# Artificial Intelligence: Foundations & Applications

## SAT solvers


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# Introduction

- SAT is one of the central problems in computer science community that has both theoretical as well as practical challenges
- This was the first NP-Complete problem
- Wide variety of application domains - formal verification, test pattern generation, planning, scheduling, time tabling, etc.
- Provides a generic framework for combinatorial reasoning and search platform
- It is based upon propositional logic (Boolean logic)
- CSP problems can be mapped to SAT
- There exists good open-source industrial strength SAT solvers

# SAT problems

- **Propositions** - $\mathcal{P} = \{a, b, c, \ldots\}$
- **Literals** - $\{a, \neg a, b, \neg b, \ldots\}$
- **Clause** - $C_1 = (a \vee b \vee \neg c), C_2 = (\neg a \vee b \vee \neg d), \ldots$
  - Clause is disjunction of literals
- **Formula** - $\mathcal{F} = C_1 \wedge C_2 \wedge \ldots$
  - Conjunctive normal form (CNF)
- **Goal is to find an assignment (interpretation) to the propositions such that $\mathcal{F}$ is *true***
  - $\mathcal{F}$ is satisfiable if there exists at least one valid interpretation
  - $\mathcal{F}$ is unsatisfiable if there exists none

# SAT tools

- Very good open-source SAT solvers are available
  - MiniSAT
  - zChaff
  - CaDiCaL
  - Glucose
  - Lingeling
  - PicoSAT
  - Cryptominisat
  - Rsat
  - Riss
  - many others
- http://www.satcompetition.org/

# Input format - DIMACS

- There is standard format to specify clauses and its literals
- To specify comments

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p cnf 3 4
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- **To specify problem, you need to provide number of variables and number of clauses**

```
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```

- **To specify CNF**

```
c list_of_literals 0
1 -2 3 0
2 4 0
-3 0
-1 2 3 -4 0
```

# Output format

- Outputs from a SAT solver are - SATISFIABLE / UNSATISFIABLE, an assignment of Boolean variables
- Typically it will be as follows

```
SAT
-1 2 -3 4 0
```

  - The last line needs to be interpreted as follows: $\neg a \wedge b \wedge \neg c \wedge d$
  - There may be additional messages to provide information on resource usage, statistics, etc.

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  - If $\mathcal{M}$ is tautology then $\mathcal{F} \wedge \bar{\mathcal{G}}$ will be false ie. $\mathcal{F} \wedge \mathcal{G} = \emptyset$
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  $\neg a \lor b$
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```
p cnf 2 3
-1 2 0
1 0
-2 0
```

```
UNSATISFIABLE
```

$(\neg a \vee b) \wedge a \wedge \neg b$

$b \wedge a \wedge \neg b = \phi$

# SAT modeling: Propositional logic - 2

- If Rajat is the Director then Rajat is well known. Rajat is not the Director. So, Rajat is not well known.
- Propositions: $a$ : Rajat is the Director, $b$ : Rajat is well known.
- Formula $(\mathcal{F})$: $a \implies b, \neg a$
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- Formula $(\mathcal{F})$: $a \implies b, \neg a$
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```
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```
SATISFIABLE
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- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**
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  - $a \implies \neg b \equiv \neg a \lor \neg b$
  - $\neg a \implies (b \land c)$

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```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
```

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-4 5 0
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```

# SAT modeling: Propositional logic - 3

- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**
- **Propositions:** $a$**-mythical,** $b$**-mortal,** $c$**-mammal,** $d$**-horned,** $e$**-magical**
- **Formula** $(\mathcal{F})$**:**
  - $a \implies \neg b \equiv \neg a \vee \neg b$
  - $\neg a \implies (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$
  - $(\neg b \vee c) \implies d \equiv (b \vee d) \wedge (\neg c \vee d)$
  - $d \implies e \equiv (\neg d \vee e)$
- **Goal** $(\mathcal{G})$**:** $a, e, d$
- **SAT modeling**

```
p cnf 5 7
-1 -2 0
1 2 0
1 3 0
2 4 0
-3 4 0
-4 5 0
-1 0 // a - SAT
-5 0 // e - UNSAT
-4 0 // d - UNSAT
```

# SAT modeling: Propositional logic - 4

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
- **Target: What else did he wear?**
- **Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie**

# SAT modeling: Propositional logic - 4

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- **Formula** $(\mathcal{F})$**:**

# SAT modeling: Propositional logic - 4

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- **Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie**
- **Formula $(\mathcal{F})$:**
  - $\neg bs \vee \neg ts$

# SAT modeling: Propositional logic - 4

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- **Formula ($\mathcal{F}$):**
  - $\neg bs \vee \neg ts$
  - $ts \vee s$

# SAT modeling: Propositional logic - 4

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- **Target: What else did he wear?**
- **Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie**
- **Formula $(\mathcal{F})$:**
  - $\neg bs \lor \neg ts$
  - $ts \lor s$
  - $(ts \land ps) \implies \neg t$

# SAT modeling: Propositional logic - 4

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  - $ts \vee s$
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  - $ts \vee s$
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  - $ts \implies (ps \vee s)$

# SAT modeling: Propositional logic - 4

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  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps$

# SAT modeling: Propositional logic - 4

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  - $s \implies ps \equiv (\neg s \vee ps)$
  - $t$

# SAT modeling: Propositional logic - 4

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- Target: What else did he wear?
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  - $ts \vee s$
  - $(ts \wedge ps) \implies \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
  - $ts \implies (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
  - $s \implies ps \equiv (\neg s \vee ps)$
  - $t$
- Goal ($\mathcal{G}$): All satisfying solutions

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  - $\neg bs \lor \neg ts$
  - $ts \lor s$
  - $(ts \land ps) \implies \neg t \equiv (\neg ts \lor \neg ps \lor \neg t)$
  - $ts \implies (ps \lor s) \equiv (\neg ts \lor ps \lor s)$
  - $s \implies ps \equiv (\neg s \lor ps)$
  - $t$
- **Goal ($\mathcal{G}$): All satisfying solutions**
- **SAT modeling**

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  - $s \implies ps \equiv (\neg s \vee ps)$
  - $t$
- **Goal $(\mathcal{G})$: All satisfying solutions**
- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
```

# SAT modeling: Propositional logic - 4

- Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.
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- SAT modeling

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

# SAT modeling: Propositional logic - 4

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
```

# SAT modeling: Propositional logic - 4

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- SAT modeling

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
```

# SAT modeling: Propositional logic - 4

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```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
Add:  -1 2 -3 -4 -5 0
```

# SAT modeling: Propositional logic - 4

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  - $s \implies ps \equiv (\neg s \vee ps)$
  - $t$
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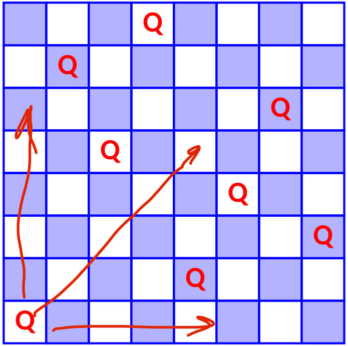
- SAT modeling

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
SAT: -1 -2 3 4 5 0
```

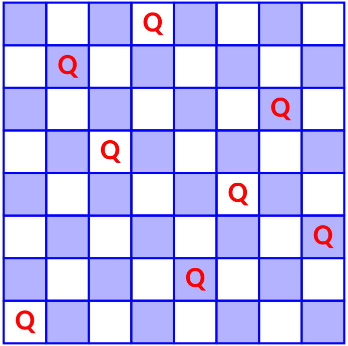
```
Add:  1 2 -3 -4 -5 0
SAT: 1 -2 3 4 5 0
Add:  -1 2 -3 -4 -5 0
UNSAT
```

# SAT modeling: 8-queens



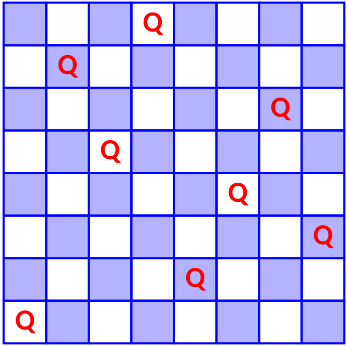
# SAT modeling: 8-queens

- Define $x_{ij}$ as $(i, j)$th cell contains a queen



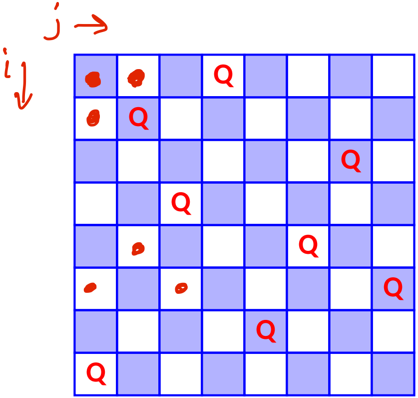
# SAT modeling: 8-queens

- **Define $x_{ij}$ as $(i, j)$th cell contains a queen**
- **Constraints**



# SAT modeling: 8-queens

- **Define $x_{ij}$ as $(i, j)$th cell contains a queen**
- **Constraints**
  - $x_{ij} \implies \neg x_{ij'}$ **(row)**
  - $x_{ij} \implies \neg x_{i'j}$ **(column)**
  - $x_{ij} \implies \neg x_{(i+k)(j+k)}$ **(diagonal)**
  - $x_{ij} \implies \neg x_{(i+k)(j-k)}$ **(diagonal)**
  - $\bigvee_i x_{ij}$ **(column)**
  - $\bigvee_j x_{ij}$ **(row)**



# SAT modeling: Sudoku

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- Define $x_{ijk}$ as $(i, j)$th cell contains $k$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- Define $x_{ijk}$ as $(i, j)$th cell contains $k$
- Constraints:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define $x_{ijk}$ as $(i, j)$th cell contains $k$**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define** $x_{ijk}$ **as** $(i, j)$**th cell contains** $k$
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define $x_{ijk}$ as $(i, j)$th cell contains $k$**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- Define $x_{ijk}$ as $(i, j)$th cell contains $k$
- Constraints:
  - $x_{ijk} \implies \neg x_{ij'k}$ $\forall i, k, j \neq j'$ (same row)
  - $x_{ijk} \implies \neg x_{i'jk}$ $\forall j, k, i \neq i'$ (same column)
  - $x_{ijk} \implies \neg x_{i'j'k}$ $\forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block
  - $\bigvee_j x_{ijk}$ $\forall i, k$ (row)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define** $x_{ijk}$ **as** $(i, j)$**th cell contains** $k$
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_j x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_i x_{ijk} \quad \forall j, k$ **(column)**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define $x_{ijk}$ as $(i, j)$th cell contains $k$**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_{j} x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_{i} x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- Define $x_{ijk}$ as $(i, j)$th cell contains $k$
- Constraints:
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block
  - $\bigvee_j x_{ijk} \quad \forall i, k$ (row)
  - $\bigvee_i x_{ijk} \quad \forall j, k$ (column)
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ (same cell)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

$x_{111}$ $x_{112}$

# SAT modeling: Sudoku

- **Define $x_{ijk}$ as $(i, j)$th cell contains $k$**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k} \quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk} \quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_{j} x_{ijk} \quad \forall i, k$ **(row)**
  - $\bigvee_{i} x_{ijk} \quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk} \quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'} \quad \forall i, j, k \neq k'$ **(same cell)**
  - $\bigvee_{k} x_{ijk} \quad \forall i, j$ **(every cell)**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Sudoku

- **Define $x_{ijk}$ as $(i, j)$th cell contains $k$**
- **Constraints:**
  - $x_{ijk} \implies \neg x_{ij'k}$ $\quad \forall i, k, j \neq j'$ **(same row)**
  - $x_{ijk} \implies \neg x_{i'jk}$ $\quad \forall j, k, i \neq i'$ **(same column)**
  - $x_{ijk} \implies \neg x_{i'j'k}$ $\quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ **- every block**
  - $\bigvee_{j} x_{ijk}$ $\quad \forall i, k$ **(row)**
  - $\bigvee_{i} x_{ijk}$ $\quad \forall j, k$ **(column)**
  - $\bigvee_{i,j} x_{ijk}$ $\quad \forall k$ **every block**
  - $x_{ijk} \implies \neg x_{ijk'}$ $\quad \forall i, j, k \neq k'$ **(same cell)**
  - $\bigvee_{k} x_{ijk}$ $\quad \forall i, j$ **(every cell)**
  - $x_{133}, x_{176}, \ldots$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | | 2 | | 6 | | |
| 9 | | | 3 | | 5 | | | 1 |
| | | 1 | 8 | | 6 | 4 | | |
| | | 8 | 1 | | 2 | 9 | | |
| 7 | | | | | | | | 8 |
| | | 6 | 7 | | 8 | 2 | | |
| | | 2 | 6 | | 9 | 5 | | |
| 8 | | | 2 | | 3 | | | 9 |
| | | 5 | | 1 | | 3 | | |

# SAT modeling: Langford sequence

- Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are $i$ numbers between the two occurrences of $i$?
  - $L(4)$ = 41312432
  - $L(3)$ = ?

# SAT modeling: Langford sequence

- **Given the bag of numbers** $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$**, can they be arranged in a sequence** $L(n)$ **such that for** $1 \leq i \leq n$ **there are** $i$ **numbers between the two occurrences of** $i$**?**
  - $L(4)$ = 41312432
  - $L(3)$ = **?**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | | 1 | | | |
| $x_2$ | | 1 | | 1 | | |
| $x_3$ | | | 1 | | 1 | |
| $x_4$ | | | | 1 | | 1 |
| $x_5$ | 2 | | | 2 | | |
| $x_6$ | | 2 | | | 2 | |
| $x_7$ | | | 2 | | | 2 |
| $x_8$ | 3 | | | | 3 | |
| $x_9$ | | 3 | | | | 3 |

# SAT modeling: Langford sequence

- **Given the bag of numbers** $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$**, can they be arranged in a sequence** $L(n)$ **such that for** $1 \leq i \leq n$ **there are** $i$ **numbers between the two occurrences of** $i$**?**
  - $L(4) = 41312432$
  - $L(3) = ?$
- **Constraints:**
  - $x_1 \vee x_2 \vee x_3 \vee x_4$
  - $x_k \implies \neg x_{k'} \quad 1 \leq k < k' \leq 4$
  - **Similarly for the other numbers**
  - $x_1 \vee x_5 \vee x_8$
  - $x_1 \implies \neg x_5, \ldots$
  - **Similarly for the other columns**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $x_1$ | 1 | | 1 | | | |
| $x_2$ | | 1 | | 1 | | |
| $x_3$ | | | 1 | | 1 | |
| $x_4$ | | | | 1 | | 1 |
| $x_5$ | 2 | | | 2 | | |
| $x_6$ | | 2 | | | 2 | |
| $x_7$ | | | 2 | | | 2 |
| $x_8$ | 3 | | | | 3 | |
| $x_9$ | | 3 | | | | 3 |

Thank you!