Indian Institute of Technology Kharagpur
End-Autumn Semester Examination 2017-18

Date of Examination: **27 Nov. 2017** **Session:** **AN** **Duration:** **3 Hours**
Subject No.: **EE41013, EE60033** **Subject:** **Digital Signal Processing**
Department/Center/School: **Electrical Engineering** **Credits:** **4** **Full marks:** **100**

Instructions

1. This question paper contains 4 pages and 10 questions. All questions are compulsory. Marks are indicated in parentheses. This question paper has been cross checked and no errors exist.

2. Detach the pages 3-4, fill the answers on them and attach with the answer script.

3. Please write your name, roll number, subject name and code, date and time of examination on the answer script before attempting any solution.

4. Use of electronic calculators only is permitted. No extra resources viz. graph papers, log-tables, trigonometric tables would be required.

5. **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page or across the answer script without a clear ordering will receive no marks.

6. **Mysterious or unsupported answers will not receive full marks**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no marks; an incorrect answer supported by substantially correct calculations and explanations may receive partial marks. Fundamental theorems may be stated and would be considered while properties assumed would not be awarded.

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1. (a) (5 points) Given that two real valued length-N sequences $g[n]$ and $h[n]$ are used to define $x[n] = g[n] + jh[n]$, and $x[n] \overset{DFT}{\longleftrightarrow} X[k]$, $g[n] \overset{DFT}{\longleftrightarrow} G[k]$ and $h[n] \overset{DFT}{\longleftrightarrow} H[k]$, then prove that

$$G[k] = \frac{1}{2}\left\{X[k] + X^*[\langle -k\rangle_N]\right\} \tag{1}$$

$$H[k] = \frac{1}{2j}\left\{X[k] - X^*[\langle -k\rangle_N]\right\} \tag{2}$$

(b) (5 points) Given two length-N sequences $g[n] = v[2n]$ and $h[n] = v[2n+1]$ represent the length-$2N$ sequence $v[n]$, and $v[n] \overset{DFT}{\longleftrightarrow} V[k]$, prove that

$$V[k] = G[\langle k\rangle_N] + W_{2N}^{k} H[\langle k\rangle_N], 0 \leq k \leq 2N-1 \tag{3}$$

2. (10 points) Prove that $x[n] \overset{DCT}{\longleftrightarrow} X_{DCT}[k]$ when

$$X_{DCT}[k] = \sum_{n=0}^{N-1} 2x[n] \cos\left(\frac{\pi k(2n+1)}{2N}\right), 0 \le k \le N-1 \tag{4}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \alpha[k] X_{DCT}[k] \cos\left(\frac{\pi k(2n+1)}{2N}\right), 0 \le n \le N-1 \tag{5}$$

where

$$\alpha[k] = \begin{cases} \frac{1}{2}, & k = 0 \\ 1, & 1 \le k \le N-1 \end{cases} \tag{6}$$

3. (10 points) Derive and draw the complete signal flow graph of decimation-in-frequency FFT algorithm for $N = 8$.

4. (a) (3 points) Derive the z-transform of $(r^n \sin \omega_0 n)\, \mu[n]$ and state its ROC.
   (b) (3 points) Derive the sequence $h[n]$ given $H(z) = z/(z + 0.6)$ with ROC $|z| > 0.6$.
   (c) (2 points) Given $g[n] \leftrightarrow G(z)$, prove that $g[-n] \leftrightarrow G(1/z)$.
   (d) (2 points) Let $y[n] = mean\{x[n], x[n-1], \cdots, x[n-N+1]\}$, then compute $y[19]$ and $y[20]$ given that the histogram of $\{x[19], \cdots, x[0]\}$ is $h[k] = \{4, 6, 7, 3\}$ where $x \in \mathbb{Z}^+ \cap [0, 3]$ and $k = \{0, 1, 2, 3\}$, $x[0] = 3$ and $x[20] = 2$?

5. (a) (4 points) Given $H_0(z) = 0.5(1 + z^{-1})$, derive its frequency response $H_0(e^{j\omega})$ and using the plot of frequency response comment on the nature of $H_0(z)$ as a filter.
   (b) (4 points) Given $H_1(z) = 0.5(1 - z^{-1})$, derive its frequency response $H_1(e^{j\omega})$ and using the plot of frequency response comment on the nature of $H_1(z)$ as a filter.
   (c) (2 points) Given $x[n] \overset{DFT}{\longleftrightarrow} X[k]$, prove that $x^*[\langle -n \rangle_N] \overset{DFT}{\longleftrightarrow} X^*[k]$.

6. (a) (2 points) Identify the passband nature and impulse response nature of the filter defined as $H(z) = \frac{K(1+z^{-1})}{1-\alpha z^{-1}}$.
   (b) (2 points) Derive the coefficient K in terms of α for which $|H(e^{j\omega})|_{\max} = 1$.
   (c) (3 points) Derive the value of α if the filter has a 3-dB cutoff frequency of 0.8π.
   (d) (3 points) Draw the structure of this filter using only $\boxed{z^{-1}}$, $\oplus$ and $\triangleright$ blocks.

7. (a) (5 points) In the filter designed in Q. 6(d) state the minimum bit resolution of each of the blocks used to obtain unsaturated output if input signal and filter coefficients are each represented in 8-bits.
   (b) (5 points) If $y[n] = x_1[n] \textcircled{N} x_2[n]$ is defined for $0 \le n \le N-1$, then prove that when N is even

$$\sum_{n=0}^{N-1} (-1)^n y[n] = \left(\sum_{n=0}^{N-1} (-1)^n x_1[n]\right)\left(\sum_{n=0}^{N-1} (-1)^n x_2[n]\right) \tag{7}$$

Name: ______________________ **Roll no:** ______________________

Fill up the answers on this sheet before submitting. Extra calculations and explanations if necessary are to be performed on the answer script and not on this sheet. Remember to fill in your name and roll no. before attempting any solution. Anonymous sheets will not be evaluated. Write all answers and draw plots using a pen.

Detach this sheet and attach with answer script before submission.

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8. Identify the following for the filter function

$$y[n] = x[n-1] - 1.2x[n-2] + x[n-3] + 1.3y[n-1] - 1.04y[n-2] + 0.222y[n-3] \quad (8)$$

   (a) (2 points) $H(z) =$ ______________________

   (b) (1 point) Impulse response type: ______________________

   (c) (1 point) Is it canonical form realizable: ______________________

   (d) (2 points) Zeroes: ______________________

   (e) (3 points) Poles: ______________________

   (f) (1 point) Gain factor: ______________________

9. (a) (5 points) Identify the DFT property indicated by these equations

   1. $\alpha g[n] + \beta h[n] \overset{DFT}{\longleftrightarrow} \alpha G[k] + \beta H[k]$ ______________________
   2. $g[\langle n - n_0 \rangle_N] \overset{DFT}{\longleftrightarrow} W_N^{kn_0} G[k]$ ______________________
   3. $W_N^{-k_0 n} g[n] \overset{DFT}{\longleftrightarrow} G[\langle k - k_0 \rangle_N]$ ______________________
   4. $Ng[\langle -k \rangle_N] \overset{DFT}{\longleftrightarrow} G[n]$ ______________________
   5. $g[n] \textcircled{N} h[n] \overset{DFT}{\longleftrightarrow} G[k]H[k]$ ______________________

   (b) (3 points) When $h(t) = \frac{1}{\pi t}$, $H(j\Omega) =$ ______________________

   and this is known as ______________________ transform

   (c) (2 points) If $x_u[n]$ is a zero inserted factor-2 upsampled version of $x[n]$, then express $y[n]$ in terms of $x_u[n]$ such that $y[n]$ is a smoothened interpolated version of $x[n]$.

10. (a) (4 points) Identify the type of FIR transfer function in the following:

1. $\breve{H}(\omega) = h\left[\frac{N}{2}\right] + 2\sum_{n=1}^{N/2} h\left[\frac{N}{2} - n\right]\cos(\omega n).$ Type ______

2. $\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h\left[\frac{N+1}{2} - n\right]\cos\left(\omega\left(n - \frac{1}{2}\right)\right).$ Type ______

3. $\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h\left[\frac{N+1}{2} - n\right]\sin\left(\omega\left(n - \frac{1}{2}\right)\right).$ Type ______

4. $\breve{H}(\omega) = 2\sum_{n=1}^{N/2} h\left[\frac{N}{2} - n\right]\sin(\omega n).$ Type ______

(b) (3 points) Using tabular approach compute $y[n] = x[n] \, \textcircled{4} \, h[n]$ where $x[n] = \{1, 0, 2, 1\}$ and $h[n] = \{2, 1, 2, 1\}$. Detail calculations in the space below.

$y[n] = \{$ ______________________________ $\}$

(c) (3 points) Using the tabular approach compute $y[n] = x[n] \circledast h[n]$ where $x[n] = \{3, \underset{\uparrow}{-1}, 4\}$ and $h[n] = \{4, \underset{\uparrow}{1}, -3, 4\}$. Detail calculations in the space below.

$y[n] = \{$ ______________________________ $\}$