LOGICAL DEDUCTION IN AI

PROPOSITIONAL LOGIC

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Logic in Ancient Times

Indic

Geometry, Calculations
Nyaya, Vaisisekha
Theory of Argumentation
Sanskrit language with Binary-Level arguments
Logical Argumentation: Chatustoki
Buddhist and Jain Philosophies
Formal Systems
Vedanta

China

Confucious, Mozi,
Master Mo (Mohist School)
Basic Formal Systems
Buddhist Systems from India

Greek

Theorems

Thales, Pythagoras (Propositions and Geometry)
Heraclitus, Parmenides (Logos)
Plato (Logic beyond Geometry)
Aristotle (Syllogism, Syntax)
Stoics

Middle East

Ancient Egypt, Babylon
Arab (Avisennian Logic)
Inductive Logic

Medieval Europe

Post Aristotle
Precursor to First Order Logic

Today

Propositional
Predicate
Higher Order
Logic, Numbers & Computation
Psychology
Philosophy

Circuits
Networks
Brain/Neural Networks
PROPOSITIONAL

First Few Examples

Propositional
Boolean

- If I am the President then I am well-known. I am the President. So I am well-known ✓
- If I am the President then I am well-known. I am not the President. So I am not well-known. Not correct
- If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well known. ✓
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

Deduction Using Propositional Logic: Steps

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

Boolean Formulae developed using well defined connectors ~, ∧, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of Sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a Combined Formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Deduction Using Propositional Logic: Example 1

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$ $\quad a \rightarrow b \equiv \neg a \vee b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is:

$((\underbrace{a \rightarrow b}_{F1}) \wedge \underbrace{a}_{F2}) \rightarrow \underbrace{b}_{G}$

Deduction Using Propositional Logic: Example 1

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: a → b

F2: a

G: b

The final formula for deduction: (F1 ∧ F2) → G, that is: ((a → b) ∧ a) → b

Truth Table Method

a, b 4 possible combinations

Tautology

a	b	a → b	(a → b) ∧ a	((a → b) ∧ a) → b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Interpretation

Deduction Using Propositional Logic: Example 2

Boolean variables a, b, c, d, … which can take values true or false.

Boolean formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If I am the President then I am well-known. I am not the President. So I am not well-known

Coding: Variables

a: I am the President

b: I am well-known

$a = $ False

$b = $ True

Coding the sentences:

F1: a → b

F2: ~a

G: ~b

The final formula for deduction: (F1 Λ F2) → G, that is: ((a → b) Λ ~a) → ~b

Truth Table

a	b	a → b	(a → b) Λ ~a	((a → b) Λ ~a) → ~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Deduction Using Propositional Logic: Example 3

If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is: $((a \rightarrow b) \wedge a) \rightarrow b$

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well known

Coding: Variables

a: Rajat is the President ✓

b: Rajat is well-known ✓

Coding the sentences:

F1: $a \rightarrow b$ ✓

F2: a ✓

G: b ✓

The final formula for deduction:

$(F1 \wedge F2) \rightarrow G$,

that is: $((a \rightarrow b) \wedge a) \rightarrow b$

Both formulae are identical

Deduction Using Propositional Logic: Example 4 & 5

If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.

a: Asha is elected VP
b: Rajat is chosen G-Sec
c: Bharati is chosen Treasurer

F1: $(a \rightarrow (b \wedge c))$

F2: $\neg b$

G: $\neg a$

Truth Table will have 8 rows

$[(F1 \wedge F2) \rightarrow G]$ is a tautology or not

If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

F1: $(a \rightarrow (b \wedge c))$ $\rightarrow$ $(a \rightarrow b) \wedge (a \rightarrow c)$

F2: b

G: a

$[(F1 \wedge F2) \rightarrow G]$ is true under all interpretations or not.

More Examples

If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer

a: Asha is elected VP

b: Rajat is chosen G-Sec

c: Bharati is chosen Treasurer

F1: $a \to (b \vee c)$

F2: $\neg b$

G: $(a \to c)$

$[(F1 \wedge F2) \to G]$

If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer

F1: $[a \to ((b \wedge \neg c) \vee (\neg b \wedge c))]$

$b \oplus c$

F2: $\neg b$

G: $(a \to c)$

$\boxed{(F1 \wedge F2) \to G}$

Methods for Deduction in Propositional Logic

Interpretation of a Formula → false under all interpretation

Valid, non-valid, Satisfiable, Unsatisfiable → if there is an interpretation for which it is true

Decidable but NP-Hard

Truth Table Method

Faster Methods for validity checking:-

Tree Method

Data Structures: Binary Decision Diagrams (BDD) → AXIOMS

Symbolic Method: Natural Deduction

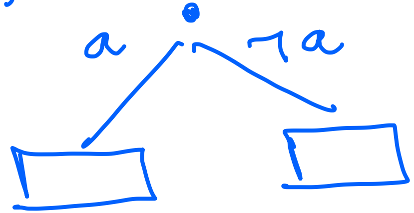
Soundness and Completeness of a Method

Interpretation is the Truth of a formula under assignment of Boolean variables to True or False

valid: If it is true under all interpretation

non-valid: if there is an interpretation for which it is false

a, b, c F



Methods for Deduction in Propositional Logic

Interpretation of a Formula

Valid, non-valid, Satisfiable, Unsatisfiable

Decidable but NP-Hard

Truth Table Method

Faster Methods for validity checking:-
Tree Method

Data Structures: Binary Decision Diagrams

Symbolic Method: Natural Deduction

Soundness and Completeness of a Method

NATURAL DEDUCTION:

Modus **Ponens: (a → b), a** :- therefore b

Modus Tollens: **(a → b), ~b :-** therefore ~a

Hypothetical Syllogism: (a → b), (b → c):- therefore (a → c)

Disjunctive Syllogism: (a V b), ~a:- therefore b

Constructive Dilemma: (a → b) Λ (c → d), (a V c) :- therefore (b V d)

Destructive Dilemma: (a → b) Λ (c → d), (~b V ~d) :- therefore (~a V ~c)

Simplification: a **Λ** b:- therefore a

Conjunction: a, b:- therefore a **Λ** b

Addition: a :- therefore a V b

Natural Deduction is Sound and Complete

Insufficiency of Propositional Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

Predicate Logic

Thank you