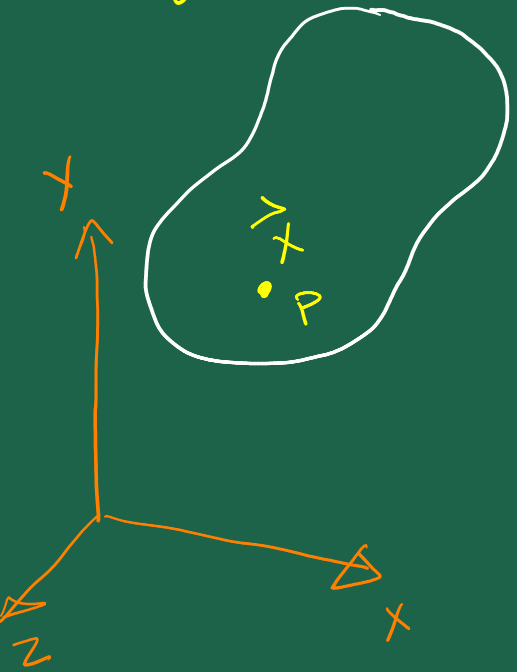
# Strain and Strain Transformation

$\Delta \quad \nabla$ nabla

## Deformation





$$\vec{x} \equiv \vec{x}(\vec{X})$$

Deformation map

$$\nabla \varphi$$

$$\nabla \varphi \equiv \frac{\partial \varphi}{\partial x}\hat{i} + \frac{\partial \varphi}{\partial y}\hat{j} + \frac{\partial \varphi}{\partial z}\hat{k}$$

$$[\nabla \varphi] \equiv \begin{bmatrix} \frac{\partial \varphi}{\partial x} \\ \frac{\partial \varphi}{\partial y} \\ \frac{\partial \varphi}{\partial z} \end{bmatrix} \qquad [\vec{V}] \equiv \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

$$[\nabla \vec{V}]$$

$$[\nabla \vec{v}] = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

$$\vec{a} \cdot \vec{b} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}^T \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = a_x b_x + a_y b_y + a_z b_z$$

$\vec{a} \cdot \nabla \vec{v} \rightarrow$ vector entity

$\left([\vec{a}]^T [\nabla \vec{v}]\right)^T \rightarrow 1 \times 3$ (row matrix) X

$[\nabla \vec{v}][\vec{a}] \rightarrow 3 \times 1$ ✓

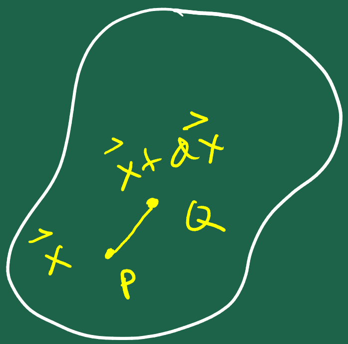
$[\nabla \vec{v}]^T [\vec{a}] \rightarrow 3 \times 1$ X

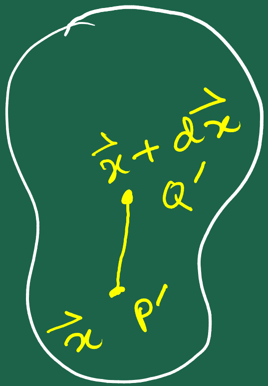
$$d\vec{x} \cdot \nabla \vec{u} = [\nabla \vec{u}][d\vec{x}]$$

## Displacement

$$\vec{u} := \vec{x} - \vec{X}$$

## Quantification of deformation




$$\vec{u}(\vec{X}+d\vec{X}) = (\vec{x}+d\vec{x}) - (\vec{X}+d\vec{X})$$

$$= \vec{x}-\vec{X} + d\vec{x} - d\vec{X}$$

$$= \vec{u}(\vec{X}) + d\vec{x} - d\vec{X}$$

$$\therefore \; d\vec{x} = d\vec{X} + \vec{u}(\vec{X}+d\vec{X}) - \vec{u}(\vec{X})$$

Taylor expansion

$$f(x+h) = f(x) + \frac{h}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \cdots$$

$$f(x+h, y+k) = f(x,y) + \frac{h}{1!}\frac{\partial f}{\partial x} + \frac{k}{1!}\frac{\partial f}{\partial y} + \frac{h^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{k^2}{2!}\frac{\partial^2 f}{\partial y^2} + 2\frac{hk}{2!}\frac{\partial^2 f}{\partial x \partial y} + \cdots$$

$$f(x+h, y+k, z+m) = f(x,y,z) + \frac{h}{1!}\frac{\partial f}{\partial x} + \frac{k}{1!}\frac{\partial f}{\partial y} + \frac{m}{1!}\frac{\partial f}{\partial z} + \cdots$$

$$f(\vec{X}+d\vec{X})$$

$$\vec{u}(\vec{x}+d\vec{x}) = \vec{u}(x+dx, y+dy, z+dz)$$

$$= \vec{u}(x,y,z) + \frac{dx}{1!}\frac{\partial \vec{u}}{\partial x} + \frac{dy}{1!}\frac{\partial \vec{u}}{\partial y} + \frac{dz}{1!}\frac{\partial \vec{u}}{\partial z} + \dots$$

$$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= \vec{u}(\vec{x}) + dx\frac{\partial \vec{u}}{\partial x} + dy\frac{\partial \vec{u}}{\partial y} + dz\frac{\partial \vec{u}}{\partial z}$$

$$= \vec{u}(\vec{x}) + \left\{ \left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)\cdot\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\right\}\vec{u}$$

$$= \vec{u}(\vec{x}) + d\vec{x}\cdot\nabla\vec{u}$$

$$\therefore d\vec{x} = d\vec{x} + \vec{u}(\vec{x}+d\vec{x}) - \vec{u}(\vec{x}) = d\vec{x} + \cancel{\vec{u}(\vec{x})} + d\vec{x}\cdot\nabla\vec{u} - \cancel{\vec{u}(\vec{x})}$$

$$= d\vec{x} + d\vec{x}\cdot\nabla\vec{u}$$

$$[d\vec{x}] = [d\vec{X}] + [d\vec{X} \cdot \nabla \vec{u}]$$

$$= [d\vec{X}] + [\nabla \vec{u}][d\vec{X}] = [d\vec{X}]([I] + [\nabla \vec{u}])$$

$$|d\vec{x}| = \sqrt{dx^2 + dy^2 + dz^2}$$

$$|d\vec{X}| = \sqrt{dX^2 + dY^2 + dZ^2}$$

$$|d\vec{x}| \longleftrightarrow |d\vec{X}|$$

$|d\vec{x}|^2$ with $|d\vec{X}|^2$

$$|d\vec{x}|^2 = d\vec{x} \cdot d\vec{x} \; ; \; |d\vec{X}|^2 = d\vec{X} \cdot d\vec{X}$$

$\sigma_{nn}$ or $\tau_{nn}$

$\sigma_{ns}$ or $\tau_{ns}$

$$\varepsilon := \frac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|}$$

$$|d\vec{x}|^2 = d\vec{x}\cdot d\vec{x}$$
$$= (d\vec{x} + d\vec{x}\cdot\nabla\vec{u})\cdot(d\vec{x} + d\vec{x}\cdot\nabla\vec{u})$$
$$= [d\vec{x} + d\vec{x}\cdot\nabla\vec{u}]^T[d\vec{x} + d\vec{x}\cdot\nabla\vec{u}]$$
$$= \left([d\vec{x}] + [\nabla\vec{u}][d\vec{x}]\right)^T\left([d\vec{x}] + [\nabla\vec{u}][d\vec{x}]\right)$$
$$= \left([d\vec{x}]^T + [d\vec{x}]^T[\nabla\vec{u}]^T\right)\left([d\vec{x}] + [\nabla\vec{u}][d\vec{x}]\right)$$
$$= [d\vec{x}]^T[d\vec{x}] + [d\vec{x}]^T[\nabla\vec{u}][d\vec{x}] + [d\vec{x}]^T[\nabla\vec{u}]^T[d\vec{x}]$$
$$+ \underbrace{[d\vec{x}]^T[\nabla\vec{u}]^T[\nabla\vec{u}][d\vec{x}]}_{\text{neglect}}$$

$$\approx |d\vec{x}|^2 + [d\vec{x}]^T\left([\nabla\vec{u}] + [\nabla\vec{u}]^T\right)[d\vec{x}]$$

$$\vec{a}\cdot\vec{b}$$
$$[\vec{a}]^T[\vec{b}]$$

$$[d\vec{x}\cdot\nabla\vec{u}]$$
$$= [\nabla\vec{u}][d\vec{x}]$$

$$([A][B])^T$$
$$= [B]^T[A]^T$$

$$\left(\frac{\partial u}{\partial x}\right)^2, \quad \frac{\partial u}{\partial x}\frac{\partial v}{\partial x}$$
$$\frac{\partial v}{\partial y}\quad \frac{\partial w}{\partial z}$$

$$|d\vec{x}|^2 = |d\vec{x}|^2 + [d\vec{x}]^T\left([\nabla\vec{u}] + [\nabla\vec{u}]^T\right)[d\vec{x}]$$

Define the infinitesimal or small strain tensor

$$\underset{\sim}{\varepsilon} := \frac{1}{2}\left\{\nabla\vec{u} + (\nabla\vec{u})^T\right\}$$

$$[\underset{\sim}{\varepsilon}] := \frac{1}{2}\left\{[\nabla\vec{u}] + [\nabla\vec{u}]^T\right\}$$

↳ Symmetric part of $[\nabla\vec{u}]$

$$\frac{1}{2}\left([\nabla\vec{v}] + [\nabla\vec{v}]^T\right)$$

$= \vec{E}$ (strain rate tensor)

$$[A] = \frac{1}{2}\left([A] + [A]^T\right) + \frac{1}{2}\left([A] - [A]^T\right)$$

↳ Symmetric part

$$\left\{\frac{1}{2}\left([A] + [A]^T\right)\right\}^T = \frac{1}{2}\left([A]^T + [A]\right)$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

= In terms of derivatives of $u$, $v$, $w$

$$\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) = \frac{\partial u}{\partial x}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \ldots$$

$$|d\vec{x}|^2 = |d\vec{X}|^2 + [d\vec{X}]^T\left([\nabla\vec{u}] + [\nabla\vec{u}]^T\right)[d\vec{X}]$$

$$\Rightarrow |d\vec{x}|^2 = |d\vec{X}|^2 + 2[d\vec{X}]^T[\underset{\sim}{\varepsilon}][d\vec{X}]$$

$$\Rightarrow |d\vec{x}|^2 = |d\vec{X}|^2 + 2|d\vec{X}|^2[\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]$$

$$\Rightarrow \frac{|d\vec{x}|^2}{|d\vec{X}|^2} = 1 + 2[\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]$$

$$\Rightarrow (1+\varepsilon)^2 = 1 + 2[\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]$$

$$\Rightarrow \cancel{1} + 2\varepsilon + \varepsilon^2 = \cancel{1} + 2[\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]$$

Neglect $\varepsilon^2$ compared $\varepsilon$

$$\therefore \boxed{\varepsilon_N = [\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]}$$

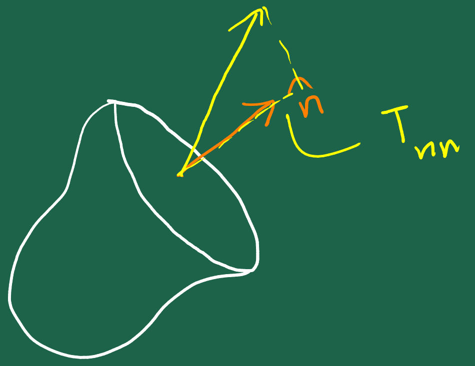
(Normal strain along a particular dir$^n$ $\rightarrow \hat{N}$)

$$d\vec{X} = |d\vec{X}|\,\hat{N}$$

$$[d\vec{X}] = |d\vec{X}|[\hat{N}]$$

$$[d\vec{X}]^T = |d\vec{X}|[\hat{N}]$$

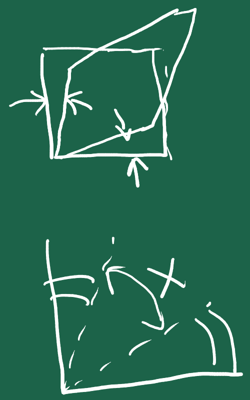
$$\varepsilon := \frac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|}$$

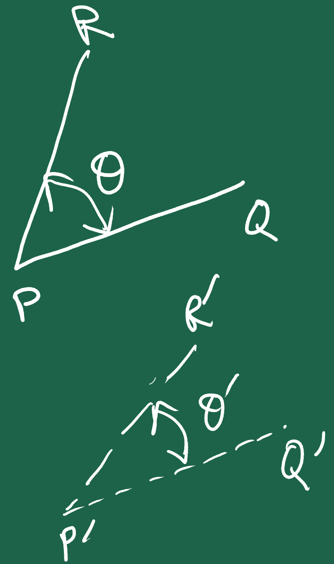


# Change in dir$^n$ of an elemental line segment


Given: $\hat{N}$

Find : $\hat{n}$






$\theta - \theta'$

$$d\vec{x} = d\vec{X} + d\vec{X}\cdot\nabla\vec{u}$$

$$\Rightarrow |d\vec{x}|\,\hat{n} = |d\vec{X}|\,\hat{N} + |d\vec{X}|\,\hat{N}\cdot\nabla\vec{u}$$

$$\Rightarrow \frac{|d\vec{x}|}{|d\vec{X}|}\,\hat{n} = \hat{N} + \hat{N}\cdot\nabla\vec{u}$$

$$\Rightarrow (1+\varepsilon_N)\,\hat{n} = \hat{N} + \hat{N}\cdot\nabla\vec{u}$$

$$\Rightarrow \boxed{\hat{n} = \frac{1}{1+\varepsilon_N}\left(\hat{N} + \hat{N}\cdot\nabla\vec{u}\right)}$$

# Change in the angle bet$^n$ 2 elemental line segments


Given: $\theta$ between $\hat{N}^{(1)}$ & $\hat{N}^{(2)}$

Find: $\theta'$ between $\hat{n}^{(1)}$ & $\hat{n}^{(2)}$

$$\hat{n}^{(1)} \cdot \hat{n}^{(2)} = \left\{ \frac{1}{\left(1+\varepsilon_N^{(1)}\right)} \left( \hat{N}^{(1)} + \hat{N}^{(1)} \cdot \nabla \vec{u} \right) \right\} \cdot \left\{ \frac{1}{\left(1+\varepsilon_N^{(2)}\right)} \left( \hat{N}^{(2)} + \hat{N}^{(2)} \cdot \nabla \vec{u} \right) \right\}$$

$$\Rightarrow \left(1+\varepsilon_N^{(1)}\right)\left(1+\varepsilon_N^{(2)}\right) \cos\theta' = \left[ \hat{N}^{(1)} + \hat{N}^{(1)} \cdot \nabla \vec{u} \right]^T \left[ \hat{N}^{(2)} + \hat{N}^{(2)} \cdot \nabla \vec{u} \right]$$

$$= \left\{ \left[\hat{N}^{(1)}\right]^T + \left( \left[\nabla \vec{u}\right] \left[\hat{N}^{(1)}\right] \right)^T \right\} \left\{ \left[\hat{N}^{(2)}\right] + \left[\nabla \vec{u}\right] \left[\hat{N}^{(2)}\right] \right\}$$

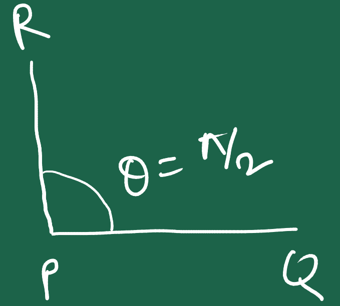
$$= \left[\hat{N}^{(1)}\right]^T \left[\hat{N}^{(2)}\right] + \left[\hat{N}^{(1)}\right]^T \left[\nabla \vec{u}\right] \left[\hat{N}^{(2)}\right] + \left[\hat{N}^{(1)}\right]^T \left[\nabla \vec{u}\right]^T \left[\hat{N}^{(2)}\right] + \text{h.o.t}$$

$$\cos\theta' \approx \cos\theta + \left[\hat{N}^{(1)}\right]^T \left( \left[\nabla \vec{u}\right] + \left[\nabla \vec{u}\right]^T \right) \left[\hat{N}^{(2)}\right]$$

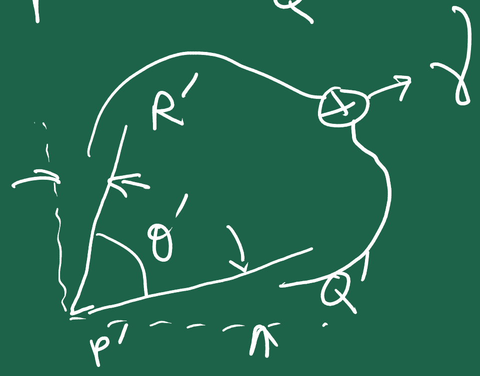
$$\cos\theta' \approx \cos\theta + [\hat{N}^{(1)}]^T\left([\nabla\vec{u}] + [\nabla\vec{u}]^T\right)[\hat{N}^{(2)}]$$

$$\Rightarrow \cos\theta' = \cos\theta + 2[\hat{N}^{(1)}]^T[\underset{\sim}{\varepsilon}][\hat{N}^{(2)}]$$

When $\theta = \pi/2$




$$\cos\left(\frac{\pi}{2} - \gamma\right) = 0 + 2[\hat{N}^{(1)}]^T[\underset{\sim}{\varepsilon}][\hat{N}^{(2)}]$$

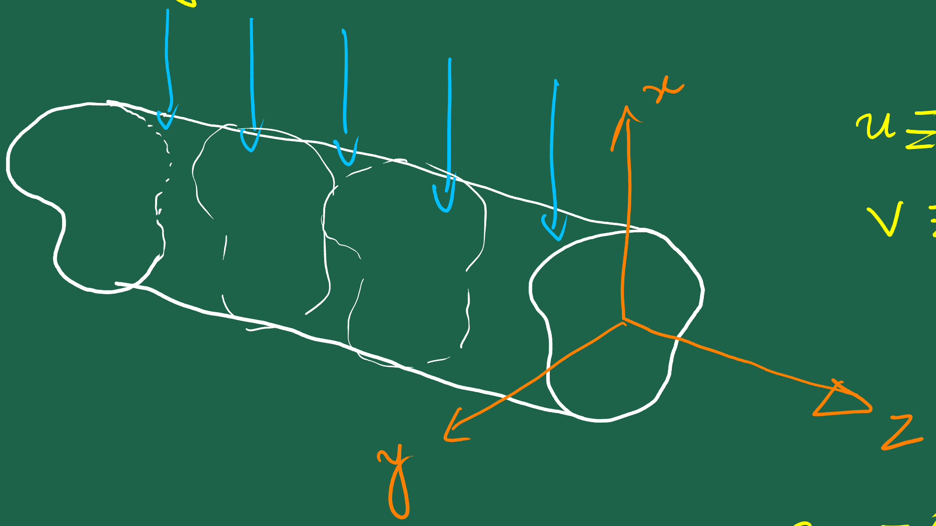



$$\Rightarrow \sin\gamma \simeq \boxed{\gamma = 2[\hat{N}^{(1)}]^T[\underset{\sim}{\varepsilon}][\hat{N}^{(2)}]}$$

$$\theta' = \frac{\pi}{2} - \gamma$$

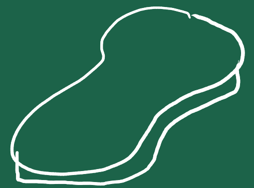
$$\boxed{\varepsilon_N = [\hat{N}]^T[\underset{\sim}{\varepsilon}][\hat{N}]}$$

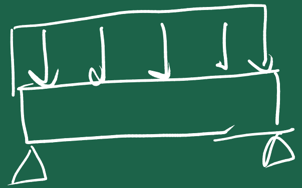
Simplifications under the consideration of PLANE STRAIN

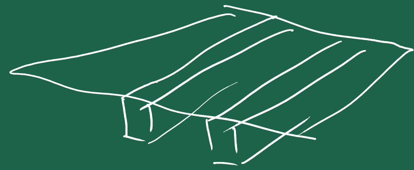



$u = u(x, y)$

$v = v(x, y)$



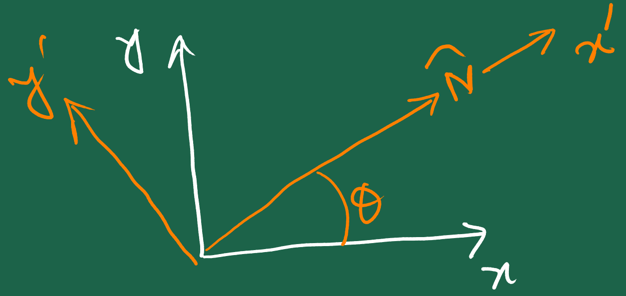




PLANE STRAIN : $\varepsilon_{zz} = 0$ , $\varepsilon_{zx} = 0$ , $\varepsilon_{yz} = 0$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_N = [\hat{N}]^T[\varepsilon][\hat{N}]$$




$$[\hat{N}] = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\cancel{\varepsilon_N} = \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

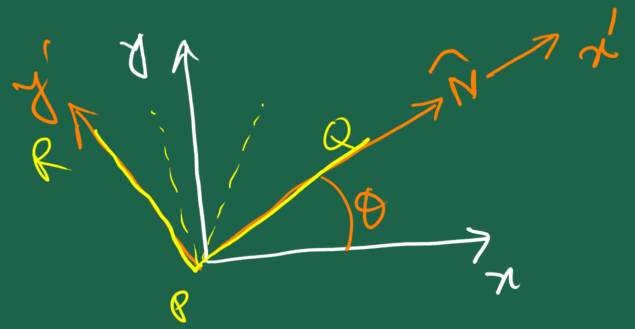
$$\frac{d\varepsilon_{x'x'}}{d\theta}$$

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 2\theta + \varepsilon_{xy}\sin 2\theta$$

$\theta \rightarrow \theta + 90^\circ$

$$\varepsilon_{y'y'} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} - \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 2\theta - \varepsilon_{xy}\sin 2\theta$$

$$\gamma = 2\left[\hat{N}^{(1)}\right]\left[\underset{\sim}{\varepsilon}\right]\left[\hat{N}^{(2)}\right]$$




$$\left[\hat{N}^{(1)}\right] = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\left[\hat{N}^{(2)}\right] = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\gamma_{x'y'} = 2\begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$= -\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)\sin(2\theta) + \varepsilon_{xy}\cos(2\theta)$$

$$\left[\gamma_{xy} = 2\,\varepsilon_{xy}\right]$$

Principal strains

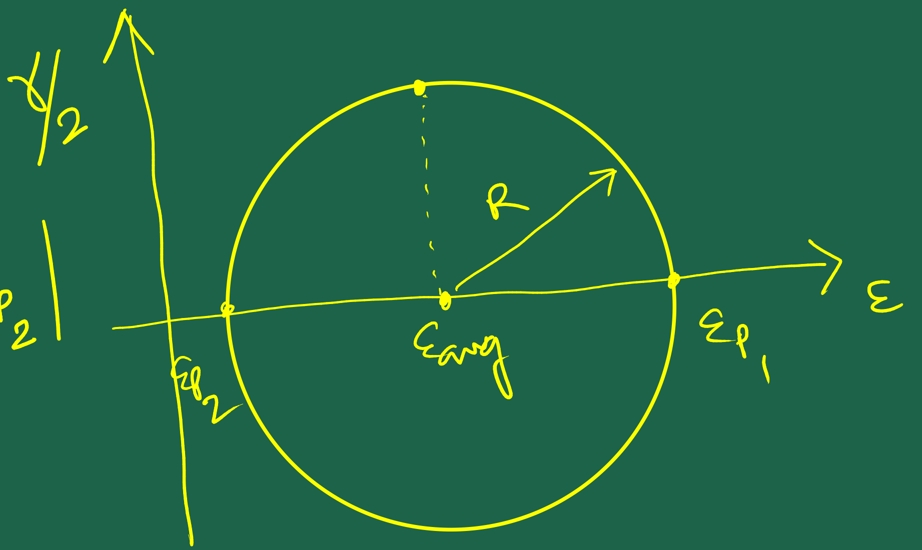
$$\varepsilon_{p_1,p_2} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2}$$

( ← Shamelessly lifted this from principal stress formulae )

$\gamma = 2\varepsilon$

$\gamma_{max} = 2R = |\varepsilon_{p_1} - \varepsilon_{p_2}|$

↳ in-plane



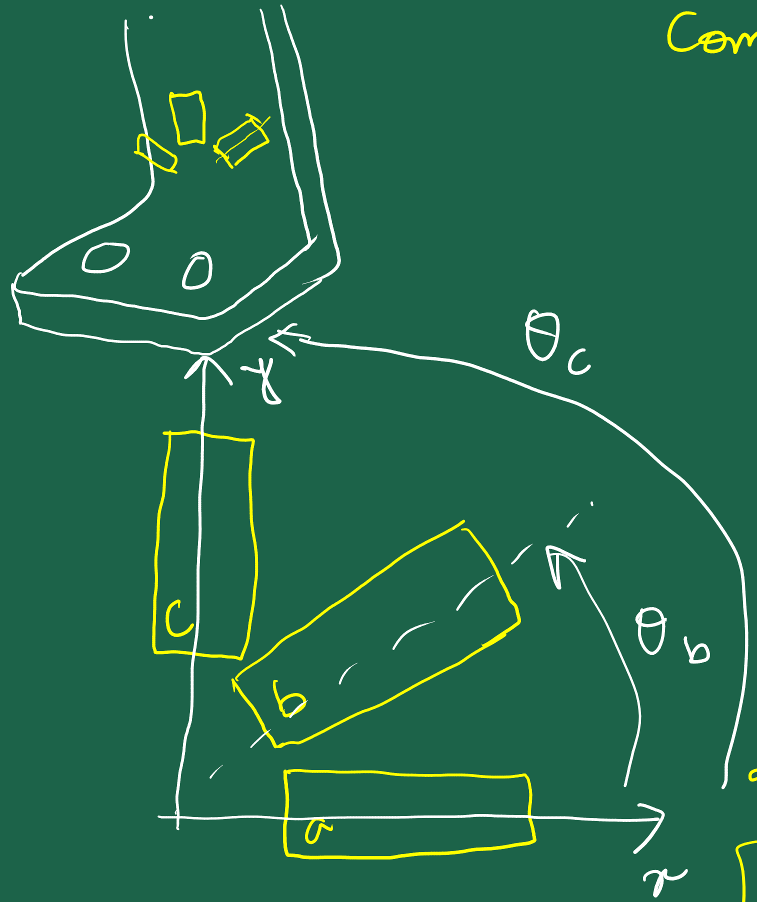

In 3D

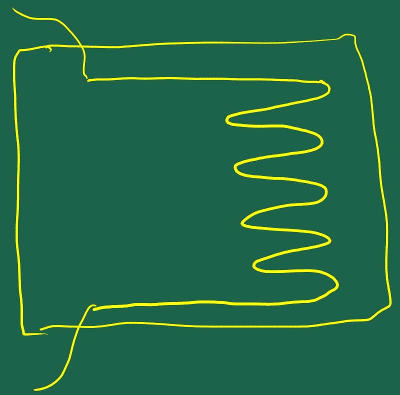
$$\gamma_{max,abs} = \max\left(|\varepsilon_{p_1}-\varepsilon_{p_2}|,\ |\varepsilon_{p_2}-\varepsilon_{p_3}|,\ |\varepsilon_{p_3}-\varepsilon_{p_1}|\right)$$

$$\varepsilon_{avg} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}, \quad R = \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2}$$

# STRAIN ROSETTE

Combination of strain gages



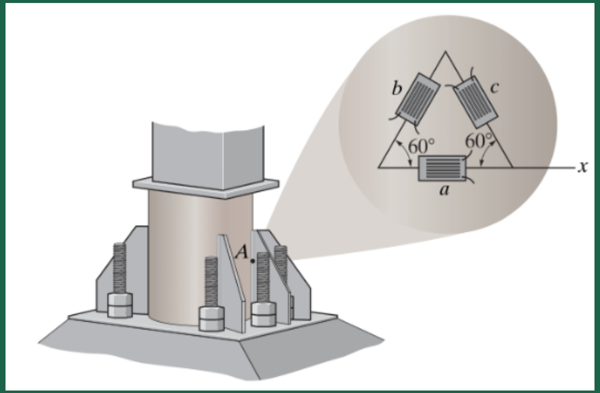
$$R = \frac{\rho l}{A}$$

$$\begin{cases} \varepsilon_a = \varepsilon_{xx}\cos^2\theta_a + \varepsilon_{yy}\sin^2\theta_a + 2\varepsilon_{xy}\sin\theta_a\cos\theta_a \\ \varepsilon_b = \varepsilon_{xx}\cos^2\theta_b + \varepsilon_{yy}\sin^2\theta_b + 2\varepsilon_{xy}\sin\theta_b\cos\theta_b \\ \varepsilon_c = \varepsilon_{xx}\cos^2\theta_c + \varepsilon_{yy}\sin^2\theta_c + 2\varepsilon_{xy}\sin\theta_c\cos\theta_c \end{cases}$$

→ Linear simultaneous eqns with unknowns $\varepsilon_{xx}$, $\varepsilon_{yy}$ & $\varepsilon_{xy}$

4. The strain rosette is attached to point A on the surface of the support. The readings from the strain gauges are: $\varepsilon_a = 300\mu$, $\varepsilon_b = -150\mu$, and $\varepsilon_c = -450\mu$. Determine (a) the in-plane principal strains, and (b) the maximum in-plane shear strain and (c) the average normal strain associated with the maximum in-plane shear strain. Specify the orientation of each element that has these states of strain with respect to the $x$-axis.*

[ (a) $336\mu$, $11.7°$; $-536\mu$, $101.7°$, (b) $872\mu$, $-33.3°$ (c) $-100\mu$ ]




$336 \times 10^{-6}$

$\theta_a = 0°, \quad \theta_b = 60°, \quad \theta_c = 120°$

$\rightarrow \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}$ ✓

(c) $\dfrac{\varepsilon_{xx} + \varepsilon_{yy}}{2}$ or $\dfrac{\varepsilon_{p_1} + \varepsilon_{p_2}}{2}$

??

We will come back to it !

$\dfrac{\sigma_{xx} + \sigma_{yy}}{2} = \dfrac{\sigma_{x'x'} + \sigma_{y'y'}}{2} = \dfrac{\sigma_{p_1} + \sigma_{p_2}}{2}$

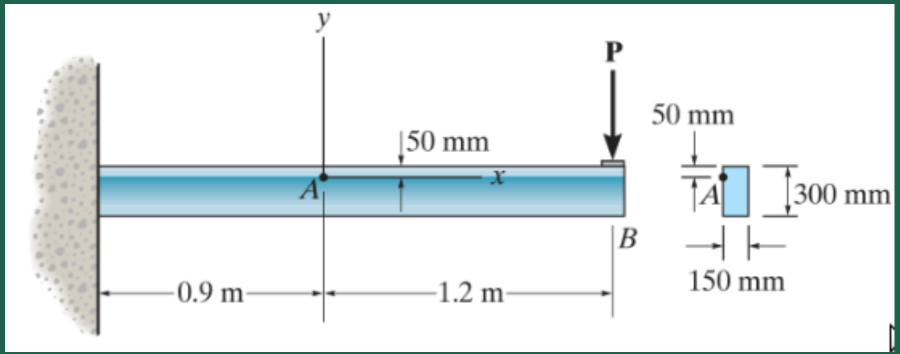
$\tan 2\theta_p = \dfrac{2\varepsilon_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$

$\theta_{p_1} = 11.705°, \quad \theta_{p_2} = \theta_{p_1} + 90°$

$\tan 2\theta_s = -\dfrac{\varepsilon_{xx} - \varepsilon_{yy}}{2\varepsilon_{xy}}$

$\theta_s$ ✓

9. The strain the $x$-direction at point A on the structural steel beam ($E = 203$ GPa and $G = 76$ GPa) is measured and found to be $\varepsilon_{xx} = 100\mu$. Determine the applied load $P$. What is the shear strain $\gamma_{xy}$ at point A? [$P = 57$ kN; $\gamma_{xy} = -13.91\mu$ ]




$\varepsilon_{xx}$ ✓

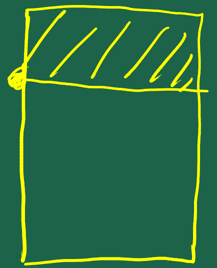
P → Bending Moment → Flexural stress (or Bending stress)

$$\sigma = \frac{My}{I} \qquad M \leftarrow P$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\cancel{\sigma_{yy}} + \cancel{\sigma_{zz}})\right]$$

$$\sigma_{xx} = E\,\varepsilon_{xx}$$

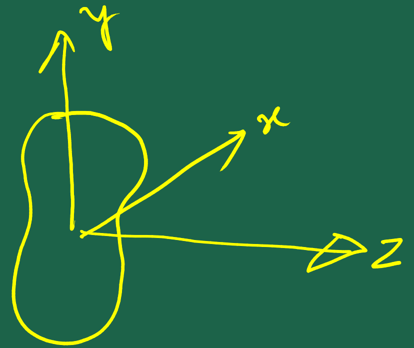
$$P() \leftarrow \frac{My}{I} = E\,\varepsilon_{xx}$$



$$\tau_{xy} = \frac{\cancel{V}^{\to P}Q}{It}$$

$$\gamma_{xy} = 2\varepsilon_{xy} = \cancel{2}\,\frac{\tau_{xy}}{\cancel{2}G}$$

8. Deduce that in the case of plane strain ($xy$-plane) for a body made of a material that follows the generalized Hooke's law, the stress component $\sigma_{zz}$ itself is a principal stress.




$$\varepsilon_{zz} = 0, \quad \varepsilon_{yz} = 0, \quad \varepsilon_{zx} = 0$$

$$\tau_{yz} = 2G\,\varepsilon_{yz} = 0$$

$$\tau_{zx} = 2G\,\varepsilon_{zx} = 0$$

$\sigma_{zz}$ ?

$$\underbrace{\varepsilon_{zz}}_{0} = \frac{1}{E}\left[\sigma_{zz} - \nu(\underbrace{\sigma_{xx}}_{\neq 0} + \underbrace{\sigma_{yy}}_{\neq 0})\right]$$

$$\Rightarrow \sigma_{zz} \neq 0$$

6. For a material that behaves according to the generalized Hooke's law:

(a) Considering the case of plane stress ($xy$-plane), derive the strain transformation equations from the stress transformation equations.

(b) How does the strain component $\varepsilon_{zz}$ transform in part (a)?

(c) Considering the case of plane strain ($xy$-plane), derive the stress transformation equations from the strain transformation equations.

(d) How does the stress component $\sigma_{zz}$ transform in part (c)?

(d) $\sigma_{zz} \neq 0$

Observe that $\sigma_{z'z'}$ will coincide with $\sigma_{zz}$

(a) Plane stress: $\sigma_{zz} = 0, \ \tau_{zx} = 0, \ \tau_{yz} = 0$

$$\sigma_{x'x'} = \sigma_{xx} \cos^2\theta + \sigma_{yy} \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \cancel{\sigma_{zz}})\right] \\ \varepsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \cancel{\sigma_{zz}})\right] \end{aligned}\right\} \quad \begin{aligned} \sigma_{xx} &= f_1(\varepsilon_{xx}, \varepsilon_{yy}) \\ \sigma_{yy} &= f_2(\varepsilon_{xx}, \varepsilon_{yy}) \end{aligned}$$

We'll obtain: $\varepsilon_{x'x'} = \varepsilon_{xx} \cos^2\theta + \varepsilon_{yy} \sin^2\theta + 2\varepsilon_{xy} \sin\theta \cos\theta$

(b) $$\varepsilon_{zz} = \frac{1}{E}\left[\cancel{\sigma_{zz}} - \nu(\sigma_{xx} + \sigma_{yy})\right] \neq 0$$

Observe that $\varepsilon_{z'z'}$ is the same as $\varepsilon_{zz}$.