Artificial Intelligence: Foundations & Applications

SAT solvers

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Introduction

- SAT is one of the central problems in computer science community that has both theoretical as well as practical challenges
- This was the first NP-Complete problem
- Wide variety of application domains - formal verification, test pattern generation, planning, scheduling, time tabling, etc.
- Provides a generic framework for combinatorial reasoning and search platform
- It is based upon propositional logic (Boolean logic)
- CSP problems can be mapped to SAT
- There exists good open-source industrial strength SAT solvers

SAT problems

- **Propositions** - $\mathcal{P} = \{a, b, c, \ldots\}$
- **Literals** - $\{a, \neg a, b, \neg b, \ldots\}$
- **Clause** - $C_1 = (a \vee b \vee \neg c), C_2 = (\neg a \vee b \vee \neg d), \ldots$
  - **Clause is disjunction of literals**
- **Formula** - $\mathcal{F} = C_1 \wedge C_2 \wedge \ldots$
  - **Conjunctive normal form (CNF)**
- **Goal is to find an assignment (interpretation) to the propositions such that $\mathcal{F}$ is** *true*
  - $\mathcal{F}$ **is satisfiable if there exists at least one valid interpretation**
  - $\mathcal{F}$ **is unsatisfiable if there exists none**

SAT tools

- Very good open-source SAT solvers are available
  - MiniSAT
  - zChaff
  - CaDiCaL
  - Glucose
  - Lingeling
  - PicoSAT
  - Cryptominisat
  - Rsat
  - Riss
  - many others
- `http://www.satcompetition.org/`

Input format - DIMACS

- **There is standard format to specify clauses and its literals**
- **To specify comments**

```
c This line is comment
```

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- To specify problem, you need to provide number of variables and number of clauses

```
c p cnf num_of_variables num_of_clauses
p cnf 3 4
```

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- To specify problem, you need to provide number of variables and number of clauses

```
c p cnf num_of_variables num_of_clauses
p cnf 3 4
```

- To specify CNF

```
c list_of_literals 0
1 -2 3 0
2 4 0
-3 0
-1 2 3 -4 0
```

Output format

- **Outputs from a SAT solver are - SATISFIABLE / UNSATISFIABLE, an assignment of Boolean variables**
- **Typically it will be as follows**

```
SAT
-1 2 -3 4 0
```

  - **The last line needs to be interpreted as follows:** $\neg a \wedge b \wedge \neg c \wedge d$
  - **There may be additional messages to provide information on resource usage, statistics, etc.**

SAT modeling: Propositional logic - 1

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- **Goal** $(\mathcal{G})$**:** b**. That is** $\mathcal{M} = (\mathcal{F} \implies \mathcal{G}) \equiv ((a \implies b) \wedge a) \implies b$

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    - If $\mathcal{M}$ is tautology then $\mathcal{F} \wedge \bar{\mathcal{G}}$ will be false ie. $\mathcal{F} \wedge \mathcal{G} = \emptyset$
    - If $\mathcal{M}$ is satisfiable then so is $\mathcal{F} \wedge \bar{\mathcal{G}}$

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```
p cnf 2 3
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- **SAT modeling**

```
p cnf 2 3
-1 2 0
1 0
-2 0
```

```
UNSATISFIABLE
```

SAT modeling: Propositional logic - 2

- **If Rajat is the Director then Rajat is well known. Rajat is not the Director. So, Rajat is not well known.**
- **Propositions:** a **: Rajat is the Director,** b **: Rajat is well known.**
- **Formula** $(\mathcal{F})$**:** $a \implies b, \neg a$
- **Goal** $(\mathcal{G})$**:** $\neg b$

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- **SAT modeling**

```
p cnf 2 3
-1 2 0
-1 0
2 0
```

```
SATISFIABLE
```

SAT modeling: Propositional logic - 3

- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**
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  - $a \implies \neg b \equiv \neg a \vee \neg b$
  - $\neg a \implies (b \wedge c)$

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  - $(\neg b \vee c) \implies d$

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  - $\neg a \implies (b \land c) \equiv (a \lor b) \land (a \lor c)$
  - $(\neg b \lor c) \implies d \equiv (b \lor d) \land (\neg c \lor d)$

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  - $d \implies e$

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- **Goal** $(\mathcal{G})$**:** a, e, d

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
```

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-1 -2 0
1 2 0
1 3 0
2 4 0
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-4 5 0
-1 0 // a -
```

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  - $d \implies e \equiv (\neg d \vee e)$
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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
```

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  - $d \implies e \equiv (\neg d \vee e)$
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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
-5 0 // e -
```

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
```

```
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
```

```
-5 0 // e - UNSAT
```

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```
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1 3 0
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-1 0 // a - SAT
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```

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-1 0 // a - SAT
-5 0 // e - UNSAT
-4 0 // d - UNSAT
```

SAT modeling: Propositional logic - 4

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
- **Target: What else did he wear?**
- **Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie**

SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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- **Formula ($\mathcal{F}$):**
  - $\neg bs \lor \neg ts$

SAT modeling: Propositional logic - 4

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- **Formula ($\mathcal{F}$):**
  - $\neg bs \vee \neg ts$
  - $ts \vee s$

SAT modeling: Propositional logic - 4

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- **Formula ($\mathcal{F}$):**
  - $\neg bs \vee \neg ts$
  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t$

SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
  - $ts \implies (ps \vee s)$

SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps$

SAT modeling: Propositional logic - 4

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  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$

SAT modeling: Propositional logic - 4

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  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$
  - t

SAT modeling: Propositional logic - 4

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  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$
  - t
- **Goal $(\mathcal{G})$: All satisfying solutions**

SAT modeling: Propositional logic - 4

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  - $s \implies ps \equiv (\neg s \vee ps)$
  - t
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- **Formula** $(\mathcal{F})$:
  - $\neg bs \lor \neg ts$
  - $ts \lor s$
  - $(ts \land ps) \implies \neg t \equiv (\neg ts \lor \neg ps \lor \neg t)$
  - $ts \implies (ps \lor s) \equiv (\neg ts \lor ps \lor s)$
  - $s \implies ps \equiv (\neg s \lor ps)$
  - t
- **Goal** $(\mathcal{G})$: **All satisfying solutions**
- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
```

SAT modeling: Propositional logic - 4

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
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  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$
  - t
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- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

SAT modeling: Propositional logic - 4

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
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  - $s \implies ps \equiv (\neg s \vee ps)$
  - t
- **Goal $(\mathcal{G})$: All satisfying solutions**
- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
```

SAT modeling: Propositional logic - 4

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  - t
- **Goal** $(\mathcal{G})$**: All satisfying solutions**
- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
```

SAT modeling: Propositional logic - 4

- Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.
- Target: What else did he wear?
- Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie
- Formula $(\mathcal{F})$:
  - $\neg bs \vee \neg ts$
  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$
  - t
- Goal $(\mathcal{G})$: All satisfying solutions
- SAT modeling

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
Add:  -1 2 -3 -4 -5 0
```

SAT modeling: Propositional logic - 4

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  - $s \implies ps \equiv (\neg s \lor ps)$
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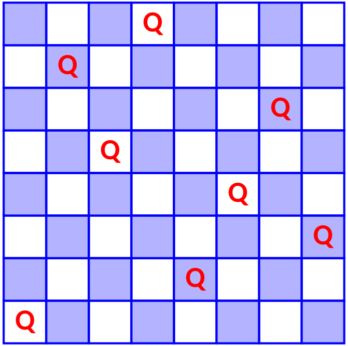
- **Goal** ($\mathcal{G}$): **All satisfying solutions**
- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
```

```
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

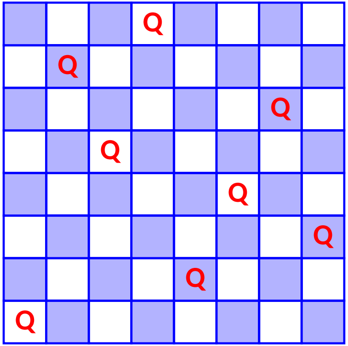
```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
Add:  -1 2 -3 -4 -5 0
UNSAT
```

SAT modeling: 8-queens



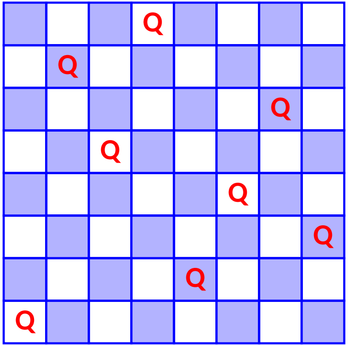
SAT modeling: 8-queens

- **Define x_{ij} as (i, j)th cell contains a queen**



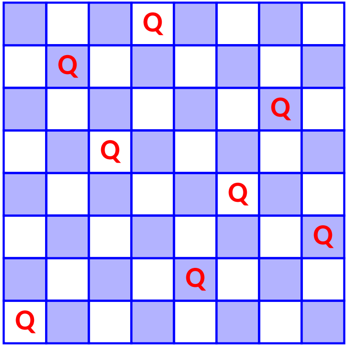
SAT modeling: 8-queens

- Define x_{ij} as (i, j)th cell contains a queen
- Constraints



SAT modeling: 8-queens

- **Define x_{ij} as (i, j)th cell contains a queen**
- **Constraints**
  - $x_{ij} \implies \neg x_{ij'}$ **(row)**
  - $x_{ij} \implies \neg x_{i'j}$ **(column)**
  - $x_{ij} \implies \neg x_{(i+k)(j+k)}$ **(diagonal)**
  - $x_{ij} \implies \neg x_{(i+k)(j-k)}$ **(diagonal)**
  - $\bigvee_i x_{ij}$ **(column)**
  - $\bigvee_j x_{ij}$ **(row)**



SAT modeling: Sudoku

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i,j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i,k,j \neq j'$ **(same row)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
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  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as (i, j)th cell contains k
- Constraints:
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block
  - $\bigvee_j x_{ijk} \quad \forall i, k$ (row)

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j, k$ **(column)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_{j} x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_{i} x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define** x_{ijk} **as** (i, j)**th cell contains** k
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_{j} x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_{i} x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ **(same cell)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ **(same cell)**
  - $\bigvee_k x_{ijk} \quad \forall i, j$ **(every cell)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as (i, j)th cell contains k
- Constraints:
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
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  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block
  - $\bigvee_j x_{ijk} \quad \forall i, k$ (row)
  - $\bigvee_i x_{ijk} \quad \forall j, k$ (column)
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ (same cell)
  - $\bigvee_k x_{ijk} \quad \forall i, j$ (every cell)
  - $x_{133}, x_{176}, \ldots$

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Langford sequence

- **Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are i numbers between the two occurrences of i?**
  - $L(4)$ = 41312432
  - $L(3)$ = **?**

SAT modeling: Langford sequence

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  - $L(4) = 41312432$
  - $L(3) =$ **?**

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

SAT modeling: Langford sequence

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  - $L(4)$ = 41312432
  - $L(3)$ = **?**

- **Constraints:**
  - $x_1 \vee x_2 \vee x_3 \vee x_4$
  - $x_k \implies \neg x_{k'} \quad 1 \le k < k' \le 4$
  - **Similarly for the other numbers**

  - $x_1 \vee x_5 \vee x_8$
  - $x_5 \implies \neg x_5, \ldots$
  - **Similarly for the other columns**

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

SAT modeling: Electric vehicle charge scheduling

- **Consider a parking area that has the facility to charge electric vehicles. There are m number of ports to charge the vehicles. Each port can charge one vehicle at a time. Let us assume that there are n number of vehicles. Each vehicle has an arrival time (a_i) in the parking area and an departure time (d_i). While the vehicle is in the parking area, it needs to be chaged uninterruptedly for a given duration (e_i). Given a set of vehicles and their arrival and departure time, does there exist a schedule such that each vehicle can be charged for its stipulated duration while it is in the parking area? A sample input will look as follows – $m = 10$ and**

Vehicle	Arrival time	Departure time	Charging time
1	4	10	3
2	7	20	6
3	8	27	10
$\vdots$	$\vdots$	$\vdots$	$\vdots$

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3	8	27	10
⋮	⋮	⋮	⋮

  - Develop a SAT based formulation to model the problem

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  - Can there be other encoding schemes to model the problem?

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  - Which encoding scheme is better?

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1	4	10	3
2	7	20	6
3	8	27	10
⋮	⋮	⋮	⋮

  - Develop a SAT based formulation to model the problem
  - Can there be other encoding schemes to model the problem?
  - Which encoding scheme is better?
  - Explore the performance of different encoding schemes using various solvers.

Thank you!