Indian Institute of Technology Kharagpur
Mid-Autumn Semester Examination 2017-18

Date of Examination: **22 Sept. 2017** **Session:** **AN** **Duration:** **2 Hours**
Subject No.: **EE41013, EE60033** **Subject:** **Digital Signal Processing**
Department/Center/School: **Electrical Engineering** **Credits:** **4** **Full marks:** **70**

Instructions

1. This question paper contains 4 pages and 7 questions. All questions are compulsory. Marks are indicated in parentheses. This question paper has been cross checked and no errors exist.
2. Detach the pages 3-4, fill the answers on them and attach with the answer script.
3. Please write your name, roll number, subject name and code, date and time of examination on the answer script before attempting any solution.
4. Use of electronic calculators only is permitted. No extra resources viz. graph papers, log-tables, trigonometric tables would be required.
5. **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page or across the answer script without a clear ordering will receive very little marks.
6. **Mysterious or unsupported answers will not receive full marks**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no marks; an incorrect answer supported by substantially correct calculations and explanations might still receive partial marks.

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1. (a) (5 points) Given that $y[n] = x_1[n] \circledast x_2[n]$ and $v[n] = x_1[n-N_1] \circledast x_2[n-N_2]$, express $v[n]$ in terms of $y[n]$.

   (b) (5 points) Prove that $\|\{x[n]\}\|_2 \leq \|\{x[n]\}\|_1$

2. (a) (5 points) Derive the convolution sum of the sequences $x[n] = \alpha^{-n}\mu[n]$ and $h[n] = \beta^n \mu[n]$ using the discrete time Fourier transferm (DTFT) based approach.

   (b) (2 points) What should be the conditions to be imposed on α and β for the solution to hold true and state its reason?

   (c) (3 points) Compute the energy of the length-N sequence $x[n] = \cos(2\pi kn/N)$, $0 \leq n \leq N-1$.

3. (a) (6 points) Let $X\left(e^{j\omega}\right)$ be the DTFT of the absolutely summable real sequence $x[n]$ such that $X\left(e^{j\omega}\right) = X_{re}\left(e^{j\omega}\right) + jX_{im}\left(e^{j\omega}\right)$, then prove that

$$X_{re}\left(e^{j\omega}\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_{im}\left(e^{j\nu}\right)\cot\left(\frac{\omega-\nu}{2}\right)d\nu + x[0] \quad (1)$$

$$X_{im}\left(e^{j\omega}\right) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} X_{re}\left(e^{j\nu}\right)\cot\left(\frac{\omega-\nu}{2}\right)d\nu \quad (2)$$

(b) (4 points) Given that $y[n] = h[n] \circledast x[n]$, then algebraically prove that

$$\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} h[n]\right)\left(\sum_{n=-\infty}^{\infty} x[n]\right) \quad (3)$$

4. (a) (6 points) The sequence of Fibonacci numbers is defined as a causal sequence $f[n] = f[n-1] + f[n-2], n \geq 2$ with $f[0] = 0,\ f[1] = 1$. Prove that if $h[n]$ is the impulse response of the causal LTI system described as $y[n] = y[n-1] + y[n-2] + x[n-1]$, then $h[n] = f[n]$.

(b) (2 points) Let the sequence $y_1[n] = median\{x[n], x[n-1], \cdots, x[n-N+1]\}$. You are provided with the histogram of $\{x[n], \cdots, x[n-N+1]\}$ as $h[k] = \{7, 3, 4, 6\}$ where $x \in \mathbb{Z}^+ \cap [0, 3]$ and $k = \{0, 1, 2, 3\}$. Compute $y_1[n]$?

(c) (2 points) Let a new sequence be $y_2[n] = mean\{x[n], x[n-1], \cdots, x[n-N+1]\}$, then using information provided above compute $y_2[19]$ and $y_2[20]$ given that $x[0] = 0$ and $x[20] = 3$?

5. (a) (6 points) Derive the matrix form for computing the $N-$point discrete Fourier transform (DFT) of an $N-$pioint sequence $x[n]$. Clearly mention the size of each matrix and the value of elements within the matrix.

(b) (2 points) Prove that if $X[k]$ represents the DFT of a length$-N$ complex valued sequence $x[n]$, then $x^*[n] \leftrightarrow X^*[\langle -k\rangle_N]$

(c) (2 points) Prove that if $X[k]$ represents the DFT of a length$-N$ complex valued sequence $x[n]$, then $x^*[\langle -n\rangle_N] \leftrightarrow X^*[k]$

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Please continue with questions on the next page.

Name: ______________________ **Roll no:** ______________________

Fill up the answers on this sheet before submitting. Extra calculations, explanations and algebraic work if necessary are to be performed on the answer script and not on this sheet. Remember to fill in your name and roll no. before attempting any solution. Anonymous sheets will not be evaluated. Write all answers and draw plots using a pen.

Detach this sheet and attach with answer script before submission.

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6. Two single slew filters with pass band frequency f_{p_1}, f_{p_2} and stop band frequency f_{s_1}, f_{s_2} are connected in cascade to form the following filters. State the comparative relations $(>, <, =, \geq, \leq)$ between them for the following filters and plot the frequency response characteristics for them in the space provided below the answer on this sheet, annotating characteristics of the curve:

   (a) (2 points) Lowpass filter: f_{p_1} ____________ f_{s_1} and f_{p_2} ____________ f_{s_2}

   (b) (2 points) Highpass filter: f_{p_1} ____________ f_{s_1} and f_{p_2} ____________ f_{s_2}

   (c) (4 points) Bandpass filter: f_{p_1} _____ f_{s_1} ; f_{p_2} _____ f_{s_2} ; f_{p_1} _____ f_{p_2} ; f_{s_1} _____ f_{s_2}

   (d) (2 points) Notch filter: f_{p_1} ____________ f_{p_2} ; f_{s_1} ____________ f_{s_2}

7. (a) (4 points) A continuous time signal $x_a(t)$ is composed of a linear combination of frequencies F_1 Hz, F_2 Hz, F_3 Hz, and F_4 Hz. The signal $x_a(t)$ is sampled at 8 kHz and the sampled signals are passed through a low-pass filter with cutoff frequency of 3.5 kHz, generating a continuous-time signal $y_a(t)$ composed of three sinusoidal signals of frequencies 150 Hz, 400 Hz, 925 Hz respectively. What are the possible values of $F_1 =$______, $F_2 =$______, $F_3 =$______ and $F_4 =$______? Justify your answer with a plot in the space provided below.

(b) (3 points) Using the tabular approach compute $y[n] = x[n] \circledast h[n]$ where $x[n] = \{3, \underset{\uparrow}{-2}, 4\}$ and $h[n] = \{4, \underset{\uparrow}{1}, 3, 4\}$. Calculate and write your answer below.
$y[n] = \{$____$\}$

(c) (3 points) What is the result of circular convolution $y[n] = x[n]$ (4) $h[n]$ where $x[n] = \{1, 2, 0, 1\}$ and $h[n] = \{2, 2, 1, 1\}$? Calculate and write your answer below.
$y[n] = \{$____$\}$