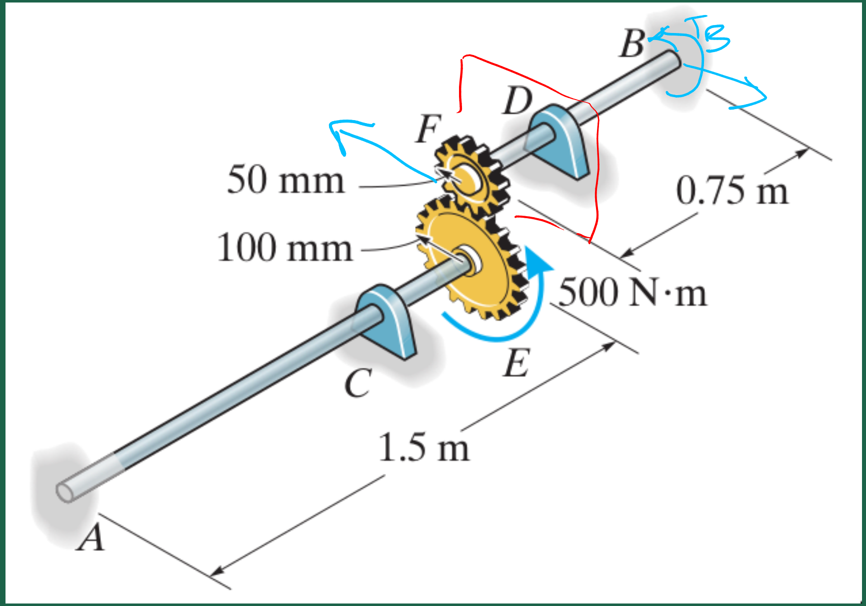
$$\Rightarrow \frac{T_{st} L}{G_{st} J_{st}} = \frac{T_{Mg} L}{G_{Mg} J_{Mg}}$$

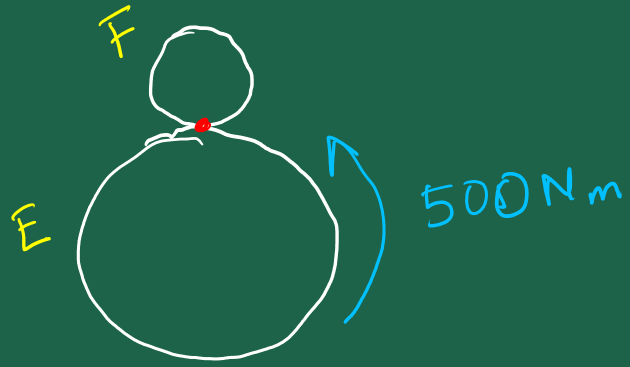
$$\tau_{allow}|_{Mg} = \frac{T_{Mg}\, r}{J_{Mg}}$$

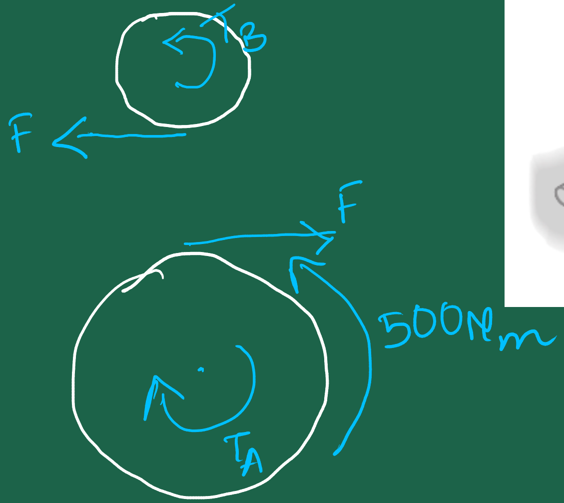
$$\tau_{allow}|_{st} = \frac{T_{st}\, r}{J_{st}}$$

$$T' = T = T_{st} + T_{Mg}$$

11. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N·m is applied to the gear at E as shown, determine the reactions at A and B. [55.6 N·m; 222 N·m]







$$T_A + (0.1)F = 500 \text{ Nm}$$

$$T_B = (0.05)F$$

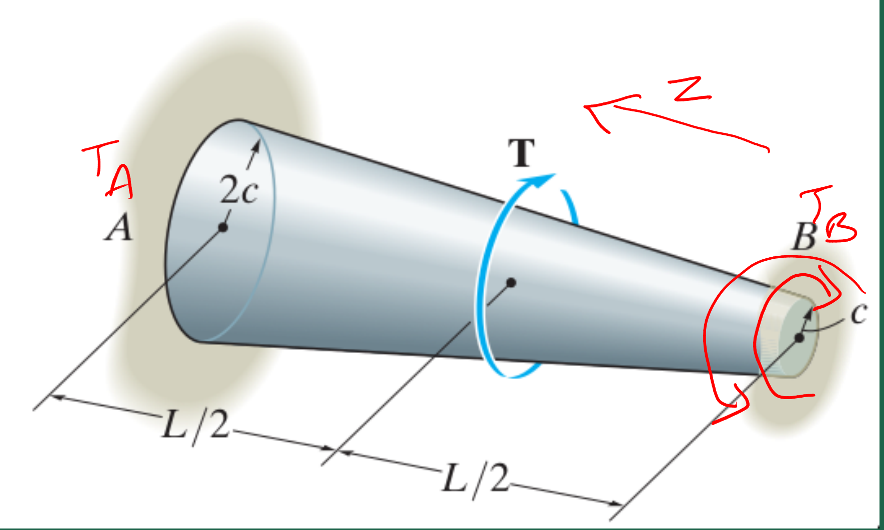
$$\Rightarrow T_A + 2T_B = 500 \text{ Nm}$$

$$\phi_F(0.05) = \phi_E(0.1)$$

$$\Rightarrow \phi_F = 2\phi_E$$

$$\Rightarrow \frac{T_B(0.75)}{GJ} = 2\frac{T_A(1.5)}{GJ}$$

12. The tapered shaft is confined by the fixed supports at A and B. If a torque T is applied at its mid-point, determine the reactions at the supports. $\left[\frac{152}{189}T; \frac{37}{189}T\right]$



ϕ_T (with no wall)

ϕ_{T_B}

$\phi_{T_B} = \phi_T$

$J = \frac{\pi r^4}{2}$