Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Digital Signal Processing Laboratory (EE39203)
Autumn, 2022-23

Experiment 3 **Frequency Analysis**

Slot: Date:

Student Name: Roll No.:

Grading Rubric

	Tick the best applicable per row			Points
	Below Expectation	Lacking in Some	Meets all Expectation	
Completeness of the report				
Organization of the report (5 pts) *With cover sheet, answers are in the same order as questions in the lab, copies of the questions are included in report, prepared in LaTeX*				
Quality of figures (5 pts) *Correctly labelled with title, x-axis, y-axis, and name(s)*				
Ability to compute Fourier series expansion and synthesize periodic signals using the expansion (15 pts) *Derivation and sketch, plots of synthesized signals, questions*				
Implementation of DTFT (25 pts) *Matlab function*				
Magnitude and Phase Response of DTFT (25 pts) *DTFT's magnitude and phase plots*				
Discrete time system analysis (25 pts) *Exercises in 3.3, completed block diagram, table of measurements, impulse and frequency response*				
			TOTAL (100 pts)	

Total Points (100): TA Name: TA Initials:

1. Introduction

In this experiment, we will use Fourier series and Fourier transforms to analyze continuous-time and discrete-time signals and systems. The Fourier representations of signals involve the decomposition of the signal in terms of complex exponential functions. These decompositions are very important in the analysis of linear time-invariant (LTI) systems, due to the property that the response of an LTI system to a complex exponential input is a complex exponential of the same frequency! Only the amplitude and phase of the input signal are changed. Therefore, studying the frequency response of an LTI system gives complete insight into its behaviour.

In this experiment and others to follow, we will use the Simulink extension to Matlab. Simulink is an icon-driven dynamic simulation package that allows the user to represent a system or a process by a block diagram. Once the representation is completed, Simulink may be used to digitally simulate the behaviour of the continuous or discrete-time system. Simulink inputs can be Matlab variables from the workspace, or waveforms or sequences generated by Simulink itself. These Simulink-generated inputs can represent continuous-time or discrete-time sources. The behaviour of the simulated system can be monitored using Simulink's version of common lab instruments, such as scopes, spectrum analysers and network analysers.

2. Background Exercises

Synthesis of Periodic Signals

Each signal given below represents one period of a periodic signal with period T_0.

1. Period $T_0 = 2$. For $t \in [0, 2]$:
$$s(t) = rect\left(t - \frac{1}{2}\right)$$
2. Period $T_0 = 1$. For $t \in \left[-\frac{1}{2}, \frac{1}{2}\right]$:
$$s(t) = rect(2t) - \frac{1}{2}$$

For each of these two signals, do the following:

i. Compute the Fourier series expansion in the form
$$s(t) = a_0 + \sum_{k=1}^{\infty} A_k \sin(2\pi k f_0 t + \theta_k)$$
where $f_0 = \frac{1}{T_0}$
Hint: You may want to use one of the following references:
Sec. 4.1 of "Digital Signal Processing", by Proakis and Manolakis, 1996;
Sec. 4.2 of "Signals and Systems", by A. Oppenheim and A. Willsky, 1983;
Sec. 3.3 of "Signals and Systems", A. Oppenheim and A. Willsky, 1997.
Note that in the expression above, the function in the summation is $\sin(2\pi k f_0 t + \theta_k)$, rather than a complex sinusoid. The formulas in the above references must be modified to accommodate this. You can compute the cos/sin version of the Fourier series, then convert the coefficients.

ii. Sketch the signal on the interval [0, T_0]

Lab Report:
Submit these background exercises with the lab report.

3. Discrete-Time Frequency Analysis

In this section of the laboratory, we will study the use of the discrete-time Fourier transform.

3.1. Discrete-Time Fourier Transform

The DTFT (Discrete-Time Fourier Transform) is the Fourier representation used for finite energy discrete-time signals. For a discrete-time signal, x(n), we denote the DTFT as the function $X(\omega)$ given by the expression

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Since $X(\omega)$ is a periodic function of ω with a period of 2π, we need only to compute $X(\omega)$ for $-\pi < \omega < \pi$.

Write a Matlab function `X = DTFT(x,n0,dw)` that computes the DTFT of the discrete-time signal `x`. Here `n0` is the time index corresponding to the 1st element of the `x` vector, and `dw` is the spacing between the samples of the Matlab vector `X`. For example, if `x` is a vector of length *N*, then its DTFT is computed by

$$X(\omega) = \sum_{n=1}^{N} x[n]e^{-j\omega(n+n0-1)}$$

where ω is a vector of values formed by `w=(-pi:dw:pi)`

Hint: In Matlab, *j* or *i* is defined as $\sqrt{-1}$. However, you may also compute this value using the Matlab expression `i = sqrt(-1)`.

For the following signals use your DTFT function to (i) Compute $X(\omega)$, and (ii) Plot the magnitude and the phase of $X(\omega)$ in a single plot using the `subplot` command.

Hint: Use the `abs()` and `angle()` commands.

1. $x[n] = \delta[n]$
2. $x[n] = \delta[n-5]$
3. $x[n] = (0.5)^n u[n]$

Lab Report:
Hand in a printout of your Matlab function. Also, hand in plots of the DTFT's magnitude and phase for each of the three signals.

3.2. Magnitude and Phase of the Frequency Response of a Discrete-Time Systems

Consider the discrete-time system described by the following difference equation:

$$y[n] = 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]$$

Assume that the system is causal.

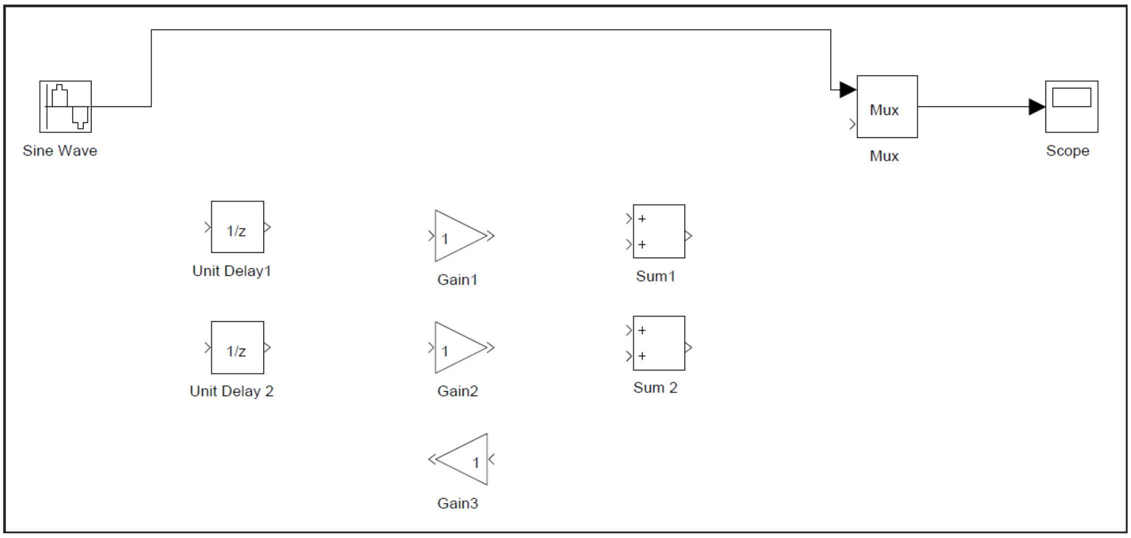
i. Draw a system diagram.
ii. Obtain the impulse response of the system by replacing $x[n]$ with $\delta[n]$ in the above equation. (Use causality to set up the initial conditions.)
iii. Use your answer in (ii) to obtain the frequency response of the system.
iv. Find the frequency response of the system using another method. Specifically, take the DTFT of the left-hand-side and right-hand-side of the difference equation, and then use linearity and the time-shifting property of the DTFT along with the fact that $H(\omega) = \frac{Y(\omega)}{X(\omega)}$
v. Use Matlab to compute and plot the magnitude and phase responses, $|H(\omega)|$and $\angle H(\omega)$, for $-\pi < \omega < \pi$. You may use Matlab commands `phase` and `abs`.

Lab Report:
Submit these exercises with the lab report.

3.3. System Analysis

Below is a figure for a model that takes a discrete-time sine signal, processes it according to a difference equation and plots the multiplexed input and output signals in a graph window. Complete this block diagram such that it implements the following difference equation given in section 3.2

$$y[n] = 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]$$



You are provided with the framework of the setup and the building blocks that you will need. You can change the values of the Gain blocks by double clicking on them. After you complete the setup, adjust the frequency of Sine Wave to the following frequencies: $\omega = \pi/16$, $\omega = \pi/8$, $\omega = \pi/4$. For each frequency, make magnitude response measurements using the input and output sequences shown in the graph window. Compare your measurements with the values of the magnitude response $|H(e^{j\omega})|$ which you computed in the background exercises at these frequencies.

An alternative way of finding the frequency response is taking the DTFT of the impulse response. Use your DTFT function to find the frequency response of this system from its impulse response. The impulse response was calculated in section 3.2 of the background exercises. Plot the impulse response, and the magnitude and phase of the frequency response in the same figure using the subplot command.

Lab Report:
Hand in the following:

- Figure of your completed block diagram
- Table of both the amplitude measurements you made and their theoretical values.
- Figure with the impulse response, and the magnitude and phase of the frequency response.

Acknowledgement: This manual is based on "Purdue University: ECE438 - Digital Signal Processing with Applications (2016)" by Prof. Charles Bouman and Prof. Mireille Boutin.