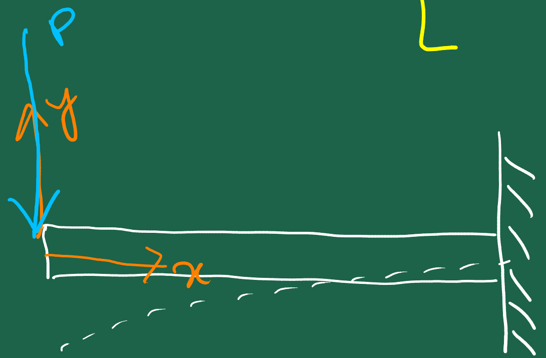
Deflection of Beams

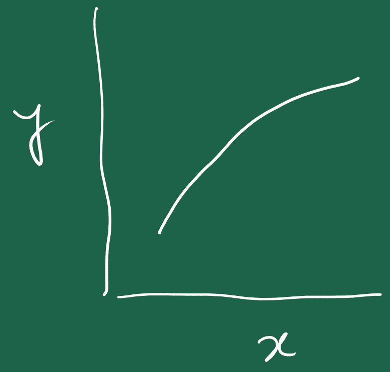
$$M = \frac{EI}{\rho}$$

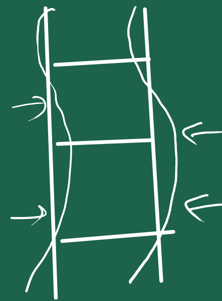
$$\frac{1}{\rho} = \kappa = \frac{\dfrac{d^2y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

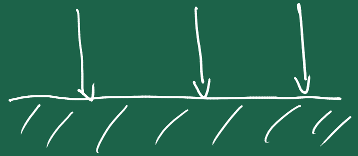


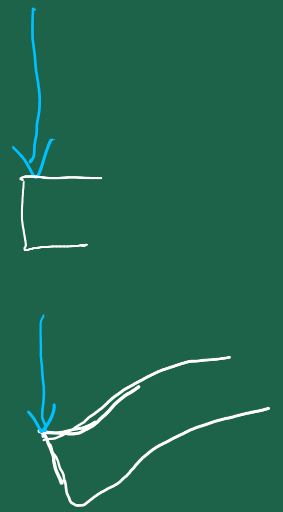
$$\left|\frac{dy}{dx}\right| << 1$$

$$\frac{1}{\rho} = \kappa \simeq \frac{d^2y}{dx^2}$$

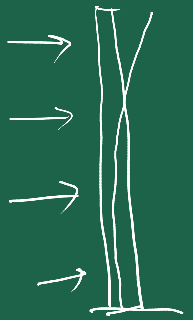








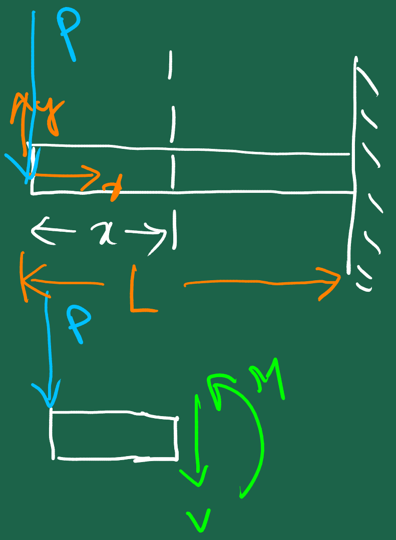
St. Venant's principle



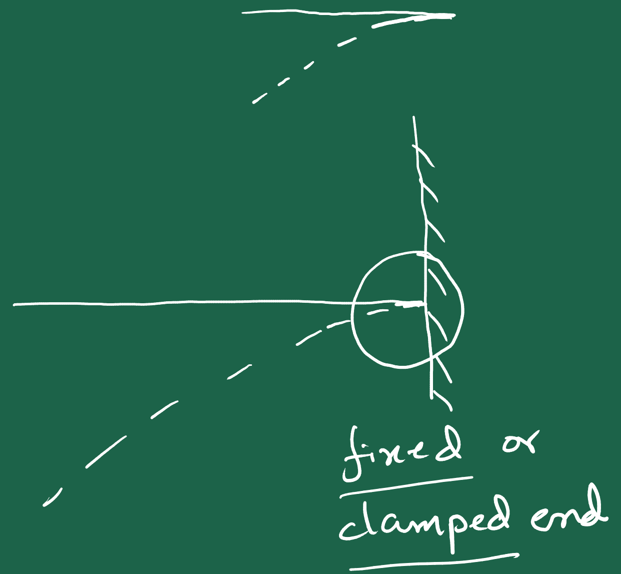
$$M = \frac{EI}{\rho}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = M$$

Ex)



$$M = -Px$$



EI : flexural rigidity

$$\therefore EI\frac{d^2y}{dx^2} = -Px$$

$$\Rightarrow EI\frac{dy}{dx} = -P\frac{x^2}{2} + C_1$$

$$\Rightarrow EI\,y = -P\frac{x^3}{6} + C_1 x + C_2$$

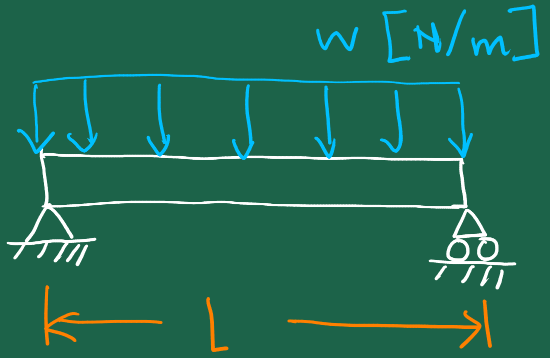
B.C.S

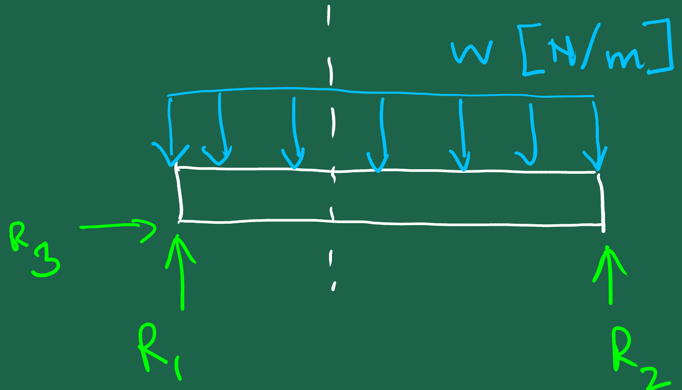
At $x = L$, $y = 0$

At $x = L$, $\frac{dy}{dx} = 0$

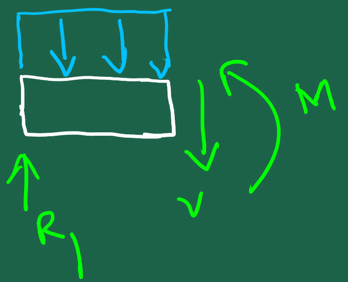
After solving $y\Big|_{x=0} = -\frac{PL^3}{3EI}$, $\frac{dy}{dx}\Big|_{x=0} = ??$

Ex 2





$$R_1 = R_2 = \frac{wL}{2}, \quad R_3 = 0$$



$$EI\frac{d^2y}{dx^2} = M$$

$$\Rightarrow EI\frac{d^3y}{dx^3} = \frac{dM}{dx} = V$$

$$\Rightarrow EI\frac{d^4y}{dx^4} = \frac{dV}{dx} = -w$$

$$\boxed{EI\frac{d^4y}{dx^4} = -w}$$

At $x=0$, $y=0$

At $x=L$, $y=0$

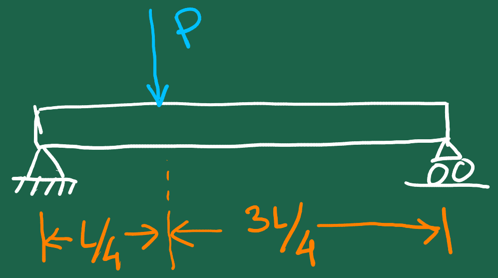
At $x=0$, $M=0 \Rightarrow EI\left.\frac{d^2y}{dx^2}\right|_{x=0} = 0$

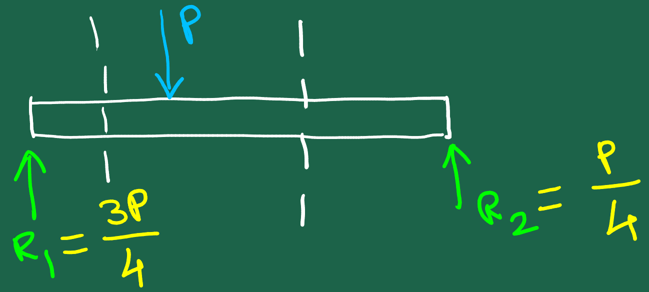
At $x=L$, $M=0 \Rightarrow EI\left.\frac{d^2y}{dx^2}\right|_{x=L} = 0$

$$y = \frac{w}{24EI} \times \left(-x^4 + 2Lx^3 - L^3x\right)$$

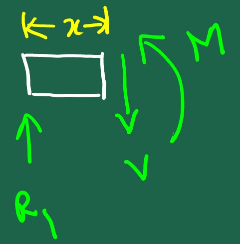
✓

Ex 3





For $0 < x < L/4$

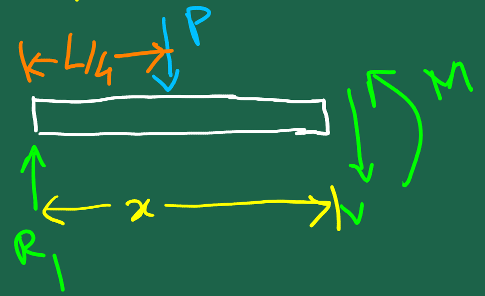


$$M - R_1 x = 0 \Rightarrow M = \frac{3P}{4} x$$

$$\boxed{EI \frac{d^2y}{dx^2} = \frac{3Px}{4}}$$

$\rightarrow C_1, C_2$

For $\frac{L}{4} \; x < L$



$$M - R_1 x + P\left(x - L/4\right) = 0$$

$$\Rightarrow M = -\frac{P}{4}(x - L)$$

$$\boxed{EI \frac{d^2y}{dx^2} = -\frac{P}{4}(x - L)}$$

$\rightarrow C_3, C_4$

To find C_1, C_2, C_3, C_4 :

@ $x=0, \quad y=0$

@ $x=L, \quad y=0$

@ $x=0, \quad M=0$

@ $x=L, \quad M=0$

They do not help us in solving for C_1, C_2, C_3, C_4

Continuity of deflection and continuity of slope must be maintained between the 2 regions

$$y\Big|_{x=L/4^-} = y\Big|_{x=L/4^+}$$

$$\frac{dy}{dx}\Big|_{x=L/4^-} = \frac{dy}{dx}\Big|_{x=L/4^+}$$

Extremely important!

$$\frac{d^2 f}{dx^2} = g(x)$$

$$\frac{d^n f}{dx^n}$$

B.C.s $\leq (n-1)$

$$\frac{d^2 f}{dx^2} = 0$$

$$f = C_1 + C_2 x$$

$x=0, \ f = \checkmark$

$$\frac{df}{dx}\Big|_{x=L} = 0 \ \checkmark$$

$$\frac{d^2 f}{dx^2}\Big|_{x=L} = \checkmark$$

Various kinds of BCs:

Clamped or Fixed End

$$y = 0$$

$$\frac{dy}{dx} = 0$$

Pinned End

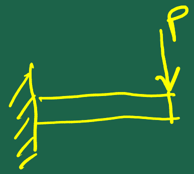
$$y = 0$$

$$M = 0 \Rightarrow EI\frac{d^2y}{dx^2} = 0$$

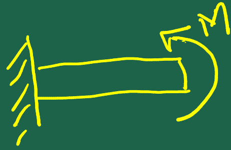
Free End

$$M = 0 \Rightarrow EI\frac{d^2y}{dx^2} = 0$$

$$V = 0 \Rightarrow EI\frac{d^3y}{dx^3} = 0$$

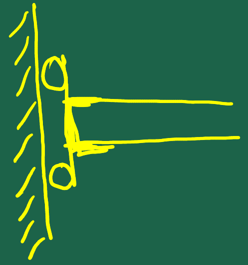


(Not free end)



(Not free end)

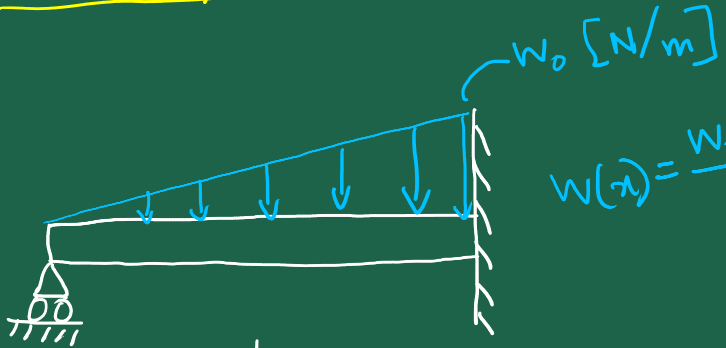
Guided support



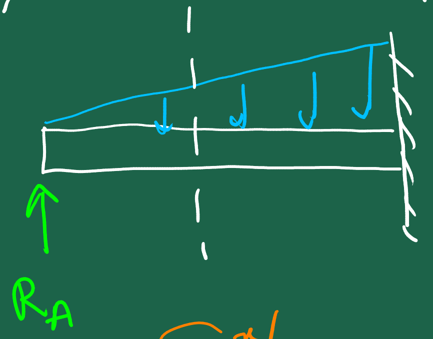
$$V = 0 \Rightarrow EI\frac{d^3y}{dx^3} = 0$$

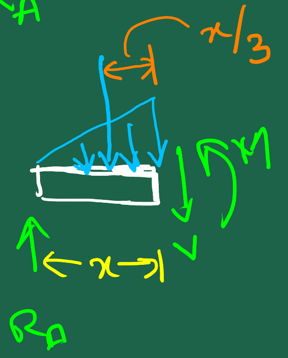
$$\frac{dy}{dx} = 0$$

Statically Indeterminate Cases



$$w(x) = \frac{w_0 x}{L}$$





$$M - R_A x + \frac{x}{3} \times \frac{w_0 x}{2} = 0$$

$$\Rightarrow M = R_A x - \frac{w_0 x^2}{6}$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x - \frac{w_0 x^2}{6}$$

$$y \rightarrow C_1, C_2 \ \& \ R_A$$

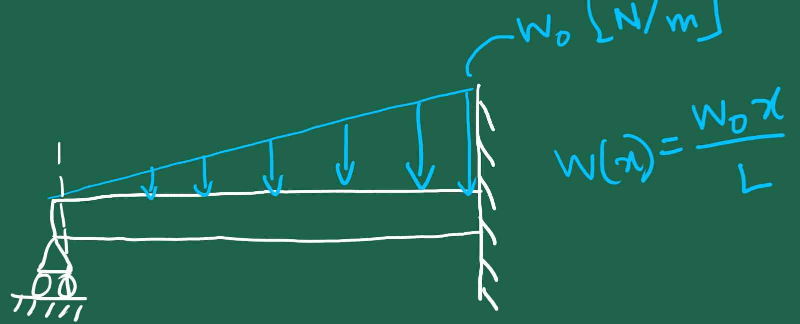
@ $x = 0$, $y = 0$, $\cancel{M = 0}$

@ $x = L$, $y = 0$

@ $x = L$, $\frac{dy}{dx} = 0$

$$EI \frac{d^4 y}{dx^4} = -w(x) = -\frac{w_0 x}{L}$$

$$EI\frac{d^4y}{dx^4} = -\frac{w_0 x}{L}$$

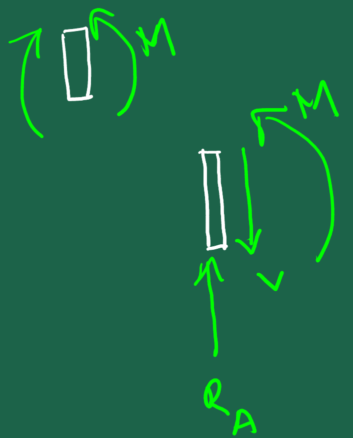


$$\Rightarrow EI\frac{d^3y}{dx^3} = -\frac{w_0 x^2}{2L} + C_1$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -\frac{w_0 x^3}{6L} + C_1 x + C_2$$

$$\Rightarrow EI\frac{dy}{dx} = -\frac{w_0 x^4}{24L} + C_1\frac{x^2}{2} + C_2 x + C_3$$

$$\Rightarrow EI\,y = -\frac{w_0 x^5}{120L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$



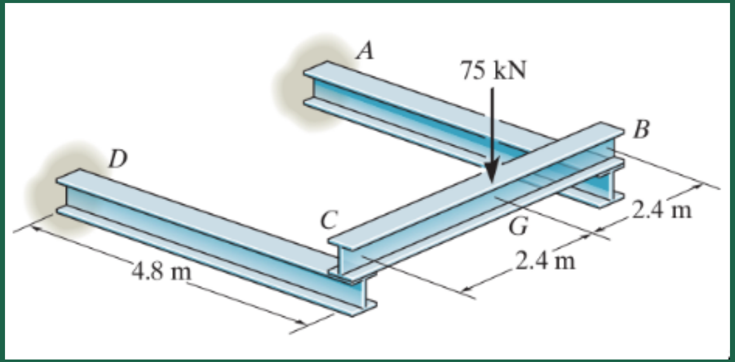
$$\sum F_y = 0$$

$$\Rightarrow V = R_A$$

$$\Rightarrow R_A = EI\left.\frac{d^3y}{dx^3}\right|_{x=0}$$

$$\left.\begin{aligned}
&@\ x=0,\ y=0\\
&@\ x=0,\ M=0 \Rightarrow EI\frac{d^2y}{dx^2}=0\\
&@\ x=L,\ y=0\\
&@\ x=L,\ \frac{dy}{dx}=0
\end{aligned}\right\}\quad C_1, C_2, C_3, C_4$$

5. The framework consists of two steel cantilevered beams CD and BA and a simply supported beam CB. If each beam has a Young's modulus of 200 GPa and a moment of inertia about its neutral axis of 46×10^6 mm^4, determine the deflection at the centre G of beam CB. [−169 mm]

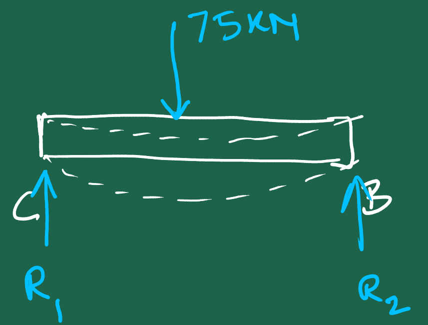


Method of superposition

$$\Delta'_c = \frac{PL^3}{3EI} = \frac{R_1 (4.8m)^3}{3EI}$$

$$\Delta'_G = \frac{PL^3}{48EI} = \frac{(75kN)(4.8m)^3}{48EI}$$

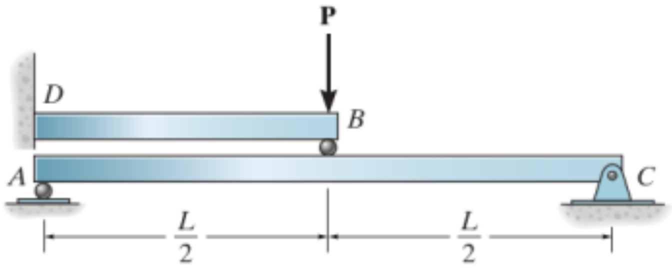
$$\Delta_G = \Delta'_G + \Delta'_c$$

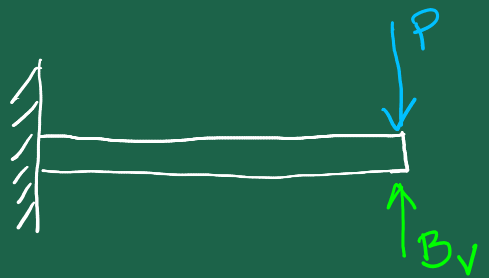


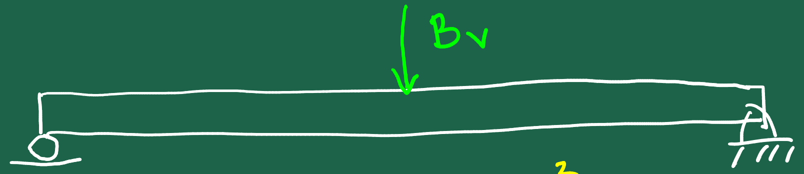
$R_1 = R_2 = 37.5$ kN

8. Determine the vertical reaction at support C in the beam arrangement shown.

[$P/3$]





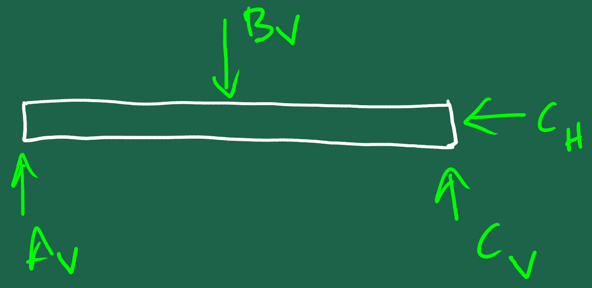


$$\Delta^B_{BD} = \frac{(P-B_V)\left(L/2\right)^3}{3EI}$$

$$\Delta^B_{AC} = \frac{B_V L^3}{48EI}$$

$$\Delta^B_{BD} = \Delta^B_{AC}$$

$$\Rightarrow B_V$$

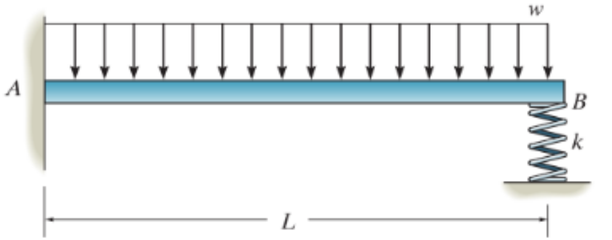


$$\sum M_A = 0$$

$$\Rightarrow C_V$$

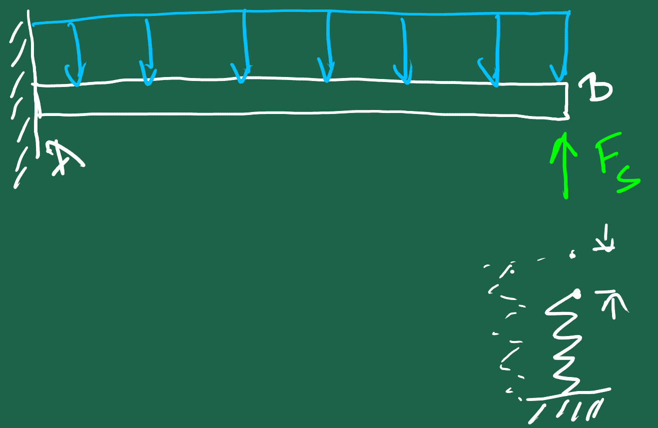
10. Determine the force in the spring.

$$\left[\frac{3kwL^4}{24EI + 8kL^3}\right]$$



$$(\downarrow)\ \Delta^{B}_{BA} = \frac{wL^4}{8EI} - \frac{F_s L^3}{3EI}$$

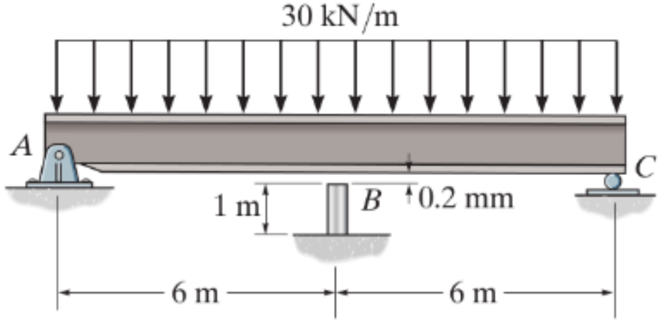
$$\Delta^{B}_{spring} = \frac{F_s}{K}$$



$$\Delta^{B}_{BA} = \Delta^{B}_{spring}$$

$$\Rightarrow F_s$$

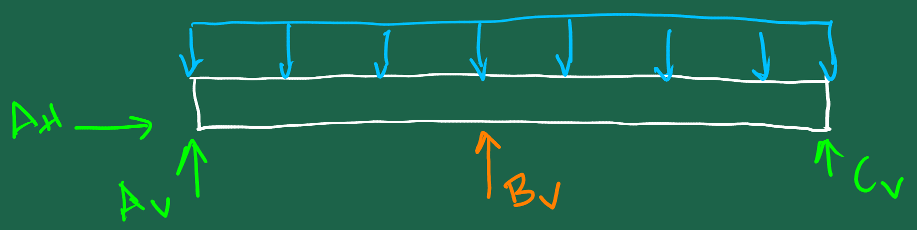
9. Before the uniformly distributed load is applied on the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is 875×10^6 mm^4. Both the post and the beam are made of steel having modulus of elasticity 200 GPa. [70.11 kN, 219.78 kN, 70.11 kN]

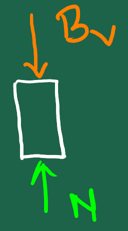


Check $\dfrac{5wL^4}{384EI} > 0.2$ mm ?

$\hookrightarrow$ 2.89 mm

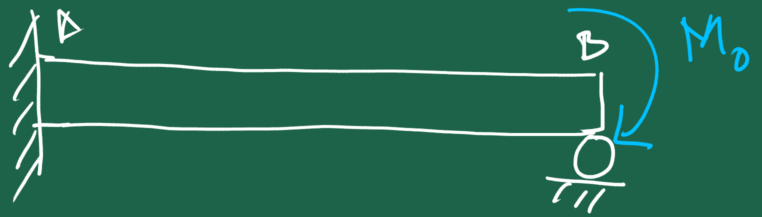
$\dfrac{PL}{AE}$

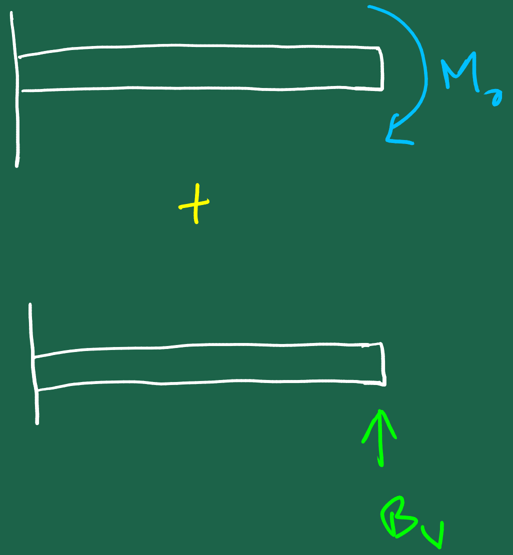




$$(\downarrow)\ \Delta^{\text{mid-point}}_{AC} = \frac{5wL^4}{384EI} - \frac{B_v L^3}{48EI} = \underbrace{(\text{Gap-height})}_{=0.2\text{ mm}} + \frac{B_v(1\text{m})}{\left(\frac{\pi d^2}{4}\right)(200\text{GPa})}$$

$\Rightarrow B_v$ ✓ $\rightarrow A_v, C_v$ ✓





$$\frac{M_0 L^2}{2EI}$$

$$\frac{B_V L^3}{3EI}$$

$$EI\frac{d^2y}{dx^2} = M = f(x)$$

$\hookrightarrow$ Linear eqn $\rightarrow$ Add solutions

$$\frac{d^2y}{dx^2} = 3$$

$$\frac{d^2y}{dx^2} = 4x$$

$$\frac{d^2y}{dx^2} = 3+4x$$

$$\frac{d^3y}{dx^2}$$

$$\frac{d^4y}{dx^4}$$