

Algorithm Analysis

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Analysis of Algorithms

- How much resource is required ?
- Measures for efficiency
 - Execution time → time complexity
 - Memory space → space complexity
- Observation :
 - The larger amount of input data an algorithm has, the larger amount of resource it requires.
 - Complexities are functions of the amount of input data (input size).

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What do we use for a yardstick?

- The same algorithm will run at different speeds and will require different amounts of space.
 - When run on different computers, different programming languages, different compilers.
- But algorithms usually consume resources in some fashion that depends on the size of the problem they solve.
 - Some parameter n (for example, number of elements to sort).

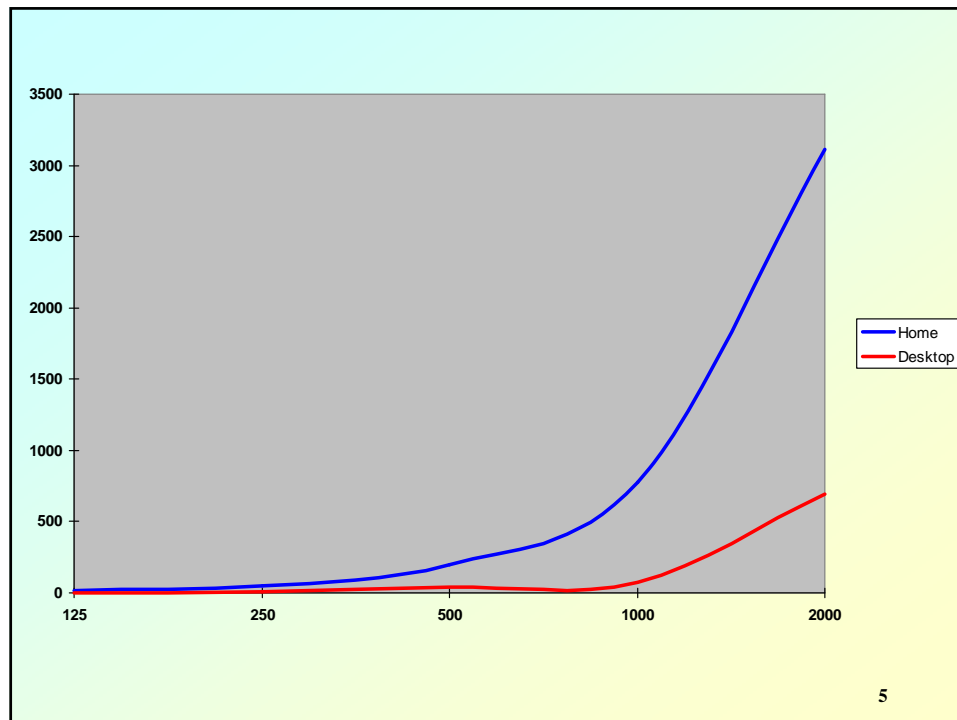
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An example of a sorting algorithm

- We run this sorting algorithm on two different computers, and note the time (in milliseconds) for different sizes of input.

Array Size n	Home Computer	Desktop Computer
125	12.5	2.8
250	49.3	11.0
500	195.8	43.4
1000	780.3	72.9
2000	3114.9	690.5

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Contd.

- Home Computer :

$$f_1(n) = 0.0007772 n^2 + 0.00305 n + 0.001$$

- Desktop Computer :

$$f_2(n) = 0.0001724 n^2 + 0.00040 n + 0.100$$

- Both are quadratic function of n .
- The shape of the curve that expresses the running time as a function of the problem size stays the same.

Complexity classes

- The running time for different algorithms fall into different **complexity classes**.
 - Each complexity class is characterized by a different family of curves.
 - All curves in a given complexity class share the same basic shape.
- The ***O-notation*** is used for talking about the complexity classes of algorithms.

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Running time of algorithms

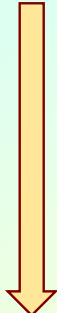
Assume speed is 10^7 instructions per second.

size n	10	20	30	50	100	1000	10000
n	.001 ms	.002 ms	.003 ms	.005 ms	.01 ms	.1 ms	1 ms
$n \log n$.003 ms	.008 ms	.015 ms	.03 ms	.07 ms	1 ms	13 ms
n^2	.01 ms	.04 ms	.09 ms	.25 ms	1 ms	100 ms	10 s
n^3	.1 ms	.8 ms	2.7 ms	12.5 ms	100 ms	100 s	28 h
2^n	.1 ms	.1 s	100 s	3 y	$3 \times 10^{13} \text{c}$	inf	inf

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- The complexity classes:

$\log_2 n$
 n
 $n \log_2 n$
 n^2
 n^3
 2^n
 $n!$



Complexity increases

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Introducing the language of O-notation

- Definition:

$f(n) = O(g(n))$ if there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n)$ when $n \geq n_0$.

- The big-Oh notation is used to categorize the complexity class of algorithms.
 - It gives an upper bound.
 - Other measures also exist, like small-Oh, Omega, Theta, etc.

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Examples

- $f(n) = 2n^2 + 4n + 5$ is $O(n^2)$.
 - One possibility: $c=11$, and $n_0=1$.
- $f(n) = 2n^2 + 4n + 5$ is also $O(n^3), O(n^4), \text{etc.}$
 - One possibility: $c=11$, and $n_0=1$.
- $f(n) = n(n-1)/2$ is $O(n^2)$.
 - One possibility: $c=1/2$, and $n_0=1$.
- $f(n) = 5n^4 + \log_2 n$ is $O(n^4)$.
 - One possibility: $c=6$, and $n_0=1$.
- $f(n) = 75$ is $O(1)$.
 - One possibility: $c=75$, and $n_0=1$.

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Complexities of Known Algorithms

Algorithm	Best-case	Average-case	Worst-case
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Bubble sort	$O(n)$	$O(n^2)$	$O(n^2)$
Quick sort	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n^2)$
Merge sort	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n \log_2 n)$
Linear search	$O(1)$	$O(n)$	$O(n)$
Binary search	$O(1)$	$O(\log_2 n)$	$O(\log_2 n)$

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Observations

- **There is a big difference between polynomial time complexity and exponential time complexity.**
- **Hardware advances affect only efficient algorithms and do not help inefficient algorithms.**

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