Artificial Intelligence: Foundations & Applications

SAT solvers

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Introduction

- SAT is one of the central problems in computer science community that has both theoretical as well as practical challenges
- This was the first NP-Complete problem
- Wide variety of application domains - formal verification, test pattern generation, planning, scheduling, time tabling, etc.
- Provides a generic framework for combinatorial reasoning and search platform
- It is based upon propositional logic (Boolean logic)
- CSP problems can be mapped to SAT
- There exists good open-source industrial strength SAT solvers

SAT problems

- **Propositions -** $\mathcal{P} = \{a, b, c, \ldots\}$
- **Literals -** $\{a, \neg a, b, \neg b, \ldots\}$
- **Clause -** $C_1 = (a \vee b \vee \neg c), C_2 = (\neg a \vee b \vee \neg d), \ldots$
  - **Clause is disjunction of literals**
- **Formula -** $\mathcal{F} = C_1 \wedge C_2 \wedge \ldots$
  - **Conjunctive normal form (CNF)**
- **Goal is to find an assignment (interpretation) to the propositions such that $\mathcal{F}$ is** *true*
  - **$\mathcal{F}$ is satisfiable if there exists at least one valid interpretation**
  - **$\mathcal{F}$ is unsatisfiable if there exists none**

SAT tools

- Very good open-source SAT solvers are available
  - MiniSAT
  - zChaff
  - CaDiCaL
  - Glucose
  - Lingeling
  - PicoSAT
  - Cryptominisat
  - Rsat
  - Riss
  - many others
- `http://www.satcompetition.org/`

Input format - DIMACS

- **There is standard format to specify clauses and its literals**
- **To specify comments**

```
c This line is comment
```

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- **To specify problem, you need to provide number of variables and number of clauses**

```
c p cnf num_of_variables num_of_clauses
p cnf 3 4
```

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```
c p cnf num_of_variables num_of_clauses
p cnf 3 4
```

- To specify CNF

```
c list_of_literals 0
1 -2 3 0
2 4 0
-3 0
-1 2 3 -4 0
```

Output format

- **Outputs from a SAT solver are - SATISFIABLE / UNSATISFIABLE, an assignment of Boolean variables**
- **Typically it will be as follows**

```
SAT
-1 2 -3 4 0
```

- **The last line needs to be interpreted as follows:** $\neg a \wedge b \wedge \neg c \wedge d$
- **There may be additional messages to provide information on resource usage, statistics, etc.**

SAT modeling: Propositional logic - 1

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- **Formula** $(\mathcal{F})$**:** $a \implies b, a$
- **Goal** $(\mathcal{G})$**:** b**. That is** $\mathcal{M} = (\mathcal{F} \implies \mathcal{G}) \equiv ((a \implies b) \land a) \implies b$

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- Goal $(\mathcal{G})$: b. That is $\mathcal{M} = (\mathcal{F} \implies \mathcal{G}) \equiv ((a \implies b) \wedge a) \implies b$
  - If $\mathcal{M}$ is tautology then $\mathcal{F} \wedge \bar{\mathcal{G}}$ will be false ie. $\mathcal{F} \wedge \mathcal{G} = \emptyset$
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```
p cnf 2 3
-1 2 0
1 0
-2 0

UNSATISFIABLE
```

SAT modeling: Propositional logic - 2

- **If Rajat is the Director then Rajat is well known. Rajat is not the Director. So, Rajat is not well known.**
- **Propositions:** a **: Rajat is the Director,** b **: Rajat is well known.**
- **Formula** $(\mathcal{F})$: $a \implies b, \neg a$
- **Goal** $(\mathcal{G})$: $\neg b$

SAT modeling: Propositional logic - 2

- If Rajat is the Director then Rajat is well known. Rajat is not the Director. So, Rajat is not well known.
- Propositions: a : Rajat is the Director, b : Rajat is well known.
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- Formula $(\mathcal{F})$: $a \implies b, \neg a$
- Goal $(\mathcal{G})$: $\neg b$
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```
p cnf 2 3
-1 2 0
-1 0
2 0
```

```
SATISFIABLE
```

SAT modeling: Propositional logic - 3

- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**
- **Propositions:** a**-mythical,** b**-mortal,** c**-mammal,** d**-horned,** e**-magical**

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  - $a \implies \neg b$

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- Formula $(\mathcal{F})$:
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- Propositions: a-mythical, b-mortal, c-mammal, d-horned, e-magical
- Formula $(\mathcal{F})$:
  - $a \implies \neg b \equiv \neg a \lor \neg b$
  - $\neg a \implies (b \land c)$

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  - $\neg a \implies (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$

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  - $(\neg b \vee c) \implies d$

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  - $d \implies e$

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
```

SAT modeling: Propositional logic - 3

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
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-4 5 0
-1 0 // a -
```

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
```

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
-5 0 // e -
```

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
-5 0 // e - UNSAT
```

SAT modeling: Propositional logic - 3

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```
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-4 5 0
-1 0 // a - SAT
-5 0 // e - UNSAT
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-1 -2 0
1 2 0
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2 4 0
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-1 0 // a - SAT
-5 0 // e - UNSAT
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```

SAT modeling: Propositional logic - 4

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
- **Target: What else did he wear?**
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  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t$

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  - $s \implies ps$

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SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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- **Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie**
- **Formula ($\mathcal{F}$):**
  - $\neg bs \lor \neg ts$
  - $ts \lor s$
  - $(ts \land ps) \implies \neg t \equiv (\neg ts \lor \neg ps \lor \neg t)$
  - $ts \implies (ps \lor s) \equiv (\neg ts \lor ps \lor s)$
  - $s \implies ps \equiv (\neg s \lor ps)$
  - t
- **Goal ($\mathcal{G}$): All satisfying solutions**
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  - t
- Goal $(\mathcal{G})$: All satisfying solutions
- SAT modeling

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
```

SAT modeling: Propositional logic - 4

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

SAT modeling: Propositional logic - 4

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
```

SAT modeling: Propositional logic - 4

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
```

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
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```

SAT modeling: Propositional logic - 4

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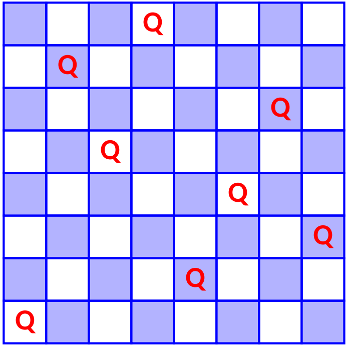
```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

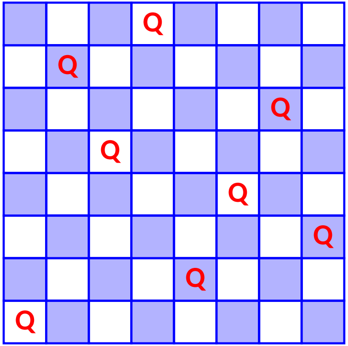
```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
Add:  -1 2 -3 -4 -5 0
UNSAT
```

SAT modeling: 8-queens



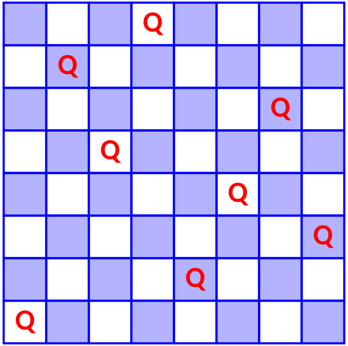
SAT modeling: 8-queens

- **Define x_{ij} as (i, j)th cell contains a queen**



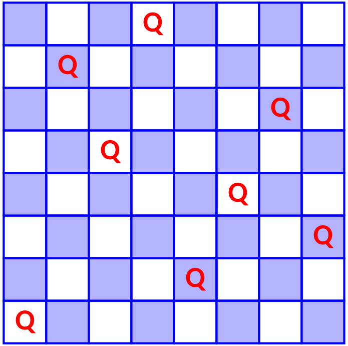
SAT modeling: 8-queens

- Define x_{ij} as (i, j)th cell contains a queen
- Constraints



SAT modeling: 8-queens

- **Define x_{ij} as (i, j)th cell contains a queen**
- **Constraints**
  - $x_{ij} \implies \neg x_{ij'}$ **(row)**
  - $x_{ij} \implies \neg x_{i'j}$ **(column)**
  - $x_{ij} \implies \neg x_{(i+k)(j+k)}$ **(diagonal)**
  - $x_{ij} \implies \neg x_{(i+k)(j-k)}$ **(diagonal)**
  - $\bigvee_i x_{ij}$ **(column)**
  - $\bigvee_j x_{ij}$ **(row)**



SAT modeling: Sudoku

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- **Define x_{ijk} as (i, j)th cell contains k**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
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		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
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  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
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  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
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		5		1		3		

SAT modeling: Sudoku

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  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j, k$ **(column)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

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  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i,k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j,k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
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  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i,k,j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j,k,i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i,i',j,j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i,k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j,k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i,j,k \neq k'$ **(same cell)**

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as (i, j)th cell contains k
- Constraints:
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block
  - $\bigvee_{j} x_{ijk} \quad \forall i, k$ (row)
  - $\bigvee_{i} x_{ijk} \quad \forall j, k$ (column)
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ (same cell)
  - $\bigvee_{k} x_{ijk} \quad \forall i, j$ (every cell)

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

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  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ **(same cell)**
  - $\bigvee_k x_{ijk} \quad \forall i, j$ **(every cell)**
  - $x_{133}, x_{176}, \ldots$

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Langford sequence

- **Given the bag of numbers** $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$**, can they be arranged in a sequence** $L(n)$ **such that for** $1 \leq i \leq n$ **there are** i **numbers between the two occurrences of** i**?**
  - $L(4)$ = 41312432
  - $L(3)$ = **?**

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- **Given the bag of numbers** $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$**, can they be arranged in a sequence** $L(n)$ **such that for** $1 \le i \le n$ **there are** i **numbers between the two occurrences of** i**?**
  - $L(4)$ = 41312432
  - $L(3)$ = **?**

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

SAT modeling: Langford sequence

- **Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are i numbers between the two occurrences of i?**
  - $L(4)$ = 41312432
  - $L(3)$ = **?**
- **Constraints:**
  - $x_1 \lor x_2 \lor x_3 \lor x_4$
  - $x_k \implies \neg x_{k'} \quad 1 \leq k < k' \leq 4$
  - **Similarly for the other numbers**

  - $x_1 \lor x_5 \lor x_8$
  - $x_1 \implies \neg x_5, \ldots$
  - **Similarly for the other columns**

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

Thank you!