Introduction[*]

1 Solids and Fluids

Almost all of us have an intuitive understanding about what a solid is and how it differs from a fluid. Let us try to put this understanding on a scientific footing. First, consider the question, *"What is a fluid?"* A fluid is defined as:

> *Fluid:* A substance that deforms continuously under the application of shear stresses *no matter how small the shear stresses may be.*

The italicized part of the above definition is the key to demarcating fluids from solids. Thus a solid can be defined as:

> *Solid:* A substance that can support some amount of shear stresses before it begins to deform continuously.

Furthermore, while solids can support both tensile and compressive stresses, fluids can support only compressive stresses. However, there is often no clear-cut distinction between fluids and solids. There are many materials which can best be described as something between fluids and solids; for instance, toothpaste, jelly, putty. In this course, however, we will primarily be concerned with materials that do not exhibit any fluid-like behaviour.

Fluid Mechanics is concerned with the velocity, forces, and stresses in fluid flows. Similarly, Mechanics of Solids is concerned with displacements, forces, stresses, and various limiting conditions associated with the stresses within solids, in general, as well as within solids that constitute various structures of engineering importance.

The course, Mechanics of Solids, will directly build on what you learnt in the second half of first year Mechanics.

[*]Notes prepared by Jeevanjyoti Chakraborty. Contact: jeevan@mech.iitkgp.ac.in

Again, just like in Fluid Mechanics, we will be working within the purview of the Continuum Hypothesis in Mechanics of Solids too.

2 Continuum hypothesis

Matter (including fluids) is composed of a large number of molecules in constant motion and undergoing collisions with each other. Matter is, therefore, discontinuous or discrete at microscopic scales. It is possible – at least in principle – to study the mechanics of fluids and solids by studying the motion of molecules themselves. This is what is done in kinetic theory[†] or statistical mechanics. Often recourse is taken to computer simulations referred to as molecular dynamics simulations. This approach basically involves solving Newton's second law for each molecule for an agglomeration of molecules. But limitations of computational resources restrict the agglomeration to be not larger than a billion or so molecules (even that requires very good computational facilities). Note that since the Avogadro number is 6.02×10^{23}, even a billion molecules is actually not much!

However, more often than not, we are interested in the gross behaviour of fluids and solids, i.e. we want to see what the combined molecular motion manifests in, in an average sense. Thus, we can ignore the discreteness due to the presence of molecules at the microscopic scale, and think of the fluid or solid as a continuous distribution called the continuum. The continuum hypothesis tells us this that this abstraction from the real molecular structure to a continuous structure should work all right for most scenarios as long as we are careful about the length scales under consideration. Specifically, we have to keep an eye on the mean free path of molecules. For the continuum hypothesis to be valid, the size of the system must be larger than the mean free path of the molecules. Ordinarily, this is not a problem but if the system, for instance, is of the dimensions of 1 nm or so then the discrete molecular structure cannot be replaced by the continuum. Again in the upper altitudes of the atmosphere, where air is very rarefied, the mean free path can be quite large;

[†] You did a bit of kinetic theory at the senior secondary level; jut take a look back at H. C. Verma's book.

so there too, we have to take recourse to kinetic theory.

3 Why study mechanics of solids?

Mechanics of Solids has a mind-boggling array of applications. These have been summarized by one of the giants of solid mechanics, Professor James Robert Rice in the following[‡]

> Here is a sampling of some of the issues addressed using solid mechanics concepts: How do flows develop in the Earth's mantle and cause continents to move and ocean floors to subduct (i.e., be thrust) slowly beneath them? How do mountains form? What processes take place along a fault during an earthquake, and how do the resulting disturbances propagate through the Earth as seismic waves, shaking, and perhaps collapsing, buildings and bridges? How do landslides occur? How does a structure on a clay soil settle with time, and what is the maximum bearing pressure that the footing of a building can exert on a soil or rock foundation without rupturing it? What materials should be chosen, and how should their proportion, shape, and loading be controlled, to make safe, reliable, durable, and economical structures – whether airframes, bridges, ships, buildings, chairs, artificial heart valves, or computer chips – and to make machinery such as jet engines, pumps, and bicycles? How do vehicles (cars, planes, ships) respond by vibration to the irregularity of surfaces or mediums along which they move, and how are vibrations controlled for comfort, noise reduction, and safety against fatigue failure? How rapidly does a crack grow in a cyclically loaded structure, whether a bridge, engine, or airplane wing or fuselage, and when will it propagate catastrophically? How can the deformability of structures during impact be controlled so as to design crashworthiness into vehicles? How are the materials and products of a technological civilization formed – e.g., by extrud-

[‡]Taken from this Encyclopaedia Britannica article written by him.

> ing metals or polymers through dies, rolling material into sheets, punching out complex shapes, and so on? By what microscopic processes do plastic and creep strains occur in polycrystals? How can different materials, such as fibre-reinforced composites, be fashioned together to achieve combinations of stiffness and strength needed in specific applications? What is the combination of material properties and overall response needed in downhill skis or in a tennis racket? How does the human skull respond to impact in an accident? How do heart muscles control the pumping of blood in the human body, and what goes wrong when an aneurysm develops?

It is thus patently clear that so far as human beings need to interact with any real-world thing (whether natural or man-made) they can hardly do so *completely* without the application of mechanics of solids. Considering all fields of engineering, this is one of the most fundamental subjects which is why the basic principles were taught to all students in first year. As mechanical engineers, of course, you will need to know more than those basic principles.

And, this is the course where you begin to learn them.