

Artificial Intelligence: Foundations & Applications

Reasoning under uncertainty



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Need for probabilistic model

- Till now, we have considered that intelligent agent has
 - Known environment
 - Full observability
 - Deterministic world
- Reasons for using probability
 - Example: *Toothache* \implies *Cavity*
 - **Toothache can be caused by many other ways also, eg.**
Toothache \implies *GumDisease* \vee *Cavity* \vee *WisdomTeeth* \vee ...
 - Specifications become too large
 - Theoretical ignorance
 - Practical ignorance

Probabilistic reasoning

- Useful for prediction
 - Analyzing causal effect, predicting outcome
 - Given that I have cavity, what is the chance that I will have toothache?
- Useful for diagnosis
 - Analyzing causal effect, finding out reasons for a given effect
 - Given that I have toothache, what is the chance that it is caused by a cavity?
- Require a methodology to analyze both the scenarios

Axioms of probability

- All probabilities lie between 0 and 1, ie., $0 \leq P(A) \leq 1$
- $P(\text{True}) = 1$ and $P(\text{False}) = 0$
- $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
- $P(A \wedge B) = P(A|B) \times P(B) = P(B|A) \times P(A)$
 - $P(A) = \frac{P(A|B) \times P(B)}{P(B|A)}$
 - Bayes' rule

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- What is the probability of a vehicle to be EV and Red?

Joint probability

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- What is the probability of a vehicle to be EV and Red?
- What is the probability of a Red vehicle?

Joint probability

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- What is the probability of a vehicle to be EV and Red?
- What is the probability of a Red vehicle?
 - Marginal probability: $\sum_T P(C = \text{Red} \wedge T = *)$

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- What is the probability of a vehicle to be EV and Red?
- What is the probability of a Red vehicle?
 - Marginal probability: $\sum_T P(C = Red \wedge T = *)$
- What is the probability of a vehicle to be EV given that it is Red?

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- What is the probability of a vehicle to be EV and Red?
- What is the probability of a Red vehicle?
 - Marginal probability: $\sum_T P(C = Red \wedge T = *)$
- What is the probability of a vehicle to be EV given that it is Red?
 - Conditional probability: $\frac{P(C = Red \wedge T = EV)}{P(C = Red)}$

Independence

- **Two variables are independent if $P(X, Y) = P(X) \times P(Y)$ holds**
 - It means that their joint distribution factors into a product two distributions
 - This can be expressed as $P(X|Y) = P(X)$
- **Independence is a simplifying modeling assumption**
 - Empirical joint distributions can be at best close to independent

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

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- Are T and W independent?

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- Are T and W independent?
- Find marginal probabilities

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- Are T and W independent?
- Find marginal probabilities
 - $P(T = \text{Hot}) = ?$, $P(T = \text{Cold}) = ?$
 - $P(W = \text{Sun}) = ?$, $P(W = \text{Rain}) = ?$

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- Are T and W independent?
- Find marginal probabilities
 - $P(T = \text{Hot}) = ?$, $P(T = \text{Cold}) = ?$
 - $P(W = \text{Sun}) = ?$, $P(W = \text{Rain}) = ?$
- Now check for independence

Example: Independence

- Tossing of N fair coins

$P(X_1)$	
H	0.5
T	0.5

$P(X_2)$	
H	0.5
T	0.5

.....

$P(X_n)$	
H	0.5
T	0.5

$P(X_1, X_2, \dots, X_n)$

X_1	X_2	X_3	X_n	P
H					H	
...	
T					H	

} 2^n

Chain rule

- **Product rule**

$$P(X, Y) = P(X) \times P(Y|X)$$

- **Chain rule**

$$\begin{aligned} P(X_1, X_2, \dots, X_n) &= P(X_1) \times P(X_2|X_1) \times P(X_3|X_1, X_2) \times \dots \times P(X_n|X_1, \dots, X_{n-1}) \\ &= \prod_i P(X_i|X_1, \dots, X_{i-1}) \end{aligned}$$

Conditional independence

- $P(\textit{Toothache}, \textit{Cavity}, \textit{Catch})$
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:

$$P(+catch | +toothache, +cavity) = P(+catch | +cavity)$$

- The same independence holds if I don't have a cavity:

$$P(+catch | +toothache, -cavity) = P(+catch | -cavity)$$

- Catch is conditionally independent of Toothache given Cavity:

$$P(\textit{Catch} | \textit{Toothache}, \textit{Cavity}) = P(\textit{Catch} | \textit{Cavity})$$

Conditional independence

- Absolute independence is very rare
- Conditional independence is the most basic and robust form of knowledge about uncertain environments
- Mathematically it is defined as - X is conditionally independent of Y given Z

$$P(X, Y|Z) = P(X|Z) \times P(Y|Z)$$

- It can also be shown that

$$P(X|Z, Y) = P(X|Z)$$

Belief network

- Qualitative information
 - A belief network is a graph consist of the following:
 - Nodes - Set of random variables
 - Edges - Dependency of nodes. $X \rightarrow Y$ means X has direct influence on Y
- Quantitative information
 - Each node has conditional probability table that quantifies the effects the parents have on the node
- The network is a **directed acyclic graph (DAG)**, ie., without any cycle

Example

- Consider the following knowledge base
 - *Rain* - Raining
 - *Traffic* - There may be traffic if it rains
 - *Umbrella* - People may carry umbrella if it rains

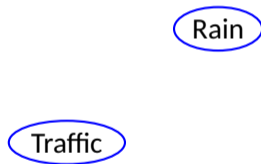
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Rain

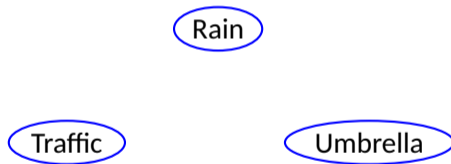
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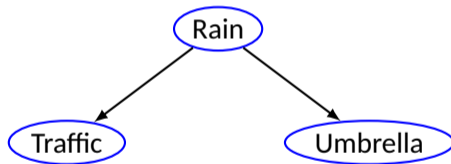
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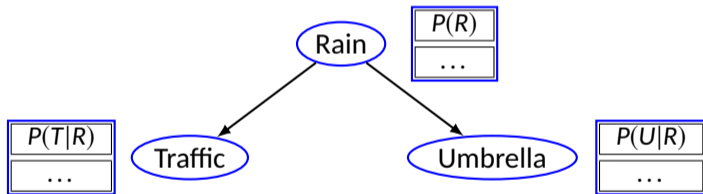
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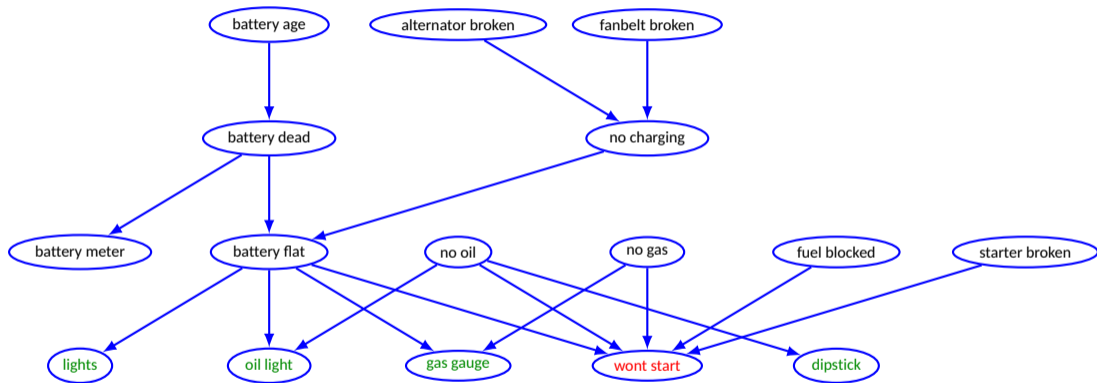


Example

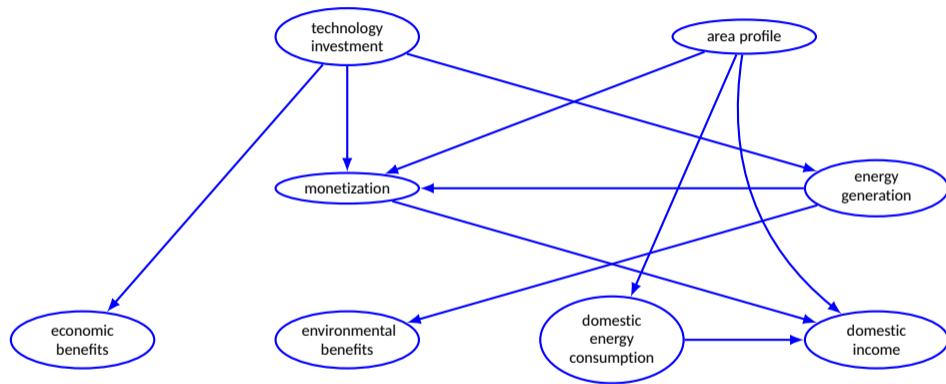
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Belief network example: car



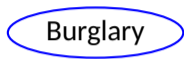
Belief network example: renewable energy



Classical example

- Burglar alarm at a home
 - It works almost perfectly
 - Sometime alarm goes off if there is a earthquake
- John calls police when he hears the alarm but sometimes he misinterpret telephone ringing as alarm and calls police too.
- Mary also calls police. But she loves loud music, therefore, she misses the alarm sometime.

Belief network example

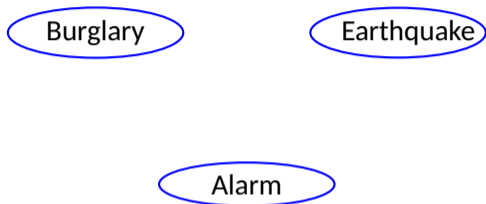


Belief network example

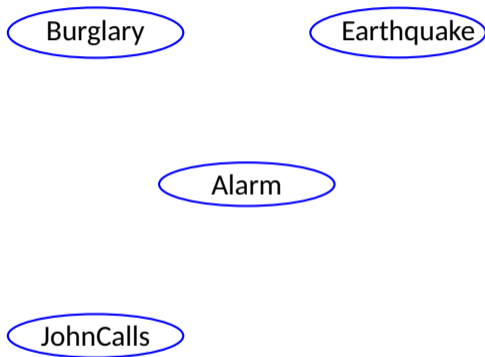
Burglary

Earthquake

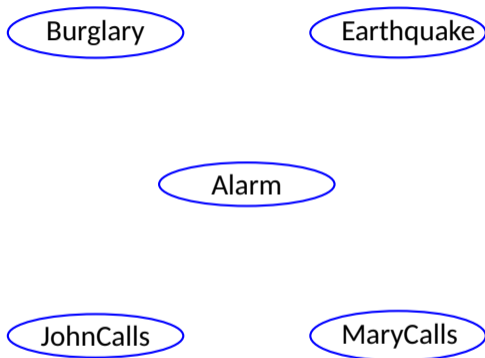
Belief network example



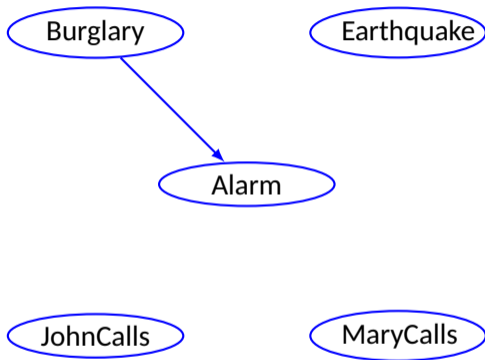
Belief network example



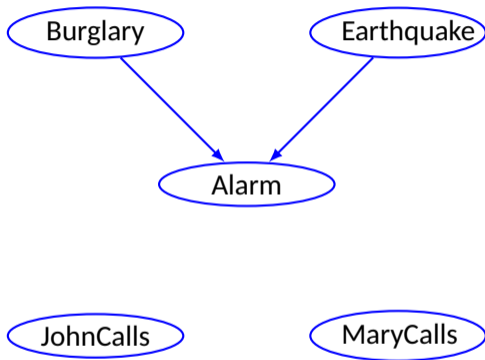
Belief network example



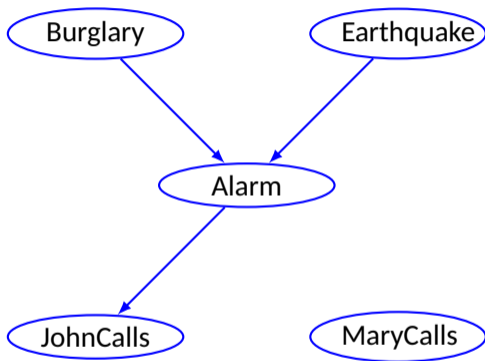
Belief network example



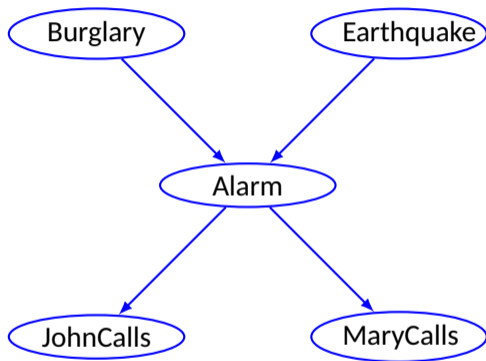
Belief network example



Belief network example

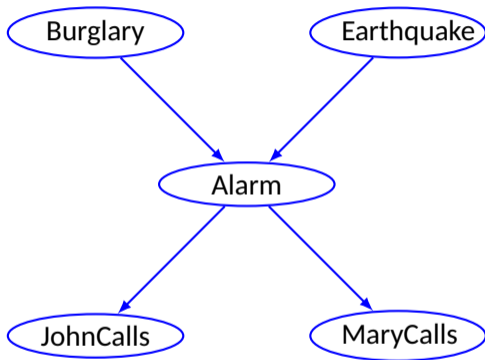


Belief network example

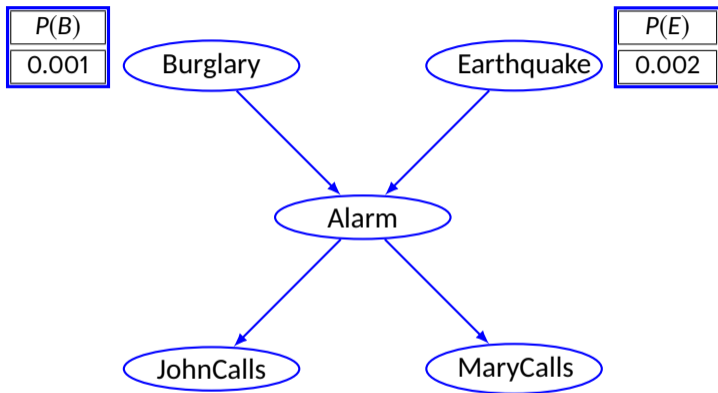


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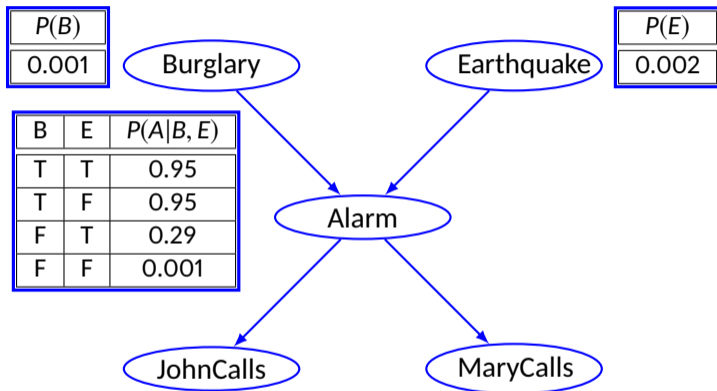
$P(B)$
0.001



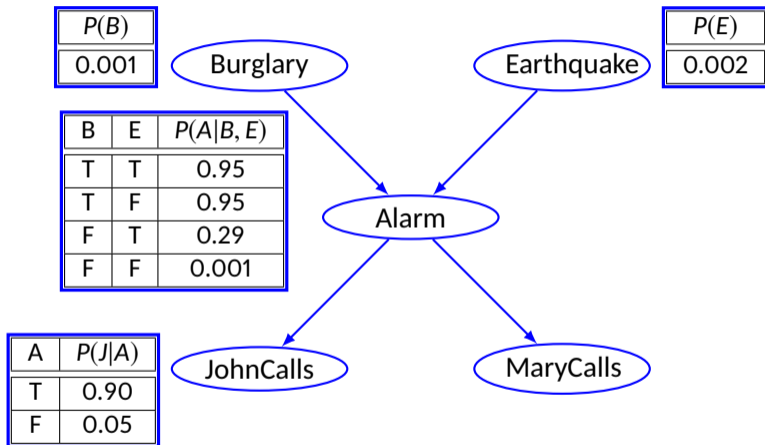
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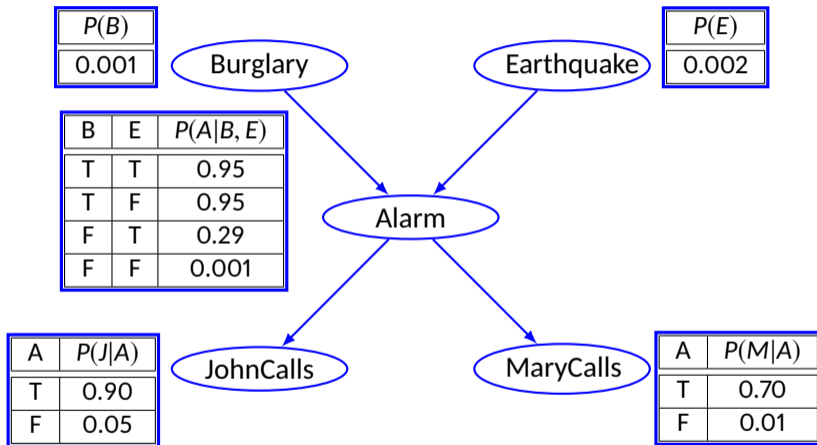
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Belief network example



Belief network example



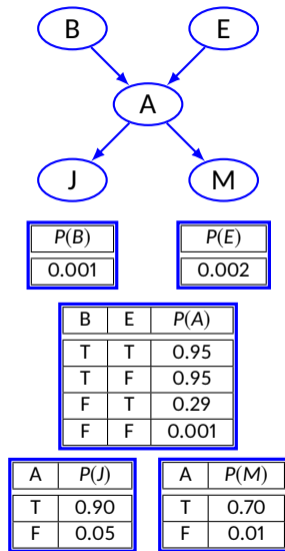
Probabilities in Belief Nets

- Bayes' net implicitly encode joint distributions
 - This can be determined from local conditional distributions
 - Need to multiply relevant conditional probabilities

- $P(X_1, X_2, \dots, X_n) = \prod_i^n P(X_i | \text{parents}(X_i))$

Joint probability distribution

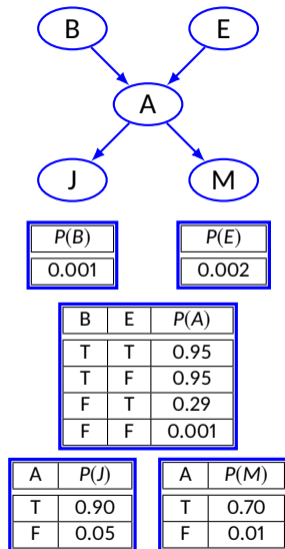
- Probability of the event that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both Mary and John call:



Joint probability distribution

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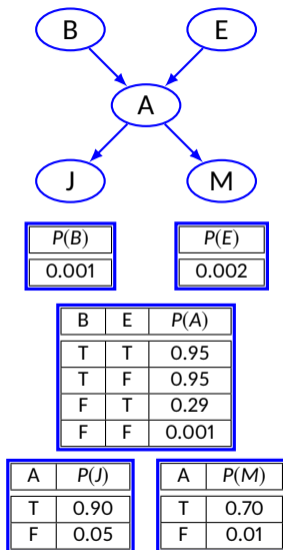
$$P(J \wedge M \wedge A \wedge \neg B \wedge \neg E)$$



Joint probability distribution

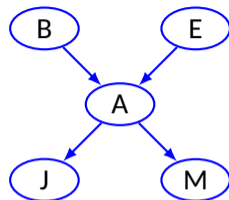
- Probability of the event that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both Mary and John call:

$$\begin{aligned}P(J \wedge M \wedge A \wedge \neg B \wedge \neg E) \\&= P(J|A) \times P(M|A) \times P(A|\neg B \wedge \neg E) \times P(\neg B) \times P(\neg E) \\&= 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \\&= 0.00062\end{aligned}$$



Joint probability distribution: $P(A)$

$P(A)$



$P(B)$
0.001

$P(E)$
0.002

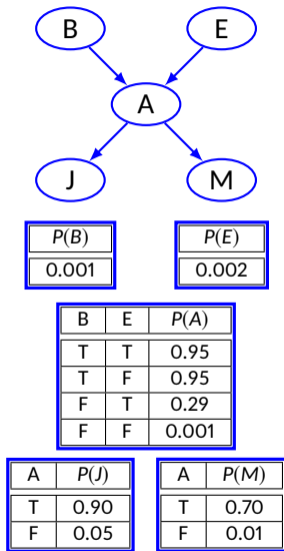
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(A)$

$$P(A) \\ = P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE)$$

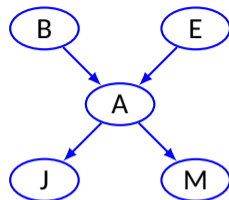


Joint probability distribution: $P(A)$

$P(A)$

$$= P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE)$$

$$= P(A|\bar{B}\bar{E}) \times P(\bar{B}\bar{E}) + P(A|\bar{B}E) \times P(\bar{B}E) + P(A|B\bar{E}) \times P(B\bar{E}) + P(A|BE) \times P(BE)$$



$P(B)$
0.001

$P(E)$
0.002

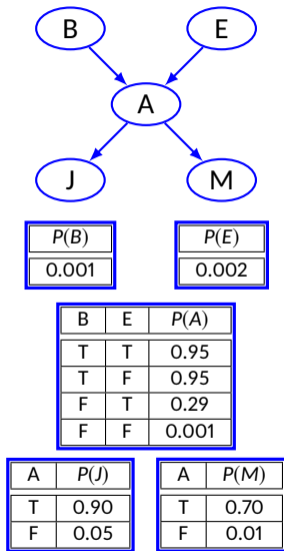
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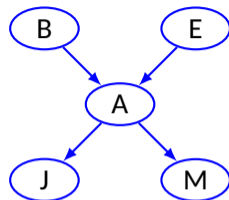
Joint probability distribution: $P(A)$

$$\begin{aligned}P(A) &= P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE) \\&= P(A|\bar{B}\bar{E}) \times P(\bar{B}\bar{E}) + P(A|\bar{B}E) \times P(\bar{B}E) + P(A|B\bar{E}) \times P(B\bar{E}) + P(A|BE) \times P(BE) \\&= 0.001 \times 0.999 \times 0.998 + 0.29 \times 0.999 \times 0.002 + 0.95 \times 0.001 \times 0.998 \\&\quad + 0.95 \times 0.001 \times 0.002 \\&= 0.0025\end{aligned}$$



Joint probability distribution: $P(J)$

$P(J)$



$P(B)$
0.001

$P(E)$
0.002

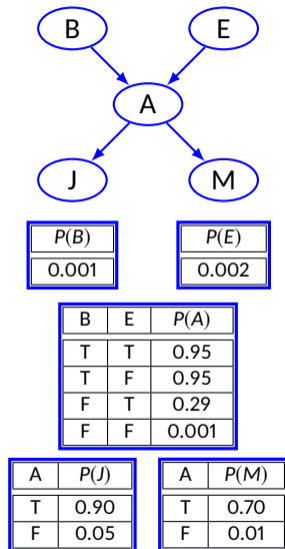
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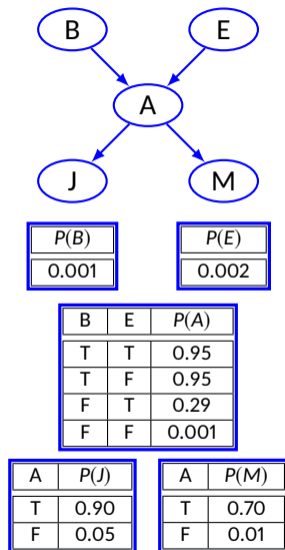
Joint probability distribution: $P(J)$

$$P(J) \\ = P(JA) + P(J\bar{A})$$



Joint probability distribution: $P(J)$

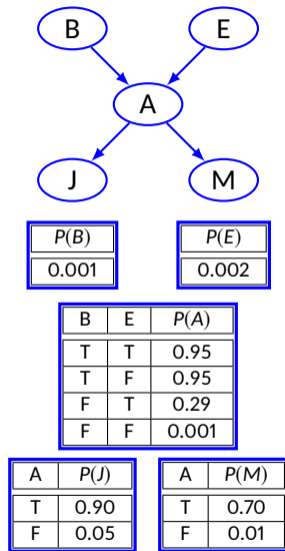
$$\begin{aligned}P(J) &= P(JA) + P(J\bar{A}) \\ &= P(J|A) \times P(A) + P(J|\bar{A}) \times P(\bar{A}) \\ &= 0.9 \times 0.0025 + 0.05 \times (1 - 0.0025) \\ &= 0.052125\end{aligned}$$



Joint probability distribution: $P(J)$

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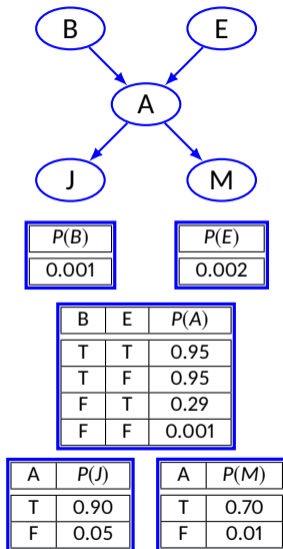
$$P(AB)$$



Joint probability distribution: $P(J)$

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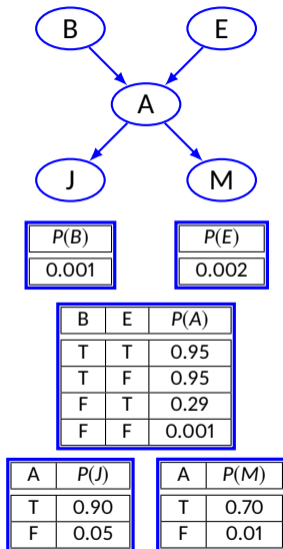
$$\begin{aligned}P(AB) &= P(ABE) + P(AB\bar{E})\end{aligned}$$



Joint probability distribution: $P(J)$

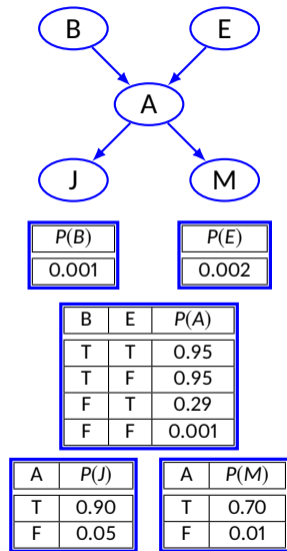
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$$\begin{aligned}P(AB) &= P(ABE) + P(AB\bar{E}) \\&= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998 \\&= 0.00095\end{aligned}$$



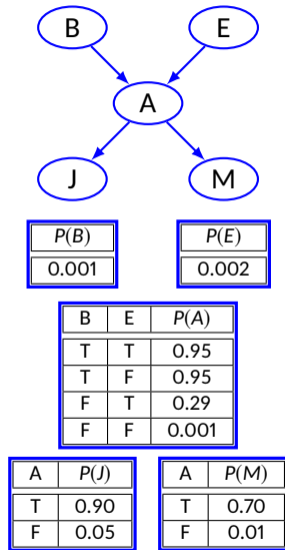
Joint probability distribution: $P(JB)$

$P(JB)$



Joint probability distribution: $P(JB)$

$$P(JB) \\ = P(JBA) + P(JB\bar{A})$$

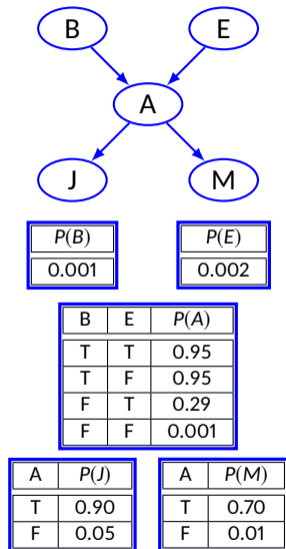


Joint probability distribution: $P(JB)$

$P(JB)$

$$= P(JBA) + P(JB\bar{A})$$

$$= P(J|AB) \times P(AB) + P(J|\bar{A}B) \times P(\bar{A}B)$$



Joint probability distribution: $P(JB)$

$$P(JB)$$

$$= P(JBA) + P(JB\bar{A})$$

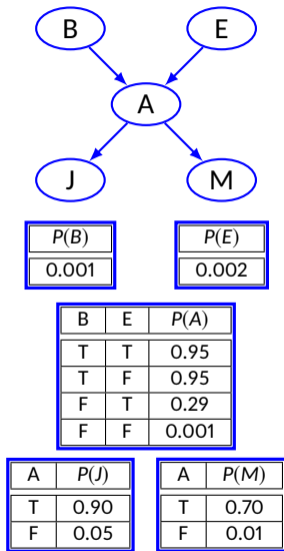
$$= P(J|AB) \times P(AB) + P(J|\bar{A}B) \times P(\bar{A}B)$$

$$= P(J|A) \times P(AB) + P(J|\bar{A}) \times P(\bar{A}B)$$

$$= 0.9 \times 0.00095 + 0.05 \times 0.00005$$

$$= 0.00086$$

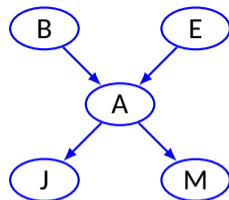
$$P(J|B)$$



Joint probability distribution: $P(JB)$

$$\begin{aligned}P(JB) &= P(JBA) + P(JB\bar{A}) \\&= P(J|AB) \times P(AB) + P(J|\bar{A}B) \times P(\bar{A}B) \\&= P(J|A) \times P(AB) + P(J|\bar{A}) \times P(\bar{A}B) \\&= 0.9 \times 0.00095 + 0.05 \times 0.00005 \\&= 0.00086\end{aligned}$$

$$\begin{aligned}P(J|B) &= \frac{P(JB)}{P(B)} = \frac{0.00086}{0.001} = 0.86\end{aligned}$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Inferences using belief networks

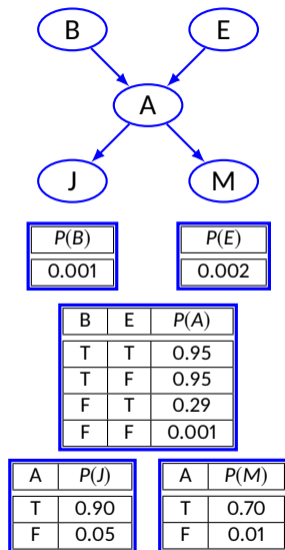
- Diagnostic inferences (effects to causes)
- Given that JohnCalls, infer that $P(\text{Burglary}|\text{JohnCalls}) = 0.016$

- Causal inferences (causes to effects)

- Given Burglary, infer that

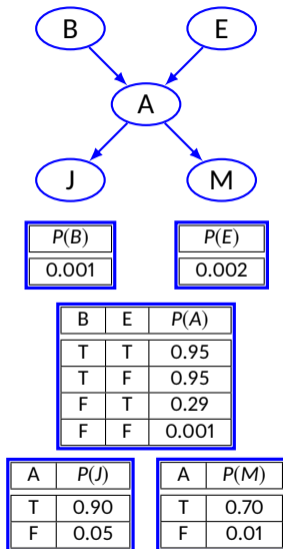
$$P(\text{JohnCalls}|\text{Burglary}) = 0.86$$

$$P(\text{MaryCalls}|\text{Burglary}) = 0.67$$



Inferences using belief networks

- Inter-causal inferences (between causes to a common effect)
- Given Alarm, we have $P(\text{Burglary}|\text{Alarm}) = 0.376$
- Also, if it is given that Earthquake is true, then $P(\text{Burglary}|\text{Alarm} \wedge \text{Earthquake}) = 0.003$
- Mixed inferences
 $P(\text{Alarm}|\text{JohnCalls} \wedge \neg\text{Earthquake}) = 0.003$



Example: traffic

- Consider the following variables
 - *R* - It is Raining
 - *T* - There is traffic

Example: traffic

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 - *R* - It is Raining
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R

T

Example: traffic

- Consider the following variables
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Example: traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic

$P(R)$
0.25



Example: traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic

$P(R)$
0.25

	$P(T R)$
R	0.25
$\neg R$	0.50



Example: traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic



$P(R)$
0.25

	$P(T R)$
R	0.25
$\neg R$	0.50

		$P(T, R)$
R	T	3/16
R	$\neg T$	1/16
$\neg R$	T	6/16
$\neg R$	$\neg T$	6/16

Example: traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic



$P(R)$
0.25

	$P(T R)$
R	0.25
$\neg R$	0.50

		$P(T, R)$
R	T	3/16
R	$\neg T$	1/16
$\neg R$	T	6/16
$\neg R$	$\neg T$	6/16

$P(T)$
9/16

Example: reverse traffic

- Consider the following variables
 - *R* - It is Raining
 - *T* - There is traffic

Example: reverse traffic

- Consider the following variables
 - R - It is Raining
 - T - There is traffic

T

R

Example: reverse traffic

- Consider the following variables
 - *R* - It is Raining
 - *T* - There is traffic



Example: reverse traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic

$P(T)$
$9/16$



Example: reverse traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic

$P(T)$
9/16

	$P(R T)$
T	1/3
$\neg T$	1/7



Example: reverse traffic

- Consider the following variables

- R - It is Raining
- T - There is traffic



$P(T)$
9/16

	$P(R T)$
T	1/3
$\neg T$	1/7

		$P(T, R)$
R	T	3/16
R	$\neg T$	1/16
$\neg R$	T	6/16
$\neg R$	$\neg T$	6/16

Example: Sprinkler

- There is some chance that tomorrow will be cloudy. If it is cloudy, then chances of rain is high. Also, there is a sprinkler to water the grass. When there is a prediction of cloud, it is less likely to run sprinkler. We want to model whether grass is wet or not.

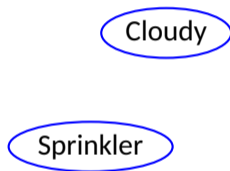
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Cloudy

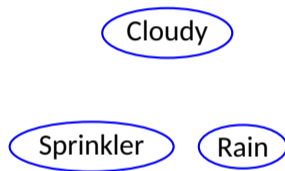
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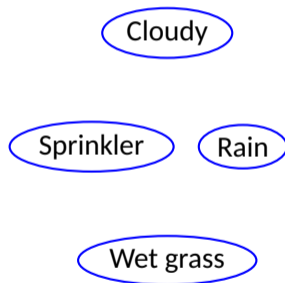
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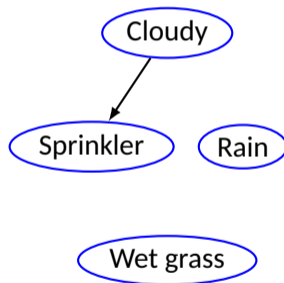
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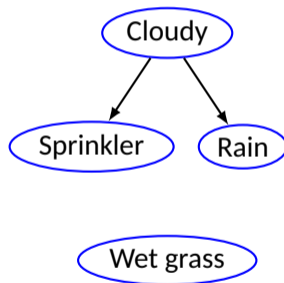
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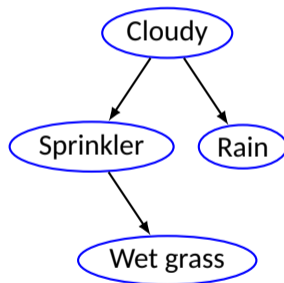
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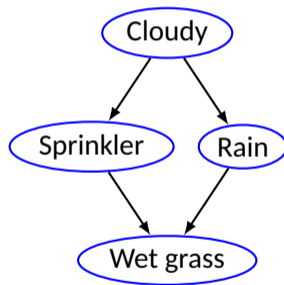
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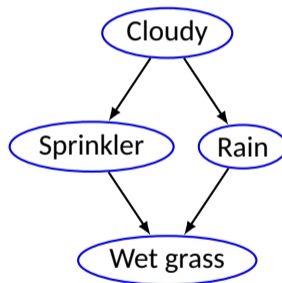
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$P(C)$
0.50

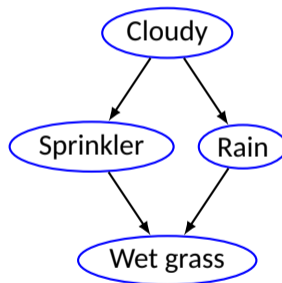


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$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50



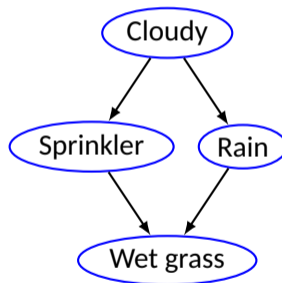
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$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20



Example: Sprinkler

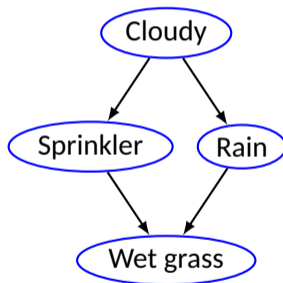
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$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



Example: Sprinkler

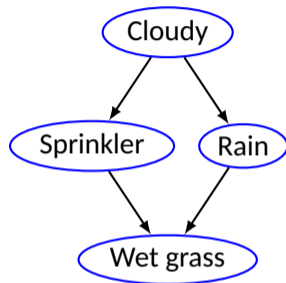
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	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00

- Find $P(W, S, R, C)$



Example: Sprinkler

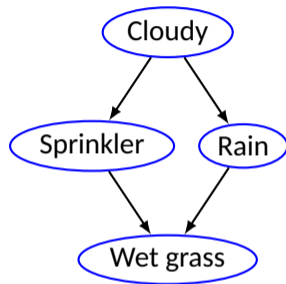
$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00

- Find $P(W, S, R, C)$
 $= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$



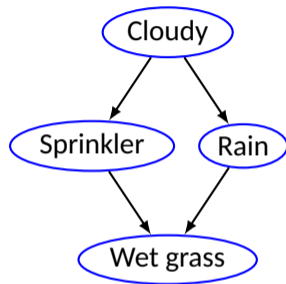
Example: Sprinkler

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S	R	0.99
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$\neg S$	$\neg R$	0.00



- Find $P(W, S, R, C)$
 $= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$
- Find $P(W = T|C = T)$

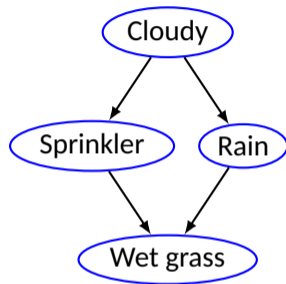
Example: Sprinkler

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	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- Find $P(W, S, R, C)$
 $= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$
- Find $P(W = T|C = T)$
 $= \frac{P(W = T, S, R, C = T)}{P(C = T)}$

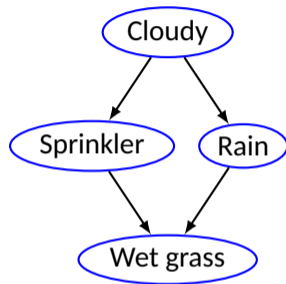
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

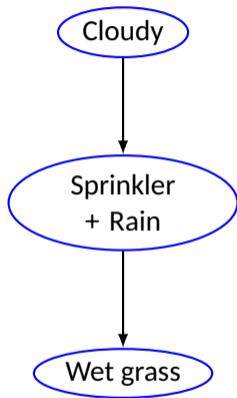
	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- Find $P(W, S, R, C)$
 $= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$
- Find $P(W = T|C = T)$
 $= \frac{P(W = T, S, R, C = T)}{P(C = T)} = 0.7452$

Example: Sprinkler - merging nodes



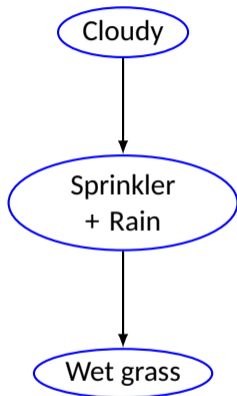
Example: Sprinkler - merging nodes

$P(C)$
0.50

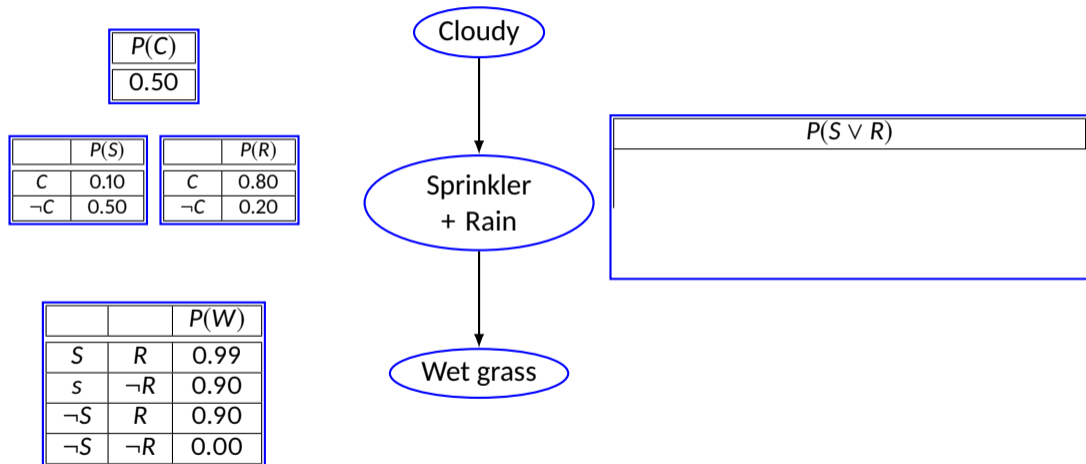
	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

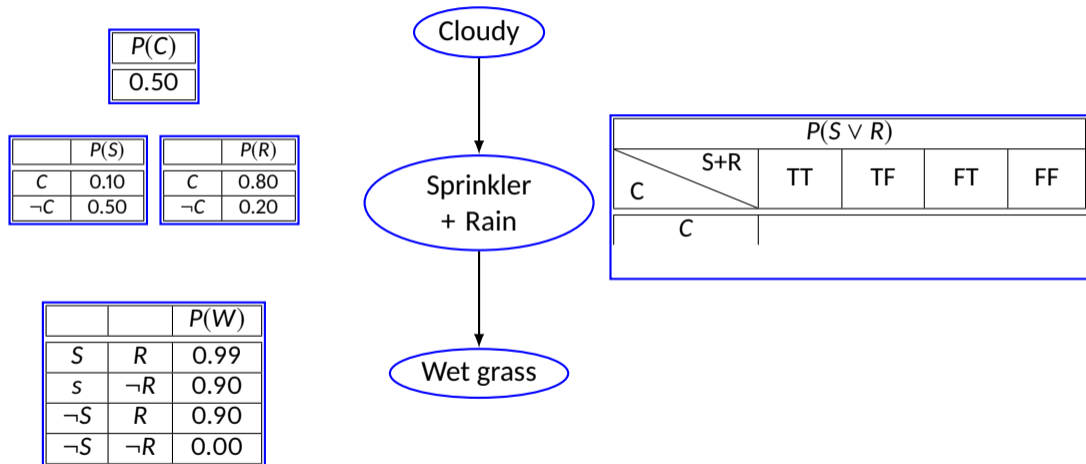
		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



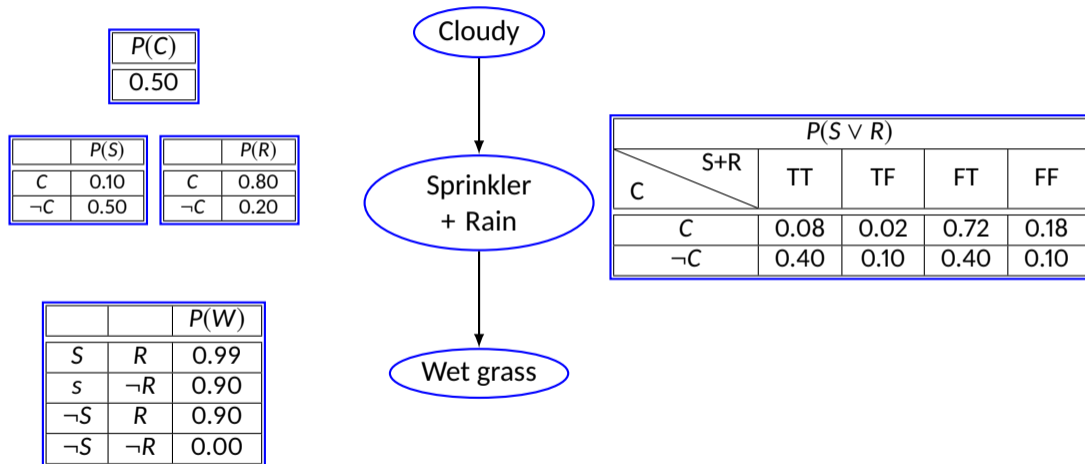
Example: Sprinkler - merging nodes



Example: Sprinkler - merging nodes



Example: Sprinkler - merging nodes



Belief network: phd admission

- Construct a belief network for the following
 - Let us assume that you are interested in pursuing phd at IITKGP!!
 - To take admission in the phd program, you may need to provide at most two recommendation letters from the course instructors or to appear for PMRF interview.
 - A course instructor can provide at most one recommendation letter based on the your grade on that subject.
 - Moreover, the grade depends on the attendance, marks obtained in the assignments and in the final examination.
 - Intelligence of the student can also play a major role in the performance of a subject or in the interview.
 - You may feel happy if you perform well in the subjects or you get admission for the phd program.

Incremental network construction

- Select the set of random variables (X_i) to describe the domains
- Select an appropriate ordering of the variables
- Steps:
 - Select a variable (X) and add a node for it
 - Set $Parents(X)$ to some minimal set of existing nodes such that the conditional independence property is satisfied
 - Define the conditional probability table for X

Burglar-Alarm

- Variable ordering
 - MaryCalls, JohnCalls, Alarm, Burglary, Earthquake

Burglar-Alarm

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 - MaryCalls, JohnCalls, Alarm, Burglary, Earthquake

MaryCalls

Burglar-Alarm

- Variable ordering
 - MaryCalls, JohnCalls, Alarm, Burglary, Earthquake

MaryCalls

JohnCalls

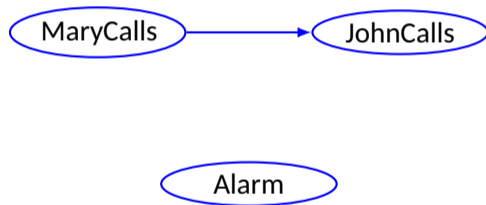
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- Variable ordering
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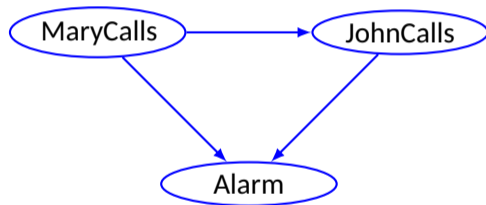
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- Variable ordering
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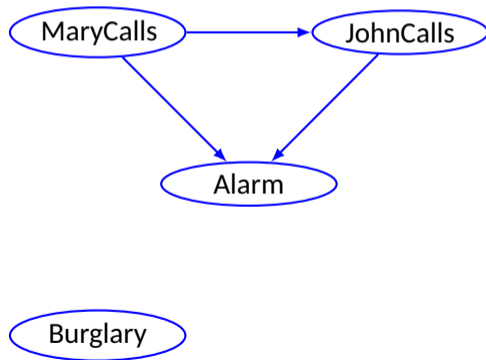
Burglar-Alarm

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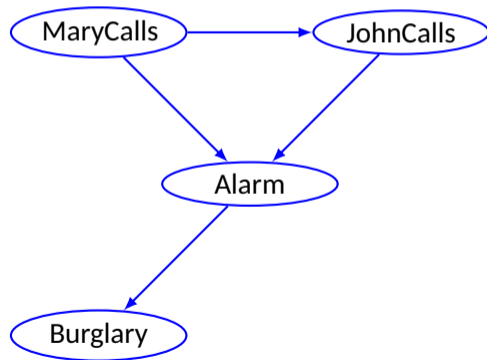
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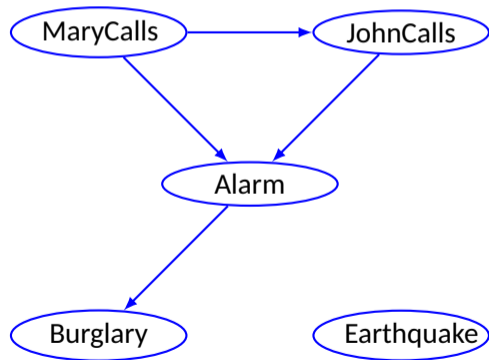
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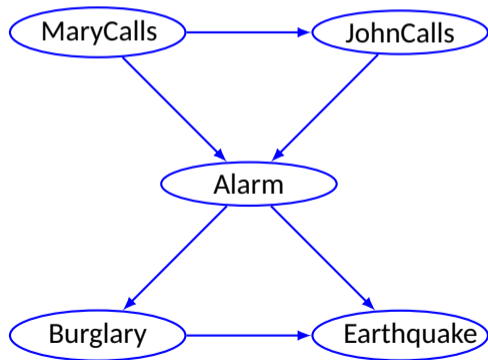
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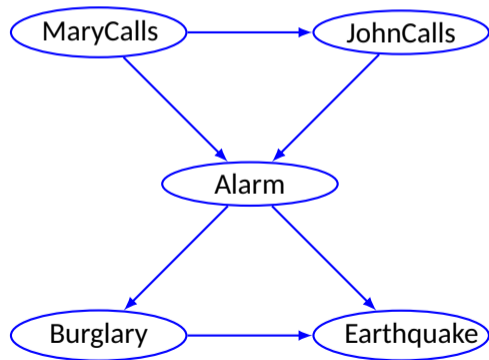
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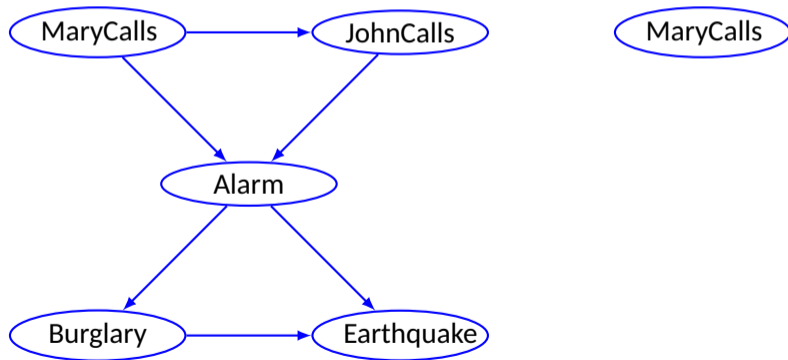
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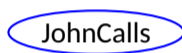
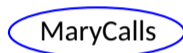
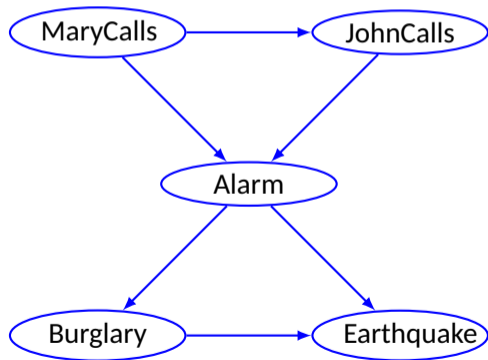
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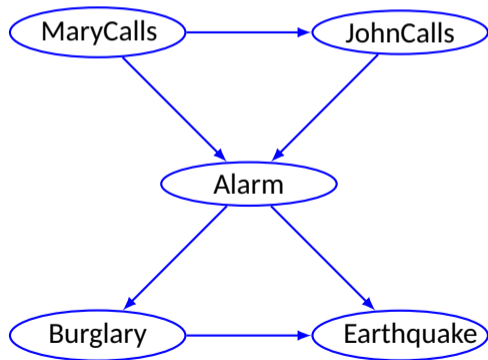
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Burglar-Alarm

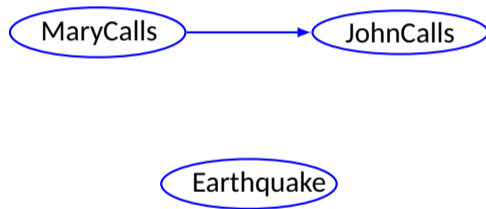
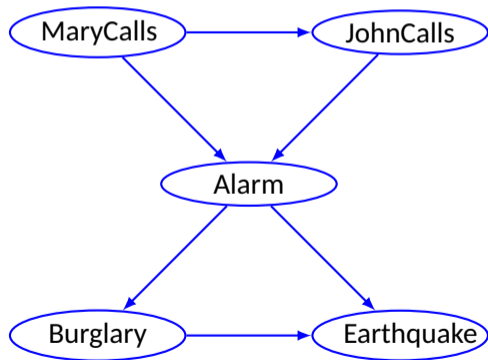
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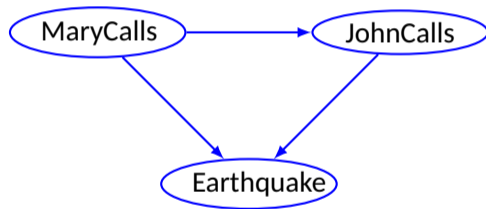
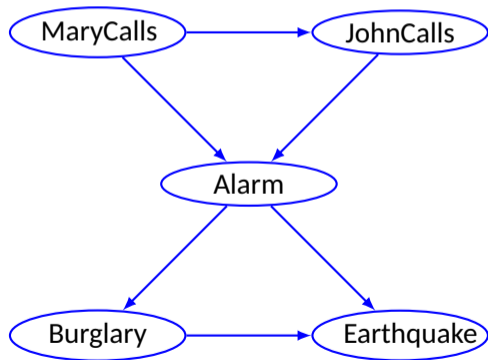
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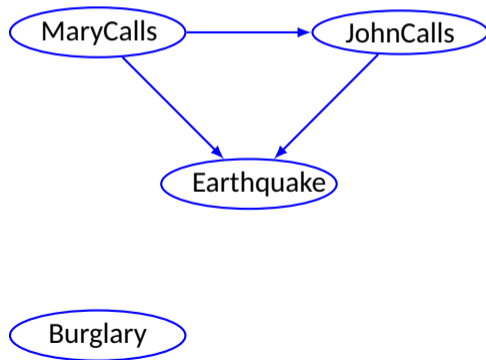
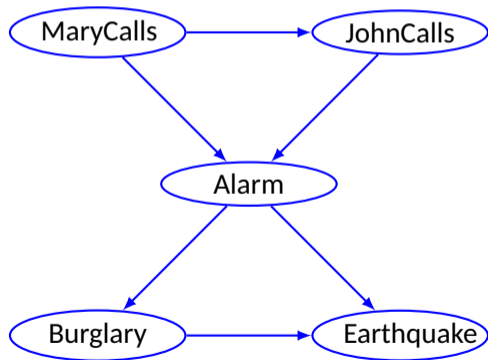
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Burglar-Alarm

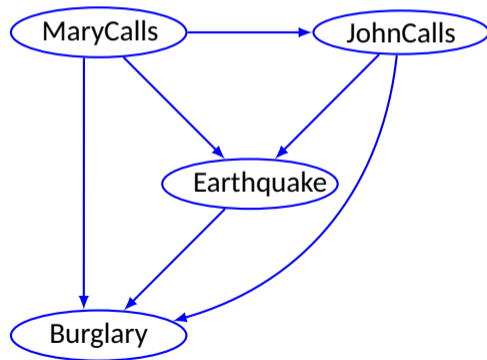
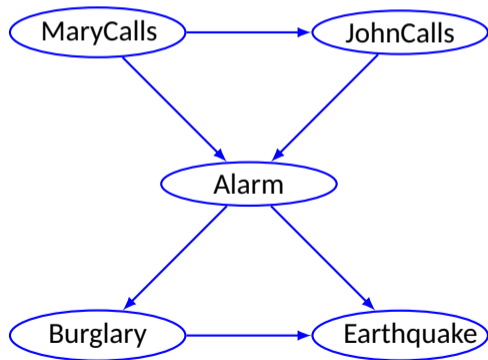
- Variable ordering
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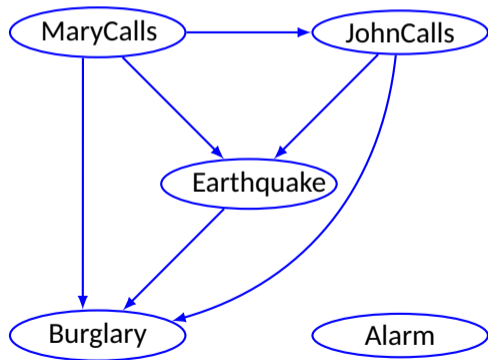
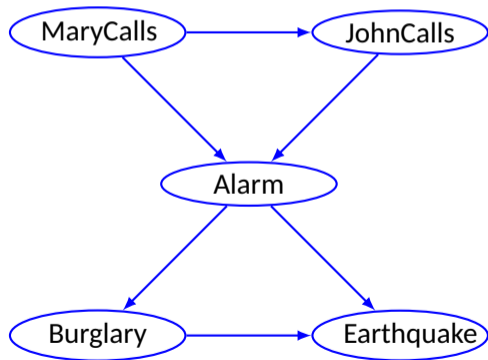
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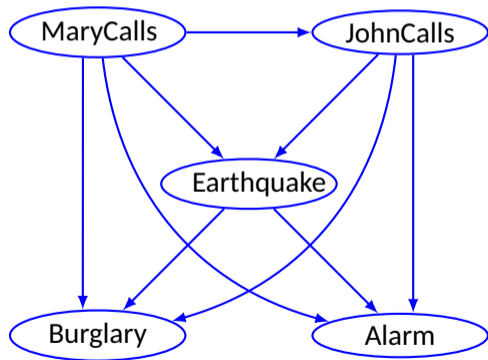
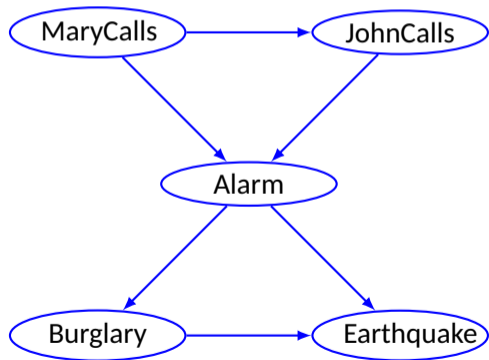
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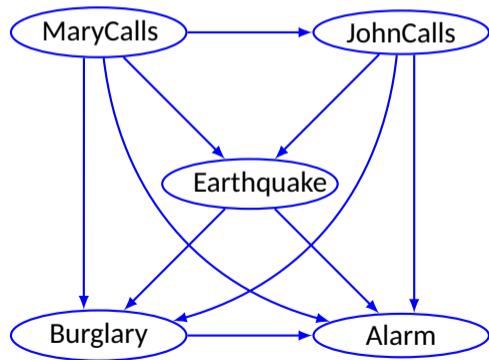
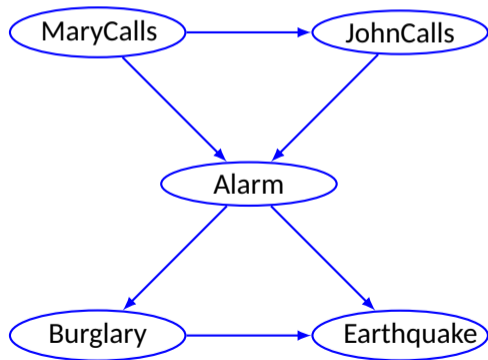
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Burglar-Alarm

- Variable ordering

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The four patterns

Q



E

Diagnostic

E



Q

Causal

E



Q



Inter Causal

E



Q

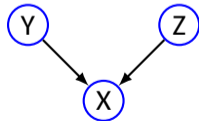
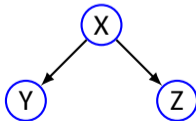


E

Mixed

Conditional independence in DAG

- Given a DAG model, how to determine conditional independent relations among nodes/variables
- DAG has three basic graphical patterns, also known as canonical networks
 - Chain - three nodes connected in a line along the same direction
 - Fork - two children of a node
 - V-structure - two parents of a node



The Bayes Ball Algorithm

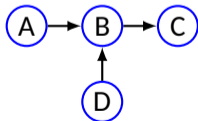
- It checks conditional independence in a network using reachability
- A ball on a node in the graph can reach other nodes by moving along edges (in both direction) according to some rules
- If the ball can reach from a start node (S) to a target node (T) with observations (shaded) (O) in the graph, then S is NOT conditionally independent of T given the O
- If the ball cannot reach from S to T then, they are conditionally independent

Rules for ball movement

- When the ball bounces into an unshaded node, then
 - If it moves in through an edge in, it can only go out of the node through an edge out
 - If it moves in through an edge out, it can go out of the node through an edge either in or out
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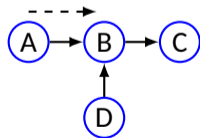
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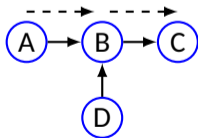
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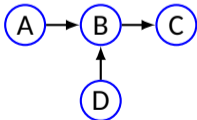
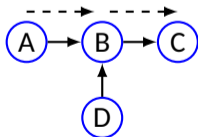
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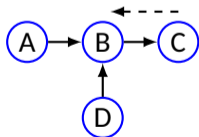
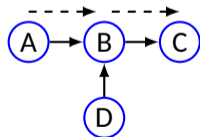
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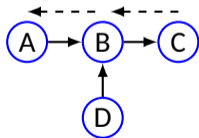
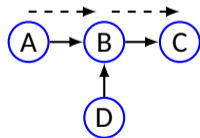
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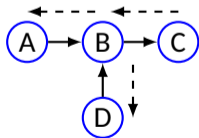
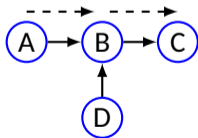
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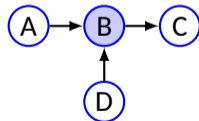
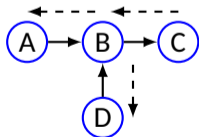
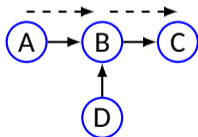
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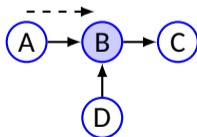
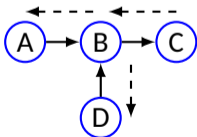
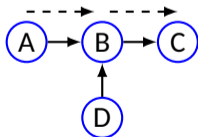
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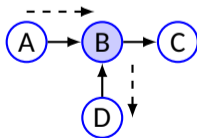
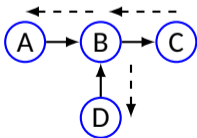
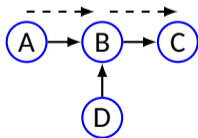
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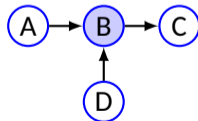
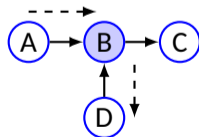
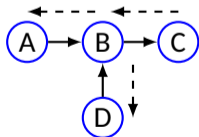
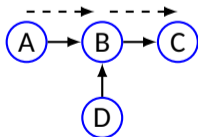
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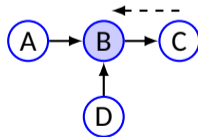
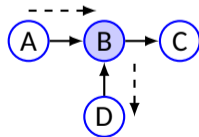
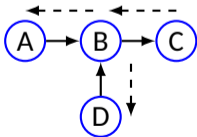
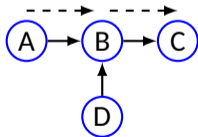
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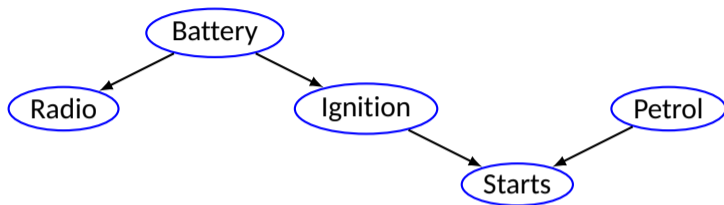


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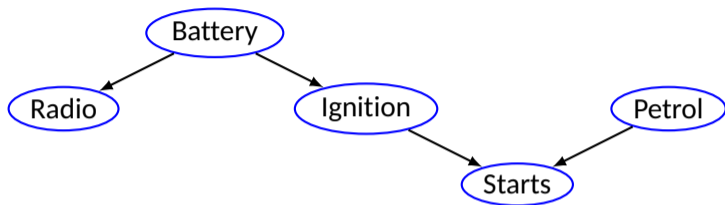
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Conditional independence in belief network

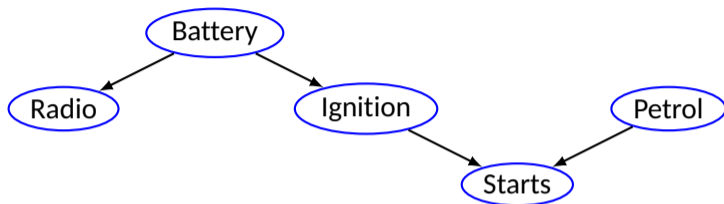


Conditional independence in belief network



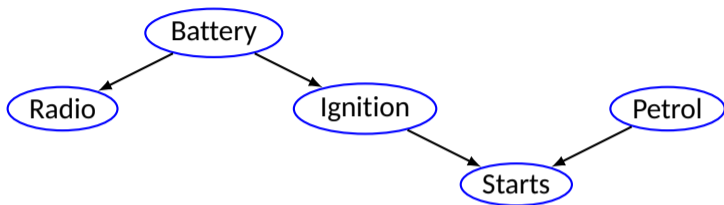
- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place

Conditional independence in belief network



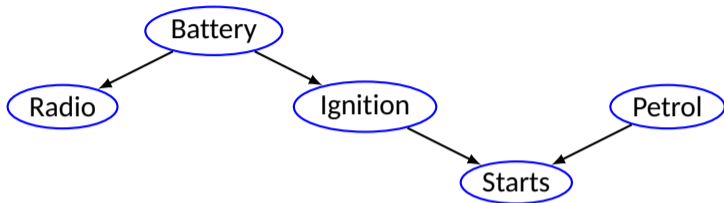
- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works

Conditional independence in belief network



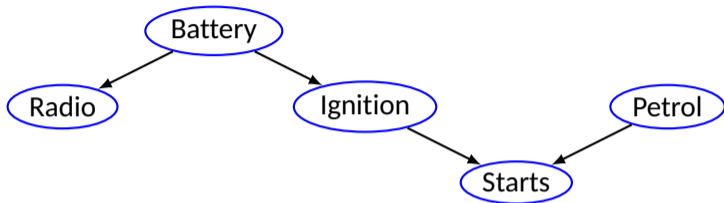
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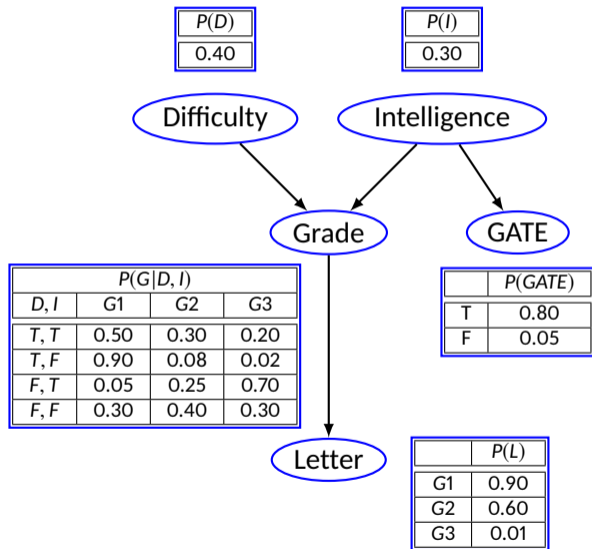
- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
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- But they are dependent given evidence about whether the car starts.

Conditional independence in belief network



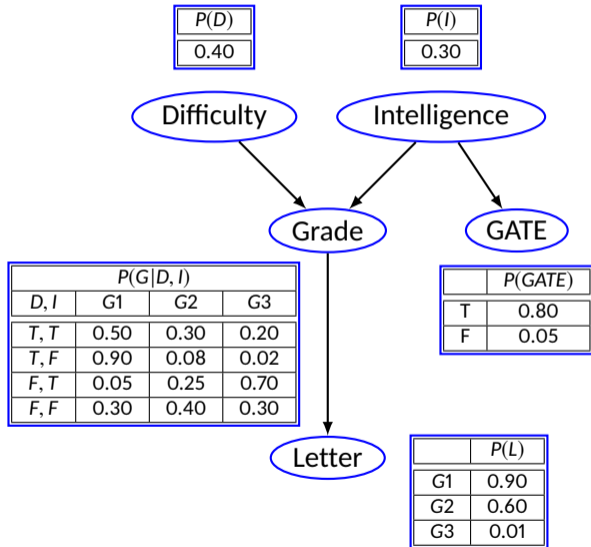
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- But they are dependent given evidence about whether the car starts.
- If the car does not start, then the radio playing is increased evidence that we are out of petrol.

Example: Grade



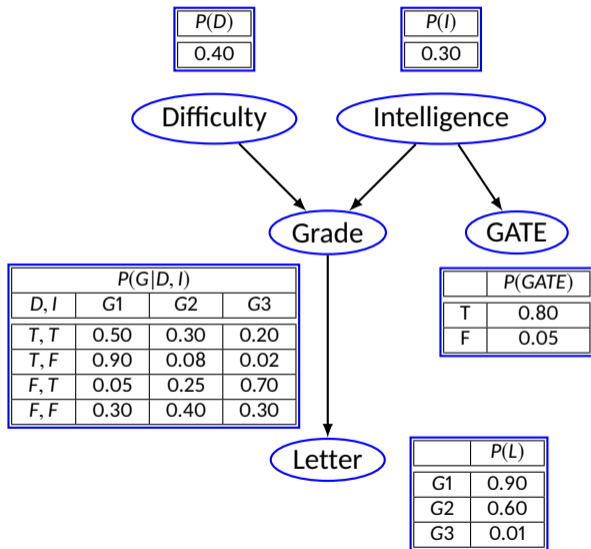
Example: Grade

- What is the probability of getting excellent recommendation letter given the semester question paper was very easy?



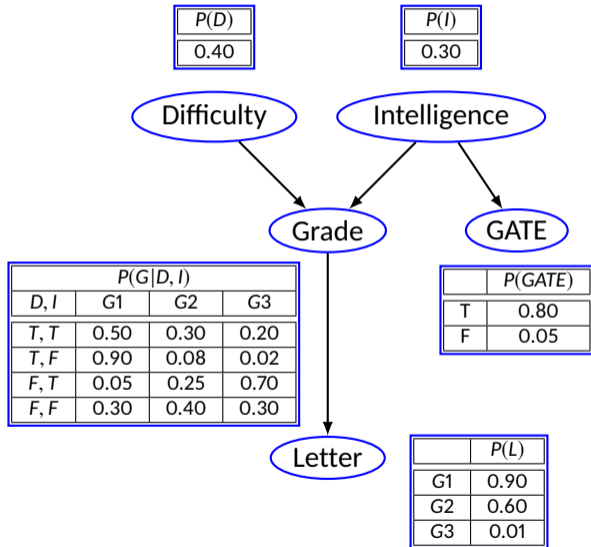
Example: Grade

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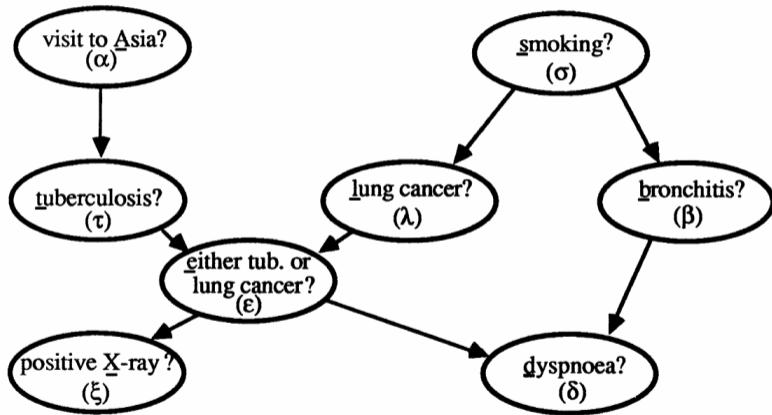
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- What is the probability of a student being very intelligent given he gets a poor recommendation letter?
- What is the probability of getting a poor GATE score given the semester grade was very good?



Belief network: medical diagnosis

- Construct a belief network for the following: [Lauritzen & Spiegelhalter, 1988]
 - Shortness-of-breath (dyspnoea) may be due to tuberculosis, lung cancer or bronchitis, or none of them, or more than one of them. A recent visit to Asia increases the chances of tuberculosis, while smoking is known to be a risk factor both lung cancer and bronchitis. The results of a single chest X-ray do not discriminate between lung cancer and tuberculosis, as neither does the presence or absence of dyspnoea.

Belief network: medical diagnosis - BN



Belief network: medical diagnosis - CPT

$$\alpha: \quad p(a) = .01$$

$$\tau: \quad p(t|a) = .05 \\ p(t|\bar{a}) = .01$$

$$\sigma: \quad p(s) = .50$$

$$\lambda: \quad p(l|s) = .10 \\ p(l|\bar{s}) = .01$$

$$\beta: \quad p(b|s) = .60 \\ p(b|\bar{s}) = .30$$

$$\varepsilon: \quad p(e|l, t) = 1$$

$$p(e|l, \bar{t}) = 1$$

$$p(e|\bar{l}, t) = 1$$

$$p(e|\bar{l}, \bar{t}) = 0$$

$$\xi: \quad p(x|e) = .98$$

$$p(x|\bar{e}) = .05$$

$$\delta: \quad p(d|e, b) = .90$$

$$p(d|e, \bar{b}) = .70$$

$$p(d|\bar{e}, b) = .80$$

$$p(d|\bar{e}, \bar{b}) = .10$$

- A patient presents at a chest clinic with dyspnoea, and has recently visited Asia
- The doctor would like to know the chance that each of the diseases is present
- Suppose information on smoking has been provided now, repeat the above step

Thank you!